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An Application of Markovian Brand Switching Model to Develop Marketing Strategies in Sunscreen Market with Special Emphasis on the Determination of Long Run Steady State Market Shares

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Abstract

Data related to the current brands of sunscreen purchased by the customers of North 24 Paraganas districts of West Bengal in India and the possible sunscreen brands to be purchased by them in their next purchase have been collected by questionnaire method. The questionnaire served to the sample of customers had several questions related to their perceptions and ratings of different sunscreen brands, frequency of usages etc., besides their demographic characteristics. Using the data thus obtained a one-step transition probability matrix has been designed/ determined. The present market shares of different brands have been estimated. Using Markovian steady state equations, the steady state probabilities of the different brands have been finally determined. These probabilities give us an idea about the predicted long run market shares of those different brands of sunscreen in the market mentioned above. Also, an effort has been made so that these brands can identify their potential competitors to whom they can lose their market shares, as well as the competitors from whom they can win back customers. Accordingly, they can decide on their strategies.

Keywords: Brand Switching, Markov Chain, Sunscreen, Market Share, Steady State

Introduction

In marketing, which is an entrepreneurial activity (Ansoff, 1987), the practice of creating a name, symbol or design that identifies and differentiates a product from other products is known as branding. Basically, brand switching is the loss of own loyal customer to other brands and winning the loyal customers of other brands.

Out of the commonly used brands switching models of marketing, one extremely popular model is the Markovian Brand switching model. A. A. Markov, a Russian Mathematician invented Markov chains in the early 1900's. It is a method based on mathematical model with the consideration of a random phenomenon. This method was constructed by imposing a crucial additional assumption on discrete time stochastic process. The concerned random phenomenon changes with time. Though the model was invented in early 1900's but the major applications of the model started in the second half of nineteenth century. The applications were principally done on brand loyalty studies.

In the year 1953, Brown first worked on brand loyalty. Next, Osgood and Tannenbaum (1955) proposed some new ideas connected to attitude change. Anderson and Goodman (1957) focused on statistical inference of Markov chains.

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Later, Harary and Lipstein (1962) worked on the dynamics of brand loyalty and Ehrenberg (1965) on the appraisal of Markov brand switching models. In this phase, researchers started experimenting on the interaction/combination of brand switching with some other established mathematical models such as stochastic models of brand switching (Farley & Kuehn, 1965), brand switching and linear learning models (Carman, 1966), Markov model for switching regressions (Goldfeld & Quandt, 1973) etc. Jacoby and Chestnut (1978) tried to measure the brand loyalty. Parker (1979) worked on brand choice behavior. Markov model was used by several researchers to study different farms size (Stavins & Stanton, 1980; Edwards et al., 1985; Disney et al., 1988) and analyze their structural change (Ethridge et al., 1985). In 1986, Geweke et al., worked on the mobility of continuous time Markov Chains.

Next, Hansen (1992), tested the likelihood ratio under nonstandard conditions of brand-switching model (Markovian). Taking Markovian mean and variance shifts Albert and Chib (1993) worked on Bayesian inference. In 1994 some of the famous works of Markov switching were time dependent transitions in a model of GNP growth of United States (Durland & McCurdy), forecast exchange rates (Engel), Markovian switching based dynamic models (Kim) and Markov switching based analysis of economic time series (McCulloch and Tsay). After that, several others areas like multivariate Markov switching factor model approach (Kim & Yoo, 1995), non-stationary Markov analysis (Zepeda, 1995), Markov-switching time series models (Hamilton, 1996) were also explored. Lyong (1998) carried out research action on applied brand loyalty.

Aypar and Tuncay (2001) also analyzed brand loyalty with Markov chains. Rundle and Mackay (2001) and Ehrenberg et al. (2002) measured the performance of brand loyalty. Boas (2004) focused on consumer learning, brand loyalty, and competition. Roy and Lahiri (2004) carried out different tests on the suitability of different brand switching models. Kolba and Rademeyer (2005) used a combination of Markov and logistic model to predict accuracy in the cellular market. Roy and Banerjee (2007) proposed a model on strategic bonding between different attributes of brands. In 2010, Puligadda and Ross studied the unique effects of branding on variety perception, where as in the same year Shaker and Basem focused on relationship marketing and organizational performance.

In the recent times, Umoh et al. (2013) used Markovian application to brand switching behaviour for toothpaste, Adebiyi et al. (2015) predicted customer churn and retention rates using Markov chain, Adeyeye et al. (2016) emphasized on performance and control of Inventory for production based firms and Bishop et al. (2017) focused on unique solution of quantum stochastic differential inclusions.

However, to the best of our information, no researches have been done on the application of Markov analysis on the brand switching related to sunscreen market.

There is no denying to the fact that sunscreen is a hugely popular and necessary product with a considerable customer base throughout the world. India with its huge population and different states like West Bengal is not an exception.

In this work, the sunscreen brands like Lakme, Lotus, Keyaseth, Parampara, Others (Patanjali & Avon) which are available in the markets of the districts of North 24 Paraganas of West Bengal, India, have been considered. However, it has been surprisingly observed that the sunscreen customers of North 24 Paraganas do not have significant affinity for the brand Loreal.

In this study a sample survey has been carried out with sunscreen purchasing customers of the districts of North 24 paraganas of West Bengal, India, by employing questionnaire method. With the data thus obtained, a sincere effort has been made to assess the present market shares of various sunscreens brands mentioned above, the probability of transition of the customers from one brand to another, and finally to assess the market shares of the same brands in long run (i.e. steady-state).

However, the calculation of the market shares in long run (i.e. steady-state) has been done on the basis of the common Markovian assumption that during the period under consideration, none of the sunscreen brands employ any radical change in their marketing strategies in comparison to the strategies they are currently using, or else, in case there is a change in their marketing strategies then that change is more or less same for all the brands.

Besides, it has also been tried to make each brand aware of their potential competitors, i.e. the competitors to whom they can lose their market shares, as well as the competitors from whom they can win back customers. Accordingly, each brand can decide on their marketing strategies.

Objectives of this Study

- To determine the present market shares of different sunscreens brands in district of North 24 Paraganas of West Bengal, India.
- To determine the one-step transition probability matrix involving the different sunscreen brands.
- To make the different brands aware about their main potential competitors and provide them suggestions to formulate appropriate promotional/advertising strategies against their rival brands.
- To determine the long run Steady State market shares of various sunscreens brands by calculating steady state probabilities of transition between them.

Methodology

In the North 24 Parganas district (comprising of areas like Barasat, Madhyamgram, Bashirhat, Bongaon etc.) of West Bengal, India, a sample of 500 sunscreen using customers has been considered. Sampling technique employed is basically convenient sampling, although it has been ensured that randomness is incorporated as far as possible. Each of these 500 customers/respondents was served with a questionnaire seeking responses related to i) their demographic characteristics, ii) the brands of sunscreen they are currently using as well as the sunscreen brands which they have used in their previous purchases, iii) their rating of different brands of sunscreen in terms of different factors in a scale of 10, and iv) the probable sunscreen brand to be purchased by them in their next purchase.

Data/response thus obtained from the respondents has been subjected to Markovian analysis involving the determination of the probabilities of retaining the same sunscreen brand and the probabilities of transition from purchasing one sunscreen brand in a purchase to purchasing any of the other sunscreen brands in the next purchase, formulation of steady state equations and calculations of steady state probabilities.

Markovian Analysis

Markov Chain refers to a specific category of discrete-time stochastic process.

If X_t denotes the value taken by a system property at time t (t can be any of the discrete points in time that may be labeled as 0, 1, 2, 3, …), then in most of the situations, it is not possible to accurately know the value of X_t until time t comes. Hence, X_t is treated as a random variable. An exhibition of the relation between X_t 's (t=0,1,2,3,…) is known as a discrete-time stochastic process.

Let there be 'k' (finite) number of states numbered as 1,2,…,k. Let us also assume that at any time t (t can take any values like 0, 1, 2, 3,…) a discrete-time stochastic process is in any one of the above mentioned states. Then in order to qualify as a Markov chain, it has to satisfy the following condition:

$$P\left(X_{t+1}=u_{t+1}\middle|X_t=u_t, X_{t-1}=u_{t-1},\ldots,X_1=u_1,X_0=u_0\right)=P\left(X_{t+1}=u_{t+1}\middle|X_t=u_t\right)$$

Further,

$P(X_{t+1} = v|X_t = u) = p_{uv}$ where p_{uv} is the probability of transition of a systemproperty from state u to state v in one time period. It is not dependent on the absolute value of t (but depends on the time gap), for stationary Markov chains

The ($k \times k$) matrix of one-step transition probabilities is given by

$$P=\begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \cdots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{bmatrix} \qquad \text{Here, } \sum_{v=1}^{k} p_{uv}=1,\ \forall\ u$$

The probability of transition of a system property from state u to state v in n time periods for a stationary Markov chain (which is in state u at time T), is given by

$P(X_{T+n} = v|X_T = u) = P(X_n = v|X_0 = u) = p_{uv}(n)$ where $p_{uv}(n)$ is known as the n–step transition probability and is independent of T.

It is known that when n takes integer values greater than 1, then $p_{uv}(n)$ = the value occupying the (u , v) th position/ cell of the matrix P^n.

Now let us discuss on the concept of steady-state probabilities. If we consider a k-state ergodic Markov chain, there has to be a vector Л=[$Л_1$ $Л_2$ $Л_3$ . . . $Л_k$] , such that

$$\lim_{m\to\infty} P^m = \begin{bmatrix} \pi_1 & \pi_2 & \cdots & \pi_k \\ \pi_1 & \pi_2 & \cdots & \pi_k \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \pi_1 & \pi_2 & \cdots & \pi_k \end{bmatrix} \quad ie.\ \lim_{m\to\infty} p_{uv}(m) = \pi_v$$

The vector $\text{Л}=[\text{Л}_1\ \ \text{Л}_2\ \ \text{Л}_3\ \ .\ .\ .\ \ \text{Л}_k]$ is known as the steady state distribution or equilibrium distribution and $\text{Л}_1,\ \text{Л}_2,\ \text{Л}_3,\ .\ .\ .\ \text{Л}_k$ are the steady state probabilities. Now, at the steady state, $p_{uv}(m+1) = p_{uv}(m) = \text{Л}_v$. In order to determine the steady state probabilities, the following equations are to be solved:

$$\text{Л} = \text{Л}P \qquad (i)$$

& $$\sum_{v=1}^{k} \pi_v = 1 \qquad (ii)$$

Analysis and Findings

The respondents were asked to mention their current sunscreen brands, as well as the sunscreen brands that they intend to purchase in their next purchase. From the responses thus obtained, the current market share of each of the sunscreen brand and the one step transition probability matrix has been calculated.

Current Market Shares

From the responses obtained, regarding the sunscreen brands currently used by the respondents of our sample, the present market shares of various sunscreens brands in the district of North 24 Parganas are estimated to be as follows (as depicted in Fig. 1): Lakme-28%, Lotus-26%, Keyaseth-22%, Parampara-16% and others (Patanjali+Avon)-8%.

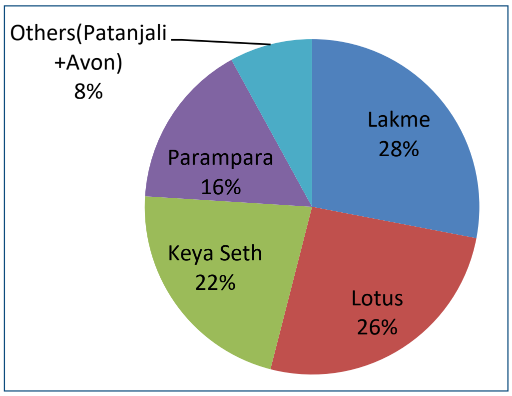

Fig. 1: Current Market Shares

Customers' Rating of Different Sunscreens

Table 1

Sunscreen	*Lakme*	*Lotus*	*Keyaseth*	*Parampara*	*Others (Patanjali + Avon)*
Score (out of 10)	7.12	6.74	6.46	4.53	3.51
Rank	1	2	3	4	5

It is evident from Table 1 that the ranking of the sunscreens brands according to customers' ratings is perfectly matching with the ranking of the sunscreens according to their current market shares as displayed in Fig. 1. This implies that the customers' rating is fully reflected in their purchases.

From the responses obtained from each of the 500 respondents regarding the sunscreen brands that they intended to purchase in their next purchase, the following data comprising of the frequency of respondent switching their brands and also the frequency of customers retaining their old brands have been obtained (As depicted in Table 2).

Table 2

Currently Purchased Brands	*Frequency of Customers Brands Purchased in the Next Purchase*					
	Lakme	*Lotus*	*Keyaseth*	*Param-para*	*Others*	*Total*
Lakme	90	30	0	0	20	140
Lotus	30	100	0	0	0	130
Keyas-eth	0	10	90	10	0	110
Param-para	23	0	11	23	23	80
Others	0	0	0	0	40	40
Total						500

Taking each row in the above table, on dividing each cell frequency by the total frequency of the corresponding row, the matrix of transition probabilities (considering one time gap/ one step) is obtained and displayed below in Table 3.

Table 3

Currently Purchased Brands	Transition Probability, $P_{uv}(1)$ Brands Purchased in the Next Purchase				
	Lakme	*Lotus*	*Keyaseth*	*Parampara*	*Others*
Lakme	0.643	0.214	0	0	0.143
Lotus	0.23	0.77	0	0	0
Keyaseth	0	0.091	0.818	0.091	0
Param-para	0.287	0	0.14	0.287	0.286
Others	0	0	0	0	1

The above transition matrix reveals some important facts. Lakme has 64% of its customers as brand loyal customers. However, it has to be careful with Lotus as its main single competitor, since it is taking away around 21% of its customers. 77% customers of Lotus appear to be brand loyal. However, 23% of Lotus's customers will probably switchover to Lakme. The above discussion makes it very clear that the main rivalry in this sunscreen market of the concerned district is between Lakme and Lotus. This necessitates that all the promotional and other advertising strategies of Lakme should be directed against Lotus and vice versa. The table further reveals that the sunscreen brand with maximum percentage of brand loyal customers is Keyaseth (81%). Another striking feature for Keyaseth is that its customers are not at all interested in purchasing Lakme in their next purchase. However, one of its main limitations is that it is incapable of winning customers from other brands apart from Parampara only. So it should focus on strategies instrumental in winning customers mainly from Lakme and Lotus.

Parampara will retain 28% of its own customers and loose 28% to Lakme, 11% to Keyaseth and 28% to other brands.

Using the transition matrix shown in Table 3 and taking the transition probability values upto 2nd decimal place, steady-state equations [equations (iii), (iv), (v), (vi), (vii) and (viii)] have been formed with the help of equations (i) and (ii). It may be noted here, that this is done on the basis of the common Markovian assumption that during the period under consideration, none of the sunscreen brands have employed any radical change in their marketing strategies in comparison to the strategies they are currently using.

Now, equation (i) may be written as,

$$[\pi_1\ \pi_2\ \pi_3\ \pi_4\ \pi_5]\begin{bmatrix} 0.64 & 0.21 & 0.00 & 0.00 & 0.14 \\ 0.23 & 0.77 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.09 & 0.81 & 0.09 & 0.00 \\ 0.28 & 0.00 & 0.14 & 0.28 & 0.28 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} = [\pi_1\ \pi_2\ \pi_3\ \pi_4\ \pi_5]$$

Or

$$0.64\pi_1 + 0.23\pi_2 + 0 + 0.28\pi_4 + 0 = \pi_1 \quad \text{(iii)}$$

$$0.21\pi_1 + 0.77\pi_2 + 0.09\pi_3 + 0 + 0 = \pi_2 \quad \text{(iv)}$$

$$0 + 0 + 0.81\pi_3 + 0.19\pi_4 + 0 = \pi_3 \quad \text{(v)}$$

$$0 + 0 + 0.09\pi_3 + 0.28\pi_4 + 0 = \pi_4 \quad \text{(vi)}$$

$$0.14\pi_1 + 0 + 0 + 0.28\pi_4 + \pi_5 = \pi_5 \quad \text{(vii)}$$

And equation (ii) may be expressed as,

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 + \pi_5 = 1 \quad \text{(viii)}$$

Solving the above set of steady state equations using software Mathematica 7.0, the following solution has been obtained. These values give the long run predicted market share (i.e. steady-state market share) of each brand.

π_1= 0.3817762 (i.e. for Lakme); π_2 = 0.47774002 (for Lotus); π_3 = 0.1085932 (for Keyaseth); π_4 = 0.0222304 (for Parampara); π_5 = 0.01001 (for Patanjali + Avon)

Thus, the market-shares of various Sunscreens brands in the long run will be as follows (as depicted in Fig. 2):

Lakme – 38.177%. ; Lotus - 47.774%. ; Keyaseth - 10.859%.; Parampara-2.223%.;

Others (Patanjali +Avon) - 1.001%.

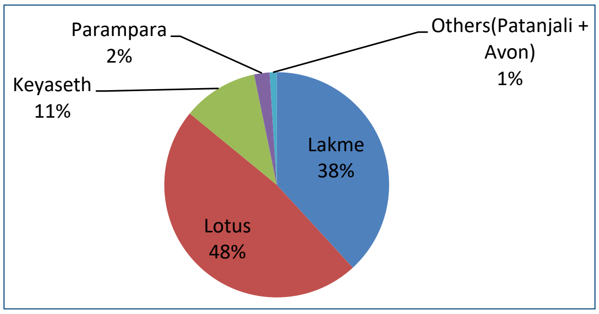

Fig. 2: Long Run Steady State Market Shares

From the values of current market shares and long run steady state market shares, it is evident that there is a gross difference between the two.

The long run market share shows that Lakme (38.177%) & Lotus (47.78%) will continue to be the main players in the sunscreen market in the future. However, Lotus will replace Lakme as the leading market player. Keyaseth will have just 10.859% market share in the future, compared to 22% if we consider its present market share. A bulk of the Keyaseth's market share will be captured by Lakme and Lotus in the future.

Conclusion

In this study we have applied Markovian brand switching model to determine the long run steady state market shares of different popular sunscreen brands in the districts of North 24 Paraganas of West Bengal, India. The matrix comprising of the probabilities of transition of customer from one sunscreen brand to another sunscreen brand in the next purchase has been determined. From these probabilities each of the brands can identify their potential competitors to whom they can lose their market shares, as well as the competitors from whom they can win back customers. Accordingly, they can decide on their marketing strategies. With the help of this one step transition matrix and using Markovian steady state equations, the steady state probabilities (representing the market shares in long run) of the different sunscreen brands have been finally determined. Further, a comparison between the current and long run (ie. the steady-state) market shares of the various brands reveals that Lotus will replace Lakme as the sunscreen brand with maximum share in the above mentioned district in future. The majority of the sunscreen market, i.e., 86% (Lotus – 48% and Lakme – 38%) will possibly be captured by the two main players, viz. Lotus and Lakme. Unless the brand Keyaseth employs any radical change in its marketing strategy so that it can win customers from Lakme and Lotus, its market share will drop down from the present 22% to just 11% in future. Rest of the local brands will be almost wiped out from this market.

For future study one may carry out similar analysis in other districts by taking other products. Furthermore, one may formulate problems by considering the transition probabilities to be imprecise and solve the same using stochastic or fuzzy or interval arithmetic approach.

References

Adebiyi, S. O., Oyatoye, E. O., & Mojekwu, J. N. (2015). Predicting customer churn and retention rates in Nigeria's mobile telecommunication industry using markov chain Modelling. *Acta Universitatis Sapientiae Economics and Business 3*(1), 67-80.

Adeyeye, J. O., Ogunnaike, O., Amaihian, A., Olokundun, M., & Inelo, F. (2016). Inventory control and performance of manufacturing firms. *Journal of Engineering and Applied Sciences, 11*(2), 199-203.

Albert, J., & Chib, S. (1993). Bayesian inference via Gibbs sampling of autoregressive time series subject to Markov mean and variance shifts. *Journal of Business & Economic Statistics, 11,* 1-15.

Anderson, T. W., & Goodman, L. A. (1957). Statistical inference about Markov chains. *Annals of Mathematical Statistics, 28,* 89-110.

Ansoff, H. I. (1987). *Corporate strategy.* Penguin Books Ltd., McGraw Hill.

Aypar, U., & Tuncay, C. (2001). *Analysis of brand loyalty with Markov chains.*

Bishop, S. A., Agarana, M. C., Okagbue, H., & Oghonyon, J. G. (2017). On unique solution of quantum stochastic differential inclusions. *Proceedings of the World Congress on Engineering and Computer Science, 1*(Oct. 2017). San Francisco, USA.

Brown, G. (1953). Brand loyalty: Fact or fiction. *Advertising Age, 24,* 28-35.

Carman, J. M. (1966). Brand switching and linear learning models. *Journal of Advertising Research*, *6,* 23-31.

Disney, W. T., Duffy, P. A., Hardy, W. E. J. (1988). A Markov Chain analysis of pork farm size distributions in the South. *Southern Journal of Agricultural Economics, 20,* 57-64.

Durland, J. M., & McCurdy, T. H. (1994). Duration-dependent transitions in a Markov model of U.S. GNP growth. *Journal of Business & Economic Statistics, 12,* 279-288.

Edwards, C., Smith, M. G., Peterson, & R. N. (1985). The changing distribution of farms by size: A Markov analysis. *Agricultural Economics Research, 37*(4), 1-16.

Ehrenberg, A. S. C., Uncles, M. D., & Goodhardt, G. J. (2002). Understanding brand performance measures: Using Dirichlet benchmarks. *Journal of Business Research*, *57*(12:9), 1307.

Ehrenberg, A. S. C. (1965). An appraisal of Markov brand switching models. *Journal of Marketing Research, 2,* 347-62.

Engel, C. (1994). Can the Markov switching model forecast exchange rates? *Journal of International Economics, 36,* 151-165.

Ethridge, D. E., Roy, S. K., & Myers, D. W. (1985). A Markov chain analysis of structural changes in the Texas high plains cotton ginning industry. *Southern Journal of Agricultural Economics, 7*(2), 11-20.

Farley, J., & Kuehn, A. (1965), Stochastic models of brand switching. In *Science in Marketing,* Schwartz, G (ed). New York: John Wiley & Sons Inc.

Geweke, J., Marshall, R., & Zarkin, G. (1986). Mobility indices in continuous time Markov chains. *Econometrica, 54,* 1407-1423.

Goldfeld, S. M., & Quandt, R. E. (1973). A Markov model for switching regressions. *Journal of Econometrics, 1,* 3-16.

Hamilton, J. D. (1996). Specification testing in Markov-switching time series models. *Journal of Econometrics, 70,* 127-157.

Hansen, B. E. (1992). The likelihood ratio test under nonstandard conditions: Testing the Markov switching model of GNP. *Journal of Applied Econometrics, 7,* S61-S82.

Harary, F., & Lipstein, B. (1962). The dynamics of brand loyalty: A marketing approach. *Operations Research,* 78-82.

Jacoby, J., & Chestnut, R. (1978). *Brand loyalty: Measurement and management.* New York: John Wiley & Sons.

Kim, C. J. (1994). Dynamic linear models with Markov-switching. *Journal of Econometrics, 60,* 1-22.

Kim, M.-J., & Yoo, J.-S. (1995). New index of coincident indicators: A multivariate Markov switching factor model approach. *Journal of Monetary Economics, 36,* 607-630.

Kolb, C., & Rademeyer, A. (2005). *An empirical analysis of Markov and logistic model predictive accuracy in the cellular market.* SAMRA Conference.

Lyong, H. (1998). The theory of research action applied brand loyalty. *Journal of Product and Brand Management, 7*(1), 114-123.

McCulloch, R. E., & Tsay, R. S. (1994). Statistical analysis of economic time series via Markov switching models. *Journal of Time Series Analysis, 15,* 523-539.

Osgood, C., & Tannenbaum, P. (1955). The principles of congruity in the prediction of attitude change. *Psychological Review, 62*(1), 42-55.

Parker, A. (1979). Brand choice behavior. *The Quarterly Review of Marketing,* summer, pp. 23-30.

Puligadda, S., & Ross, W. Jr. (2010). The unique effects of branding on variety perception. *Journal of Brand Management, 18,* 134-149.

Roy, D., & Banerjee, S. (2007). Caring strategy for integration of brand identity with brand image, Accepted for publication. *International Journal of Commerce and Management, 17,* 140-148.

Roy, D., & Lahiri, I. (2004). Some tests for suitability of brand switching model. *European Journal of Marketing*, *38*(5/6), 524-536.

Rundle-Thiele, S., & Mackay, M. (2001). Accessing the performance of brand loyalty measures. *Journal of Services Marketing, 15*(7), 529-546.

Shaker, T., & Basem, A. (2010). Relationship marketing and organizational performance indicators. *European Journal of Social Sciences, 12*(4), 545-557.

Stavins, R. N., & Stanton, B. F. (1980). *Using markov models to predict the size distribution of dairy farms.* New York State, 1968-1985. Cornell University, Department of Agricultural Economics.

Umoh, G. I., Awa, H. O., & Ebitu, P. T. (2013). Markovian application to brand switching behaviour: A survey of toothpaste. *European Journal of Business and Management, 5*(22), 110-124.

Villas Boas. (2004). Consumer learning, brand loyalty, and competition. *Marketing Science*, *23*(1), 134-145.

Zepeda, L. (1995). Technical change and the structure of production: A non-stationary Markov analysis. *European Review of Agricultural Economics, 22,* 41-60.