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General Approach of Retailer's Credit Period and Ordering Policy for Non-Instantaneous Deteriorating Items having Ramp-Type Demand Pattern

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Abstract

In present times, deterioration rate has gained importance in inventory system in the field of business. The shelf life of certain commodities such as fruits, vegetables, food packages and even electronic products has a short span; so, such products have ramp-type demand pattern. To overcome the situation, certain steps, such as offering a period of credit on such items, are prevalent these days by the retailer but the flip side of the same is that this tendering of credit remains as a default risk period for the retailer. Thus, this article formulates an ordering policy in general for ramp-type demand for retailers. This policy is formulated with the support of fundamental concepts of calculus. Also, a non-instantaneous deterioration rate is assumed and shortages aren't allowed. The main objective is to maximise the total profit of the retailer over the period of time and increase ordering quantity of the inventory system. The illustration of method is done by giving numeric examples and economic sensitivity analyses while keeping leading parameters in consideration.

Keywords: Inventory System, Non-Instantaneous Deteriorating Items, Credit Period, Ramp-Type Demand

Introduction

In many of the products available in market, the phenomena such as degradation, spoilage, evaporation, dryness, etc., are observed. All these cause decrease in usefulness of the product in terms of quantity or quality. This phenomenon is termed as deterioration. In past few decades, inventory models for deteriorating items have been widely studied. Ghare and Schrader (1963) derived optimal policies for deteriorating items in exponential decay. Covert and Phillip (1973) extended this model with the consideration of Weibull distribution deterioration. Goyal and Giri (2001) have studied the review of deteriorating inventory literatures. The articles of Whitin (1957), Shah and Jaiswal (1977), Agarwal (1978), Bhunia and Maiti (1999) are focused on the constant deterioration rate. Shah and Shukla (2009) discussed deteriorating inventory model with shortages and time dependent backlogging rate. The literature survey by Nahmias (1982), Raafat (1991), Shah and Shah (2000) and Bakker et al. (2012) cites an up-to-date review on deteriorating inventory models.

All the articles of Wu et al. (2006) and Oyang et al. (2006) have discussed that deterioration is instantaneous, i.e. deterioration starts as soon as inventory is replenished. This is not the case always. Wu et al. (2006) and Oyang et al. (2006) were the first to incorporate this observable fact into formulation of an inventory system. They termed it as 'non-instantaneous deterioration'. It is observed that foodstuffs, first-hand vegetables and fruits have a short span of maintaining fresh quality, in which there is almost no spoilage. Whereas volatile liquids, radioactive chemicals, trendy goods and electronic goods have more span of maintaining their quality or freshness. Many researchers such as Wu et al. (2009), Jaggi and Verma (2010), Soni and Patel (2012, 2013), Dye (2013), Wang et al. (2015), Jaggi et al. (2017), Soni and Suthar (2018), and Suthar and Shukla (2018) have discussed non-instantaneous deterioration in their study.

For a new brand of consumer good coming to the market, the demand rate increases in its growth stage and then remains stable in its maturity stage. Such a demand pattern is referred to as a term 'ramp-type demand'. Hill (1995) proposed an inventory model with increasing demand followed by a constant demand. Mandal and Pal (1998) considered an inventory model for exponentially decaying items by allowing shortages. Wu (2001) investigated an inventory model with ramp-type demand rate, Weibull distributed deterioration rate and partial backlogging. Giri et al. (2003) extended the ramp-type demand inventory model with a more generalized Weibull deterioration distribution. Deng et al. (2007) revisit the inventory model considered by Mandal and Pal (1998) and Wu and Ouyang (2000). Panda et al. (2007) built an inventory model for deteriorating items with generalized exponential ramp-type demand rate and complete backlogging. Skouri et al. (2009) extended the work of Deng et al. (2007) by introducing a general ramp-type demand rate and Weibull deterioration rate. Skouri et al. (2011) developed an order-level inventory model for deteriorating items deteriorated with constant rate with ramp-type demand rate under the conditions of permissible delay in payment. Chakraborty et al. (2018), Suthar and Shukla (2019) have studied inventory models having ramp-type demand.

In current business environment, offering delay in payments is an important demand-uplifting factor. A retailer offers delay period without any interest charges to attract more customers and to enhance sales. This delay in payments period is termed as 'credit period'. This credit period offered leads to a default risk for retailer when customer may not be able to pay back. Goyal (1985) was the first to develop the inventory model with permissible delay in payment. Review article on inventory system with trade credits was carried out by Shah et al. (2010). Huang and Hsu (2008) derived supply chain model under the assumption that the retailer receives credit period from the supplier and the retailer just offers the partial trade credit to his customer. Shah et al. (2011) extended the model by assuming time-dependent demand. Shah et al. (2014) studied optimal ordering and purchase quantity model for the demand depending credit period offered. Shukla and Suthar (2015) and Suthar and Shukla (2018) worked on an inventory policy for a retailer under two-level trade credits in declining market for deteriorating items with maximum lifetime. Shukla and Suthar (2017) derived retailer partial trade credit policy for trapezoidal type demand for deteriorating items. Recently, Qin et al. (2019) discussed the impact of a trade credit policy on alleviating conflicts arising on a dual-channel supply chain included one manufacturer and one retailer. Zou and Tian (2020) formulated an inventory model for a supply chain simultaneously adopting flexible trade credit contract and two-level trade credit policy. Shah et al. (2020) studied the supply chain and analysed time and credit period dependent demand. One may refer to articles by Abdulkader et al. (2015), Singh et al. (2013), Singh and Pandey (2013) which cite an up-to-date review and direct towards the scope of further research.

This article presents the general approach of retailer's credit period policy for inventory system with ramp-type demand rate. For instance, initially the demand of an item increases and reaches a certain point and then it remains constant till the time of another cycle of order. The demand of that item also depends on the period of credit offered to the customer. A non-instantaneous deterioration with constant rate is assumed and shortages are not allowed during the cycle of order. The main focus is to boost the total profit of the retailer over the period of time to its highest point along with economic quantity of the order. A general model is proposed illustrating numeric examples accompanying various ramp-type demands. And to give managerial insights, sensitivity analysis is carried out considering major parameters. Outline of the article is done section wise. Assumptions and notations are discussed in second section general mathematical formulation of an inventory system is presented in third section and numeric examples for special cases for various ramp-type demands are propounded in fourth section. Fifth section explores the sensitivity analysis and managerial insights considering the leading parameters; finally, sixth section deals with the conclusion sought after the research.

Assumptions and Notations

Following assumptions and notations are used in mathematical formulation and are considered throughout in this article.

The inventory system deals with single item.

- Replenishment rate is infinite. Lead time is zero or negligible and planning horizon is infinite.
- Shortages are not allowed in the inventory system.
- Deterioration rate $\theta(0 < \theta < 1)$ is assumed to be constant. There is no refurbish or replacement of the deteriorated items during cycle time.
- The function $I(t)$ represents an inventory level at any instant time $t(0 \le t \le T)$, where T is length of the ordering cycle time.
- To boost demand of an item, a retailer offers credit period M to his customers. The demand is assumed to be ramp type and depending upon credit period offered, say $Z(t, M)$ and it is defined as

$Z(t,M)=\begin{cases} f(t)M^{\beta} & ; \quad 0\le t\le t_1 \\ f(t_1)M^{\beta} & ; \quad t_1\le t\le T \end{cases}$, where $f(t)$ is a positively increasing function with respect to time t during $[0, t_1]$; thereafter, demand remains stable up to the length of the ordering cycle time T and $\beta > 0$ is credit period elasticity.

- It is observed that longer the credit period offered to the customer increases default risk to the retailer. Due to the credit period offered, default risk rate is considered as $H(M) = 1 - M^{-\delta}$, where, δ is a scaling parameter.
- The deteriorating item has no deterioration during $[0, t_d]$; thereafter, the item deteriorates with a constant rate $\theta(0 < \theta < 1)$ during $[t_d, T]$.
- Economic order quantity Q is an initial level of an inventory system.
- Parameters of cost components are defined as: C_o is the ordering cost per order, C_p is the purchase cost per unit, C_s is the selling price per unit and C_h is the holding cost per unit per time unit
- Average total profit of an inventory system for retailer is defined as $\Theta(T,M)$.

Mathematical Formulation

From the retailer's point of view, the general mathematical is presented in this section. As deterioration is non-instantaneous, retailer's inventory system depletes due to demand only at beginning level during $[0, t_d]$; thereafter at the end of the cycle time, it depletes due to both demand and deterioration. From Fig. 1, two cases arise which depend on the length of t_d and t_1. Case A: $0 \le t_d \le t_1 \le T$ Case B: $0<=t_1<=t_d<=T$

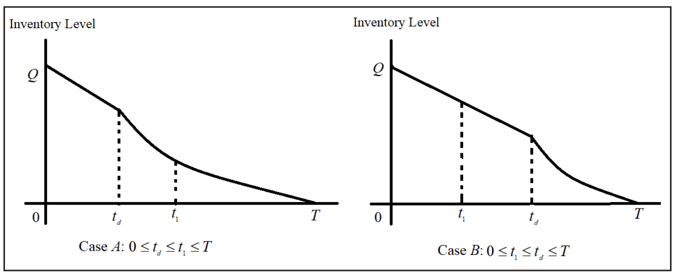

Fig. 1: Inventory Level during Ordering Cycle with Different Cases

$0 \le t_1 \le t_d \le T$.

Case A: $0 \le t_d \le t_1 \le T$

From Fig. 1, the inventory system governed by the differential equation can be written as follow:

$$\frac{dI(T)}{dt}=\begin{cases} -f(t)M^{\beta} & ; \quad 0\le t\le t_d \\ -f(t)M^{\beta}-\theta I(t) & ; \quad t_d\le t\le t_1 \\ -f(t_1)-\theta I(t) & ; \quad t_1\le t\le T \end{cases} \tag{1}$$

With boundary condition $I(T) = 0$;

Solution of the differential equation mentioned above is as under:

$$I(t)=\begin{cases} \int_{t_d}^{t_1} f(x)e^{-\theta(x-t_d)}M^{\beta}dx+\frac{M^{\beta}}{\beta}f(t_1)e^{-\theta(t_1-t_d)}\left(e^{-\theta(t_1-T)}-1\right)+\int_{t}^{t_1} f(x)M^{\beta}dx & ; \quad 0\le t\le t_d \\ \int_{t}^{t_1} f(x)e^{-\theta(x-t)}M^{\beta}dx+\frac{M^{\beta}}{\theta}e^{-\theta(t_1-t)}f(t_1)\left(e^{-\theta(t_1-T)}-1\right) & ; \quad t_d\le t\le t_1 \\ \frac{M^{\beta}}{\theta}f(t_1)\left(e^{-\theta(t-T)}-1\right) & ; \quad t_1\le t\le T \end{cases}$$

(2)

Now, retailer's initial stock is

$$Q=I(0)=\int_{t_d}^{t_1} f(x)e^{-\theta(x-t_d)}M^{\beta}dx+\frac{M^{\beta}}{\beta}f(t_1)e^{-\theta(t_1-t_d)}\left(e^{-\theta(t_1-T)}-1\right)+\int_{0}^{t_1} f(x)M^{\beta}dx \tag{3}$$

Retailer's total profit of an inventory system consists of the following cost components:

(i) Net Sale Revenue after default risk (SR):

$$SR=C_s\left(1-H(M)\right)\left\{\int_0^{t_1} f(t)M^{\beta}dt+\int_{t_1}^{T} f(t_1)M^{\beta}dt\right\}$$

(ii) Ordering Cost (OC):

$$OC=C_o$$

(iii) Purchase Cost (PC):

$$PC=C_pQ=C_p\left\{\int_{t_d}^{t_1} f(x)e^{-\theta(x-t_d)}M^{\beta}dx+\frac{M^{\beta}}{\beta}f(t_1)e^{-\theta(t_1-t_d)}\left(e^{-\theta(t_1-T)}-1\right)+\int_0^{t_1} f(x)M^{\beta}dx\right\}$$

(iv) Inventory Holding Cost (IHC):

$$IHC=C_h\left\{\int_0^{t_d} I(t)dt+\int_{t_d}^{t_1} I(t)dt+\int_{t_1}^{T} I(t)dt\right\}$$

Retailer's average total profit of an inventory system for case A is:

$$\Theta_1(T,M) = \frac{1}{T}\left(SR - OC - PC - IHC\right) \quad (4)$$

Case B: $0 \le t_1 \le t_d \le T$

From Fig. 1, the inventory system governed by the differential equation can be written as follow:

$$\frac{dI(T)}{dt} = \begin{cases} -f(t)M^{\beta} & ; \quad 0 \le t \le t_1 \\ -f(t_1)M^{\beta} & ; \quad t_1 \le t \le t_d \\ -f(t_1) - \theta I(t) & ; \quad t_d \le t \le T \end{cases} \quad (5)$$

With boundary condition $I(T) = 0$

Solution of the above differential equation is as under:

$$I(t) = \begin{cases} M^{\beta} f(t_1)\left\{t_d - t_1 + \frac{1}{\theta}\left(e^{-\theta(t_d - T)} - 1\right)\right\} + \int_t^{t_1} f(x)M^{\beta}dx & ; \quad 0 \le t \le t_1 \\ M^{\beta} f(t_1)\left\{t_d - t + \frac{1}{\theta}\left(e^{-\theta(t_d - T)} - 1\right)\right\} & ; \quad t_1 \le t \le t_d \\ \frac{M^{\beta}}{\theta} f(t_1)\left(e^{-\theta(t - T)} - 1\right) & ; \quad t_d \le t \le T \end{cases} \quad (6)$$

Now, a retailer's initial stock is

$$Q = I(0) = M^{\beta} f(t_1)\left\{t_d - t_1 + \frac{1}{\theta}\left(e^{-\theta(t_d - T)} - 1\right)\right\} + \int_0^{t_1} f(x)M^{\beta}dx \quad (7)$$

The retailer's total profit of an inventory system consists of the following cost components:

(i) Net Sale Revenue after default risk (SR):

$$SR = C_s\left(1 - H(M)\right)\left\{\int_0^{t_1} f(t)M^{\beta}dt + \int_{t_1}^{T} f(t_1)M^{\beta}dt\right\}$$

(ii) Ordering Cost (OC)

$$OC = C_o$$

(iii) Purchase Cost (PC)

$$PC = C_pQ = C_pM^{\beta}\left\{f(t_1)\left\{t_d - t_1 + \frac{1}{\theta}\left(e^{-\theta(t_d - T)} - 1\right)\right\} + \int_0^{t_1} f(x)dx\right\}$$

(iv) Inventory Holding Cost (IHC)

$$IHC = C_h\left\{\int_0^{t_1} I(t)dt + \int_{t_1}^{t_d} I(t)dt + \int_{t_d}^{T} I(t)dt\right\}$$

The retailer's average total profit of an inventory system for case B is:

$$\Theta_2(T,M) = \frac{1}{T}\left(SR - OC - PC - IHC\right) \quad (8)$$

From equations (4) and (8), the retailer's average total profit of an inventory system is defined as:

$$\Theta(T,M) = \begin{cases} \Theta_1(T,M) & ; \quad 0 \le t_d \le t_1 \\ \Theta_2(T,M) & ; \quad t_1 \le t_d \le T \end{cases}$$

To derive the optimal ordering policy, we need to find the optimal values of T say T^* and M, say M^*, which maximise the average total profit of an inventory system $\Theta(T,M)$. To find the optimal values T^* and M^*, the algorithm mentioned below has been followed.

Computational Algorithm

Step 1: Assign the values of the parameters.

Step 2: If $t_d \le t_1$ then go to step 3 otherwise go to step 4 for the case $t_1 \le t_d$.

Step 3: Solve $\frac{\partial \Theta_1}{\partial T} = 0$ and $\frac{\partial \Theta_1}{\partial M} = 0$ to find optimal values T^* and M^* for which $\frac{\partial^2 \Theta_1}{\partial T^2} < 0$ and $\frac{\partial^2 \Theta_1}{\partial T^2}\frac{\partial^2 \Theta_1}{\partial M^2} - \left(\frac{\partial^2 \Theta_1}{\partial T \partial M}\right)^2 > 0$ then go to step 5.

Step 4: Solve $\frac{\partial \Theta_2}{\partial T} = 0$ and $\frac{\partial \Theta_2}{\partial M} = 0$ to find optimal values T^* and M^* for which $\frac{\partial^2 \Theta_2}{\partial T^2} < 0$ and $\frac{\partial^2 \Theta_2}{\partial T^2}\frac{\partial^2 \Theta_2}{\partial M^2} - \left(\frac{\partial^2 \Theta_2}{\partial T \partial M}\right)^2 > 0$ then go to step 5.

Step 5: Obtain EOQ Q and maximum retailer's total profit $\Theta(T^*, M^*)$ by using optimal values T^* and M^*.

Numerical Examples

Various increasing demand rates are considered with scale parameters to obtain optimal values of time of the cycle and period of credit, supported by generalised mathematical formulation and computational algorithm.

Example 1: ($0 \le t_d \le t_1 \le T$) Consider $f(t) = a + bt + ct^2$: $a > 0$, $b > 0$ and $c > 0$ are scale parameters of demand. Also, C_o = \$100 per order, C_p = \$10 per unit, C_h = \$2 per unit time, C_S = \$12 per unit, a = 1000, b = 0.5, c = 0.3, δ = 2, θ = 0.5, β = 4, t_1 = 100/365 years and t_d = 50/365 years. With this set of parameters, optimal cycle time and credit period are $T^* = 0.41672$ years and $M^* = 0.76730$ years, respectively. Using these optimal values, EOQ Q is 141.28 units and maximized total profit Θ is \$3292.88. The graph given in ‘Fig. 2’ indicates that the total profit per time unit is strictly concave. The variation of profit with respect to T and M is exhibited too.

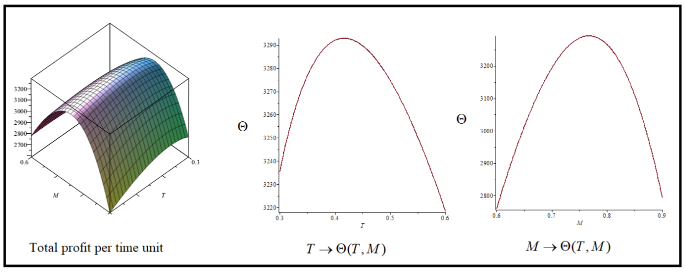

Fig. 2: Concavity of Profit Function

Example 2: $(0 \le t_1 \le t_d \le T)$ Consider $f(t) = a + bt + ct^2$: $a > 0$, $b > 0$ and $c > 0$ are scale parameters of demand. Also, C_o = \$100 per order, C_P = \$10 per unit, C_h = \$2 per unit time, C_S = \$12 per unit, $a = 1000$, $b = 0.5$, $c = 0.3$, $\delta = 2$, $\theta = 0.5$, $\beta = 4$, $t_1 = 50/365$ years and $t_d = 100/365$ years. With this set of parameters, optimal cycle time and credit period are 0.36885 years and $M^* = 0.75829$ years, respectively. Using these optimal values, EOQ Q is 122.72 units and maximized total profit Θ is \$3179.14. The graph given in ‘Fig. 3’ indicates that the total profit per time unit is strictly concave. The variation of profit with respect to T and M is exhibited too.

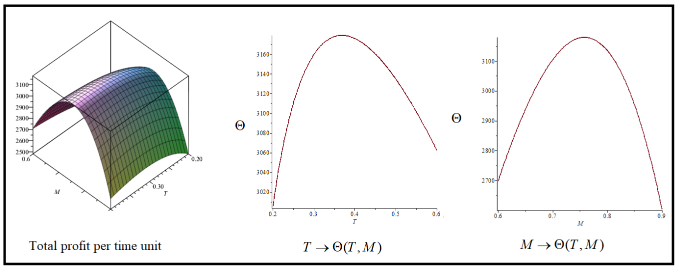

Fig. 3: Concavity of Profit Function

Example 3: $(0 \le t_d \le t_1 \le T)$ Consider $f(t) = ab^t$: $a > 0$ and $b > 0$ are scale parameters of demand. Also, C_o = \$1000 per order, C_P = \$10 per unit, C_h = \$2 per unit time, C_S = \$12 per unit, $a = 1000$, $b = 0.5$, $\delta = 2$, $\theta = 0.5$, $\beta = 4$, $t_1 = 100/365$ years and $t_d = 50/365$ years. With this set of parameters, optimal cycle time and credit period are $T^* = 1.06409$ years and $M^* = 0.70140$ years, respectively. Using these optimal values, EOQ Q is 240.56 units and maximized total profit Θ is \$1565.14. The graph given in ‘Fig. 4’ indicates that the total profit per time unit is strictly concave. The variation of profit with respect to T and M is exhibited too.

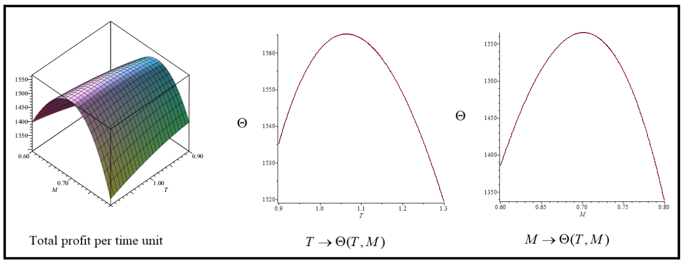

Fig. 4: Concavity of Profit Function

Example 4: $(0 \le t_1 \le t_d \le T)$ Consider $f(t) = ab^t$: $a > 0$ and $b > 0$ are scale parameters of demand. Also, C_o = \$1000 per order, C_p = \$10 per unit, C_h = \$2 per unit time, C_s = \$12 per unit, $a = 1000$, $b = 0.5$, $\delta = 2$, $\theta = 0.5$ $\beta = 4$, $t_1 = 50/365$ years and $t_d = 100/365$ years. With this set of parameters, optimal cycle time and credit period are $T^* = 1.06503$ years and $M^* = 0.68177$ years respectively. Using these optimal values, EOQ Q is 245.79 units and maximized total profit Θ is \$1613.26. The graph given in ‘Fig. 5’ indicates that the total profit per time unit is strictly concave. The variation of profit with respect to T and M is exhibited too.

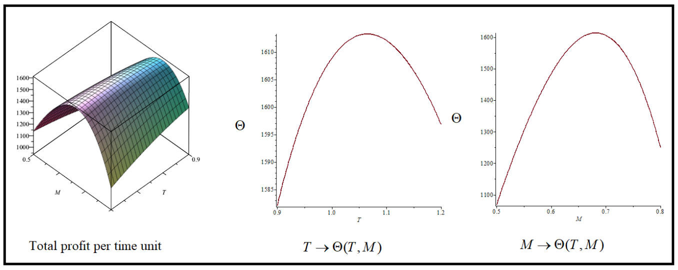

Fig. 5: Concavity of Profit Function

Sensitivity Analysis

Variations in optimal values T and M, economic order quantity Q and total profit per time unit Θ with respect to variations in parameters C_0, C_p, C_s, C_h, a, b, δ, θ, t_d and β from the example 3 are exhibited in Figs 6-9.

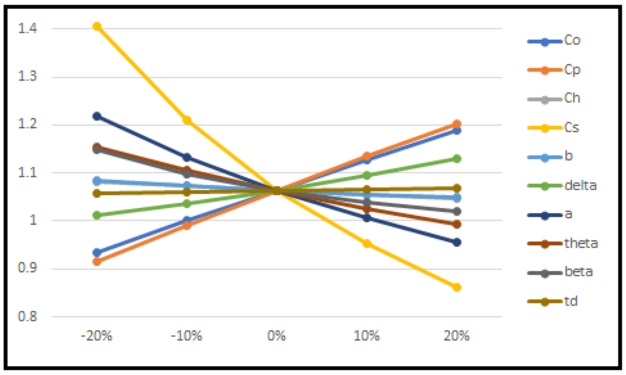

Fig. 6: Variations in T

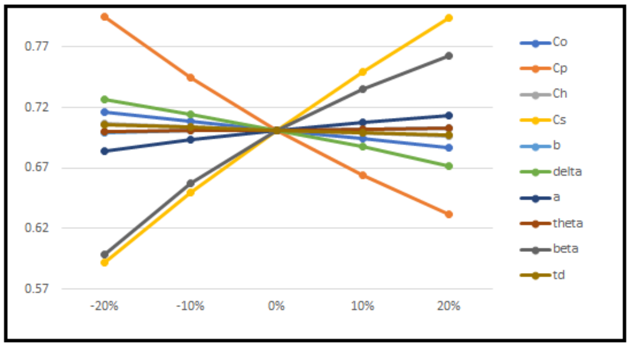

Fig. 7: Variations in M

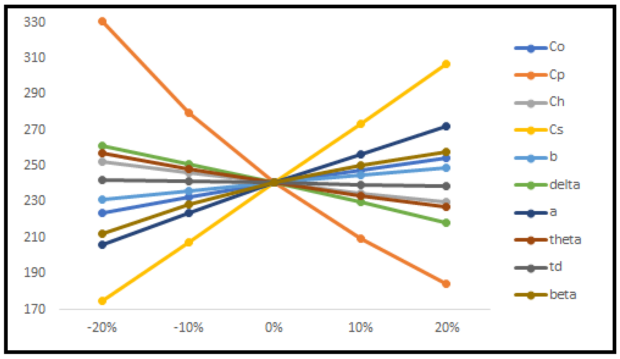

Fig. 8: Variations in EOQ Q

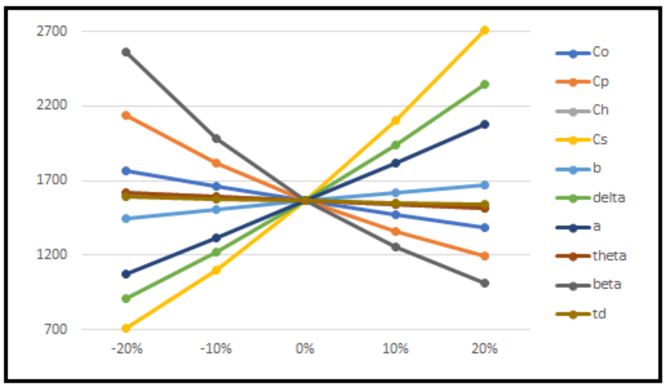

Fig. 9: Variations in Total Profit $\Theta(T, M)$

Managerial Insights

The following managerial issues are interposed by the sensitivity analysis performed above.

- It is observed from the sensitivity analysis, various parameters affect the total profit and quantity of order.
- Fig. 6 discerns that cycle time of order has a positive impact on ordering cost, purchase cost, cycle time t_d and scale parameter, δ and negative impact on other parameters.
- Fig. 7 discerns that period of credit has a positive impact on selling price, credit period elasticity, deterioration rate and parameters of demand and a negative on other parameters.
- Fig. 8 discerns that quantity of order has a positive impact on selling price, cost of order, credit period elasticity and parameters of demand and a negative on other parameters.
- Fig. 9 discerns that there is an increase in total profit due to selling price, parameters of scaling and demand parameters and decrease in other parameters.

Conclusion

This article discusses the ordering policy for non-instantaneous deteriorating items having ramp-type demand rate. For items retaining their original state and quality without getting deteriorated over a period of time in global market, a general model for non-instantaneous deteriorating items having ramp type without shortages has been proposed. Although variations might be observed by replacing different ramp-type demand rates in distinct situations. Ordering policy is illustrated by numeric examples and derived through generalised approach. The article also considers the effect of optimal values of time of cycle and period of credit offered to customers on total profit and quantity of order. The article provides a scope for the study of models related to (1) inventory system with a finite replenishment rate and (2) shortages to be allowed and partially backlogged.

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