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Analysis of Annuity and Impact of COVID 19 on Mortgage in Mathematical Finance Difference Equations Approach

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Abstract

In this article, annuity and mortgage loan in finance are analyzed through non-homogeneous linear difference equations. Some special cases of first-order linear difference equation are discussed. The impact of the contagious virus COVID 19 on mortgage loan is analyzed. The present and future values of annuity are discussed. If the annuity R is deposited at the end or beginning of each conversion period, then the effect is studied. The characteristics of the mortgage loan and types of amortized loans are discussed. The expressions for the performance measures such as monthly instalment, interest component, principal component in jth instalment and outstanding principal amount after paying certain number of annuities in mortgage loan are derived. The three-month moratorium allowed by the Reserve Bank of India on equated monthly installment (EMI) repayments due to the lockdown is analyzed. The consistency of the results obtained analytically in the proposed model is validated through numerical illustrations.

Keywords: Annuity, Mortgage, COVID 19, Present Value, Future Value, Interest, Principal

Introduction

Everybody knows that money makes money. A precise analysis of the verbal expression is an object of interest of the mathematics of finance. Financial mathematics is the product of applying mathematics to the portfolio selection theory and option pricing theory. With the rapid development of the economic situation, the products and derivatives of the financial industry are constantly optimized and made innovative, and new financial products and services are gradually increasing. The operation of financial markets, the design and pricing of financial derivatives, and the analysis and management of risk become very important, and the research and development of financial mathematics are becoming more and more important. Therefore, it is of practical significance to analyze the specific application of mathematics in the financial field.

Financial markets transfer financial resources, such as capital, equity, and credit, between various areas of the economy. Investors participating in financial markets seek to benefit from transactions taking place. The monetary value of financial securities provides a quantitative base for mathematical analysis of the markets. Mathematical finance attempts to provide mathematical explanations for the behavior of financial markets.

Financial markets are markets for financial instruments, in which buyers and sellers find each other and create or exchange financial assets. A financial instrument is a real or virtual document having legal force and embodying or conveying monetary value. Financial markets may be categorized as either money markets or capital markets. Money markets deal in short-term debt instruments, whereas capital markets trade in long-term debt and equity instruments. A financial asset is an asset whose value does not arise from its physical embodiment but from a contractual relationship. Typical financial assets are bonds, commodities, currencies, and stocks.

An adequate saving is essential for healthy economic performance; it is also critical to the economic well-being of individuals and families. The proposed mathematical model argues that how annuity markets operate, how a rational individual can make a decision pertaining to mortgage loans, and the consequences of these decisions

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for individual welfare, which are very much useful for helping to elucidate under what conditions annuitization is welfare-upgrading.

The proposed model provides an idea to a rational person in case of any natural disaster happened, how it reflects on his annuity and loans. A study on the system performance measures, such as total worth and present value of an annuity, interest component, and the outstanding principal of a mortgage loan has been discussed.

(Brown, 2007) discussed rational and behavioural perspectives on the role of annuities in retirement planning. The choice between an annuity and a lump sum: results from Swiss pension funds discussed by (Bütler & Teppa, 2007). (Brown et al., 2008) analyzed why don't people insure late-life consumption; a framing explanation of the under-annuitization puzzle. (Benartzi et al., 2011) concentrated on annuitization puzzles. How deep is the annuity market participation puzzle, is anayzed by (Inkmann, Lopes & Michaelides, 2011). (Chalmers & Reuter, 2012) studied how do retirees value life annuities? Evidence from public employees. (Previtero, 2014) studied Stock market returns and annuitization. The concepts of Mortgage loan and annuity in Mathematical Finance (2020) are retrieved from investopedia.com and en.wikipedia.org.

The basics of difference equations are given by (Elaydi, 2005). (Fulford et al., 1997) studied Modelling with Differential and Difference Equations. (Goldberg, 2010) discussed an Introduction to Difference Equations: With Illustrative Examples from Economics. (Shu Wen Pan & Jia-Qiang Pan, 2016) discussed direct solutions of linear non-homogeneous difference equations.

Many authors (Brown et al., 2000, Brown, 2001, Davidoff, 2005 and Hu et al., 2007) paid more attention on annuities in various scenarios. (Baseri & Hakaki, 2018) analyzed Financial Leverage, Operating Leverage and Capital Venture Effect on Tobin's Q Ratio of Investment and Holding Companies Listed in Tehran Stock Exchange. (Kevin, 2015) discussed Introduction to Financial Mathematics.

An Application and Comparison of Bankruptcy Models in the Indian Banking Sector was studied by (Reshma et al., 2019). In this article, the authors analyzed credit risk assessment of public, private banks and merged banks in India. (Kaur, 2019) analyzed Financial Distress and Bank Performance of selected Indian Banks. The author assessed financial performance of the banking sector in India using Altman (1968) Z-score model for the period 2012-2017. Z-score has been used as a tool to evaluate the credibility of the banks by estimating the Z-score values of the select banks in India. This value is useful when these banks demand loans from the RBI or any other funding agency. (Tesfaye, 2018) studied a review on Empirical Evidences on Structure-Conduct-Performance Relationship in Banking Sector. The author studied the behavior of banks within the given structure, banking and macro environment.

Impact of Risk Tolerance and Demographic Factors on Financial Investment Decision is discussed by (Mitali et al., 2018). In this article, the authors proposed a model for understanding the impact of investment risk tolerance, capital risk tolerance, speculative risk tolerance and six important demographic variables jointly on investment decision which could be used for designing a strategy or investment product to offer to the investors with different levels of financial risk tolerance and different demographic profiles. (Tariq et al., 2017) studied Impact of Leverage on the Profitability of Indian Banking Industry. They have made an attempt to analyze the leverage position of Indian banking industry and its impact on EPS, its risk, and return and profitability. From the real data, the value of leverages has been calculated and on the calculated leverages value Mean, Standard deviation, Skewness and Kurtosis has been calculated and the relationship between all leverages with EPS value has been calculated.

Annuity

An annuity is a sequence of payments with a fixed frequency. The term annuity originally referred to annual payments, but it is now also used for payments with any frequency. Annuities appear in many situations; for instance, interest payments on an investment can be considered as an annuity. An important application is the schedule of payments to pay off a loan. The word annuity refers in everyday language usually to a life annuity. A life annuity pays out an income at regular intervals until you die. Thus, the number of payments that a life annuity makes is not known. An annuity with a fixed number of payments is called an annuity certain, while an annuity whose number of payments depend on some other event (such as a life annuity) is a contingent annuity.

Mortgage

Mortgage lending is the major mechanism used in many countries to finance private ownership of residential and commercial property. Mortgage loans are generally structured as long-term loans, the periodic payments for which are similar to an annuity and calculated according to the time value of money formulae. The most basic arrangement would require a fixed monthly payment over a long period, depending on certain conditions. Over this period, the principal component of the loan (the original loan) would be slowly paid down through amortization.

Characteristics and Types of Amortized Loans

Several factors broadly define the characteristics of the mortgage which are as follows:

Interest: Interest may be fixed for the life of the loan or variable.

The Term: Mortgage loans generally have a maximum term, that is, the number of years, after which an amortizing loan will be repaid. Some mortgage loans may have no amortization, or require full repayment of any remaining balance at a certain date or even negative amortization.

Payment Amount and Frequency: The amount paid per period and the frequency of payments; in some cases, the amount paid per period may change or the borrower may have the option to increase or decrease the amount paid.

Prepayment: Some types of mortgages may limit or restrict prepayment of all or a portion of the loan or require payment of a penalty to the lender for prepayment.

The two basic types of amortized loans are the fixed rate mortgage or floating rate and adjustable-rate mortgage or variable rate mortgage. In some countries, such as the United States, fixed rate mortgages are the norm, but floating-rate mortgages are relatively common. Combinations of fixed and floating-rate mortgages are also common, whereby a mortgage loan will have a fixed rate for some period, for example the first five years, and vary after the end of that period.

In a fixed-rate mortgage, the interest rate remains fixed for the life (or term) of the loan. In the case of an annuity repayment scheme, the periodic payment remains the same amount throughout the loan. In the case of linear payback, the periodic payment will gradually decrease. In an adjustable-rate mortgage, the interest rate may be raised or lowered periodically as rates change.

Difference Equations

Many formulas used in financial mathematics can be derived from the recursive rules between two consecutive elements, which constitute difference equations of the first order. This includes for example simple and compound interest calculation, the present and future value of an annuity, and loan amortization. Difference equations play an important role in a growing range of applications in finance, including financial market models for interest rates, credit risk, stochastic volatility, commodities, electricity, etc.

A first-order linear difference equation is one that relates the value of a variable at a particular time in a linear fashion to its value in the previous period as well as to other exogenous variables. A difference equation can be solved fairly easily through repeated iteration.

Linear First-Order Difference Equations

A typical linear, non-homogeneous first-order difference equation is given by $x(n + 1) = a(n)x(n) + g(n)$ with an initial condition

$$x(0) = X_0 \tag{1}$$

where $a(n)$ and $g(n)$ is a known sequence.

The solution of (1) may be obtained as follows:

$$x(1) = a(0)x(0) + g(0) \tag{2}$$

$$x(2) = a(1)x(1) + g(1)$$

$$= a(1)a(0)x(0) + a(1)g(0) + g(1) \tag{3}$$

$$x(3) = a(2)x(2) + g(2)$$

$$= a(2)a(1)a(0)x(0) + a(2)a(1)g(0) + a(2)g(1) + g(2) \tag{4}$$

$$x(4) = a(3)x(3) + g(3)$$

$$= a(3)a(2)a(1)a(0) + a(3)a(2)a(1)g(0) + a(3)a(2)g(1) + a(3)g(2) + g(3) \tag{5}$$

Thus the general form of the solution for order n is

$$x(n) = x(0)\prod_{k=0}^{n-1} a(k) + \sum_{k=0}^{n-1} g(k) \prod_{i=k+1}^{n-1} a(i);$$

$$\text{where } \prod_{n}^{n-1} = 1 \text{ and } \prod_{k=j+1}^{j} = 0 \tag{6}$$

Now by the mathematical induction, we can show that (6) is true for n+1 also.

From the equation (1),

$$x(n+1) = a(n)\left(x(0)\prod_{k=0}^{n-1} a(k) + \sum_{k=0}^{n-1} g(k) \prod_{i=k+1}^{n-1} a(i)\right) + g(n)$$

$$= x(0)\prod_{k=0}^{n} a(k) + a(n)\sum_{k=0}^{n-1} g(k) \prod_{i=k+1}^{n-1} a(i) + g(n) \tag{7}$$

$$= x(0)\prod_{k=0}^{n} a(k) + \sum_{k=0}^{n-1} g(k) \prod_{i=k+1}^{n} a(i) + g(n)\prod_{i=n+1}^{n} a(i)$$

$$= x(0)\prod_{k=0}^{n} a(k) + \sum_{k=0}^{n} g(k) \prod_{i=k+1}^{n} a(i)$$

Hence, (7) is true for all values of 'n'.

Special Cases

In this section, some special cases are discussed.

Case (i): Let x(n) = ax(n) + g(n)

In this case, $\prod_{k=k_1}^{k_2} a(k) = a^{k_2-k_1+1}$

$$\text{Therefore } x(n) = x(0)a^n + \sum_{k=0}^{n-1} g(k)a^{n-k-1} \tag{8}$$

Case (ii): Let c(n) = ax(n) + g

Therefore $x(n) = x(0)a^n + \sum_{k=0}^{n-1} g\ a^{n-k-1}$

$$x(n) = \begin{cases} a^n x(0) + g\dfrac{a^n - 1}{a-1} & ; a \neq 1 \\ x(0) + gn & ; a = 1 \end{cases} \tag{9}$$

Linear Second-Order Difference Equations

The general form of second-order linear difference equations be

$$a_1 x(n) + a_2 x(n+1) + x(n+2) = g(n);\ n \geq 0 \tag{10}$$

Where a_1 and a_2 are scalars and g(n) is a known sequence, x(0) and x(1) are initial values.

Let $y_1(n) = x(n)$; $y_2(n) = x(n+1) = y_2(n+1)$; $y_3(n) = x(n+1) = y_2(n+1)$

Equation (10) becomes $y_2(n+1) = a_1 y_1(n) + a_2 y_2(n) = g(n)$

Therefore $y_2(n+1) = -a_1 y_1(n) - a_2 y_2(n) + g(n)$

In matrix notation Y(n + 1) = AY(n) + G(n)

where $Y = (y_1, y_2)$, G(n) = g(0, g(n)) and $A = \begin{bmatrix} 0 & 1 \\ -a_1 & -a_2 \end{bmatrix}$

It is clear that the initial values x(0); x(1) give the initial vector $y^0 = (x(0); x(1))$

The eigenvalues of A can be obtained by solving the equation $|A - \lambda I| = 0$

i.e., $\begin{vmatrix} -\lambda & 1 \\ -a_1 & -a_2-\lambda \end{vmatrix} = 0$ this implies $\lambda^2 + a_2\lambda + a_1 = 0$..

On solving $\lambda = \frac{-a_2}{2} \pm \frac{\sqrt{a_2^2 - 4a_1}}{2}$

Let $\lambda_1 = \frac{-a_2}{2} + \frac{\sqrt{a_2^2 - 4a_1}}{2}$ and $\lambda_2 = \frac{-a_2}{2} - \frac{\sqrt{a_2^2 - 4a_1}}{2}$

Hence the solution of (10) is $x(n) = c_1\lambda_1^n + c_2\lambda_2^n$

Future Value of an Annuity

An annuity is essentially a sequence of periodic payments, usually equal in amount, payable at equal intervals of time over the course of a fixed time period. The future value of an annuity is the total value of its periodic payments enhanced at the interest rate for giving the number of conversion periods. It is defined as the sum of the amounts of all payments and the total compound interest earned on these payments at the time of the last payment. In this section, the expressions for the total worth of an annuity and the present value of an annuity are derived.

Theorem 1: Suppose that the constant sum R is deposited at the end of each conversion period in a bank which credits interest at the annual rate r. The deposits are made 'k' times each year over 'n' conversion periods. Then the total worth of an annuity after n deposits is ; n = 0,1, 2,……..

$$x(n) = R\frac{\left(1+\frac{r}{k}\right)^n - 1}{\left(\frac{r}{k}\right)};\ n = 0, 1, 2, \ldots \tag{11}$$

Proof:

Let x(n) denote the total amount in the account at the end of a conversion period. The recursive rule for the future value of an annuity can be written as $x(n+1) = x(n) + \frac{r}{k}x(n) + R$

$x(n+1) = \left(1+\frac{r}{k}\right)x(n) + R;\ n = 0, 1, 2, \ldots$ with x(0) = 0, where $\frac{r}{k}$ is the interest rate per conversion period.

$x(n+1)-\left(1+\frac{r}{k}\right)x(n)=R;\ n=0,1,2,.......$ with initial condition x(0) = 0. (12)

In financial mathematics, the above problem (12) is solved by using the techniques of special case (ii) for a ≠ 1. Here $a=1+\frac{r}{k}$ and $g=R$.

Thus, the solution of (12) is $x(n)=R\frac{\left(1+\frac{r}{k}\right)^n-1}{\left(\frac{r}{k}\right)}$; n = 0,1,2,3,…… ; n = 0,1,2,3,……

The above relation represents the future value of an annuity formula, which gives the amount of an annuity of n payments of R at the compound rate r/k per conversion period under the assumption that the payment interval equals the conversion period. The future value of an annuity formula is used to calculate what value at a future date would be for a series of periodic payments.

Remark 01: In financial mathematics, it is common to use the following form of the formula (11) setting i = r/k where 'i' represents the interest rate per compounding interval.

Thus (11) becomes $x(n)=R\left(\frac{(1+i)^n-1}{i}\right)$; n = 0, 1, 2, 3,…….. (13)

Corollary 01: The present value of an annuity is

$$R\left(\frac{1-(1+i)^{-n}}{i}\right) \quad (14)$$

Corollary 02: Suppose that the constant sum R is deposited at the beginning of each conversion period in a bank which credits interest at the annual rate r. The deposits are made 'k' times each year over 'n' conversion periods. Then the total worth of an annuity after n deposits is $x(n)=R(1+i)\left(\frac{(1+i)^n-1}{i}\right)$;; n = 0, 1, 2,…….. (15)

Corollary 03: The present value of an annuity if the constant sum R is deposited at the beginning of each conversion period, is $R(1+i)\left(\frac{1-(1+i)^{-n}}{i}\right)$ (16)

Performance Measures of Mortgage

Mortgages make larger purchases possible for individuals lacking enough cash to purchase an asset, like a house, up front. Lenders take a risk making these loans as there is no guarantee, the borrower will be able to pay in the future. Borrowers take risk in accepting these loans, as a failure to pay will result in a total loss of the asset.

A mortgage is a loan in which property or real estate is used as collateral. The borrower enters into an agreement with the lender (usually a bank) wherein the borrower receives cash up front, then makes payments over a set time span until he pays back the lender in full. A mortgage is often referred to as home loan when it's used for the purchase of a home.

In this section, the expressions for the monthly instalment, remaining amount of principal after the payment at the end of the month, interest component and principal component are derived.

Theorem 1: Suppose that a person takes a mortgage loan for the amount L that is to be paid back over n months with equal payments x(n) at the end of each month, compounded monthly. Then the monthly instalment is

$$x(n)=\frac{Li(1+i)^n}{(1+i)^n-1};\quad \text{where } i=\frac{r}{k} \quad (17)$$

Proof :

Let 'r' be the annual interest rate and 'k' be the number of conversion periods in a year. Let 'n' equal to the number of conversion periods in the term of the deposit. Let x(n) represent the amount on deposit at the end of 'n' conversion periods and 'P' be the initial sum deposited. Then the recursive rule for the compound interest can be given as

$x(n+1)=x(n)+\frac{r}{k}x(n)$; n = 0, 1, 2, …….. ; n = 0, 1, 2, …….. (18)

Thus, the general solution of (18) is given by

$x(n)=P\left(1+\frac{r}{k}\right)^n=P(1+i)^n$, n =0,1,2,.. ….. with the initial condition P=x(0) (19)

From (19), the present value is $\frac{x(n)}{(1+i)^n}=P$

The present value of the stream of payments for 'n' equal monthly instalment of x(n) at the interest rate 'i' compounded monthly is L. Thus, we get

$$L=\frac{x(n)}{(1+i)^1}+\frac{x(n)}{(1+i)^2}+\frac{x(n)}{(1+i)^3}+......+\frac{x(n)}{(1+i)^n}$$

$$=\frac{x(n)}{(1+i)}\left(1+\frac{1}{(1+i)}+\frac{1}{(1+i)^2}+......+\frac{1}{(1+i)^{n-1}}\right)$$

$$=\frac{x(n)}{i}\left(1-\frac{1}{(1+i)^n}\right)$$

$$=\frac{x(n)}{i}\left(\frac{(1+i)^n-1}{(1+i)^n}\right)$$

Therefore $x(n)=\frac{Li(1+i)^n}{(1+i)^n-1}$; where $i=\frac{r}{k}$

Theorem 2: Let Y_j be the remaining amount of principal after the payment at the end of the month j = 0,1,2,….,n, where $Y_0 = L$ and $Y_n = 0$..

Then $Y_j=\frac{L\left(\alpha^n-\alpha^j\right)}{\alpha^n-1}$; where α = 1 + i (20)

Proof:

The principal after the payment at the end of jth month is $Y_{j+1} = Y_j(1 + i) - x(n)$; j = 0,1,2,,…n

When j = 0, $Y_1 = Y_0(1 + i) - x(n) = \alpha L - x(n)$; where $\alpha = 1 + i$

When j = 1, $Y_2 = Y_1(1 + i) - x(n) = \alpha Y_1 - x(n) = \alpha^2 L - x(n)(\alpha + 1)$

When j = 2, $Y_3 = Y_2(1 + i) - x(n) = \alpha Y_2 - x(n) = \alpha^3 L - x(n)(\alpha^2 + \alpha + 1)$ and so on.

Thus $Y_j=\alpha^j L-x(n)(\alpha^{j-1}+\alpha^{j-2}+..........\alpha^2+\alpha+1)$.
where $Y_0 = L$ and $Y_n = 0$

$$Y_j=\alpha^j L-x(n)\left(\frac{\alpha^j-1}{\alpha-1}\right)$$

From (17), we get

$$Y_j=\alpha^j L-L\alpha^n\left(\frac{\alpha^j-1}{\alpha^n-1}\right)$$

$$Y_j=L\left(\frac{\alpha^j\left(\alpha^n-1\right)-\alpha^n\left(\alpha^j-1\right)}{\alpha^n-1}\right)=L\left(\frac{\alpha^n-\alpha^j}{\alpha^n-1}\right)$$

Theorem 3: Let I_l be the interest component in jth instalment, then

$$I_j=\frac{L(\alpha-1)\left(\alpha^n-\alpha^{j-1}\right)}{\alpha^n-1}; \quad (21)$$

Proof:

By the basic principle to express interest component is

$I_j = Y_{j-1}i = Y_{j-1}(\alpha-1)$.

Therefore the interest component in 1st instalment is

$I_1 = Y_0 i = Li = L(\alpha - 1)$ Similarly,

$I_2=Y_1 i=L\left(\frac{\alpha^n-\alpha}{\alpha^n-1}\right)(\alpha-1)$; $I_3=Y_2 i=L\left(\frac{\alpha^n-\alpha^2}{\alpha^n-1}\right)(\alpha-1)$;

and so on.

Thus

$$I_j=\frac{L(\alpha-1)\left(\alpha^n-\alpha^{j-1}\right)}{\alpha^n-1}; \text{ where } \alpha=1+i \text{ and } j=0,1,2,\ldots.,n$$

Theorem 4: Let P_j be the principal component in jth instalment, then

$$P_j=\frac{L(\alpha-1)\left(\alpha^{j-1}\right)}{\alpha^n-1}; \text{ where } \alpha = 1 + i \quad (22)$$

Proof:

The relation between principal, interest and the amount is

$x(n)=P_j+I_j \Rightarrow P_j=x(n)-I_j$

From (17) and (21), we get

$$P_j=\frac{L(\alpha-1)\,\alpha^n}{\alpha^n-1}-\frac{L(\alpha-1)\left(\alpha^n-\alpha^{j-1}\right)}{\alpha^n-1}$$

$$P_j=\frac{L(\alpha-1)}{\alpha^n-1}\left(\alpha^n-(\alpha^n-\alpha^{j-1})\right)=\frac{L(\alpha-1)}{\alpha^n-1}\left(\alpha^{j-1}\right).$$

Numerical Illustration

In this section, the consistency of the theoretical results obtained in the sections 4 and 5 is justified numerically.

A study of the effect of various parameters on the system performance measures, such as total worth and present value of an annuity after n deposits made at the end as well as at the beginning of each conversion period, the monthly instalment, interest component and Principal component in jth instalment, and the remaining amount of principal after the payment at the end of the month are analyzed.

Effect of Number of Deposits and Performance Measures

The effects of number of deposits on the total worth and present value of an annuity after n deposits made at the end as well as at the beginning of each conversion period are obtained numerically. These results are tabulated in Table 1. It is observed that if the number of deposits increases, the total worth and present value of an annuity after n deposits also increases. If the deposits made at the beginning rather than at the end, will be more beneficial to the investors. These results are also given in Fig. 1.

Effect of Performance Measures on Mortgage

The effects of the monthly instalment, interest component and principal component in jth instalment, and the outstanding amount of principal against an availed mortgage loan are obtained numerically. These results are tabulated in Tables 2 and 3 (Without the effect of COVID 19 on a Loan). It is observed that if the number of instalment increases, the interest component decreases whereas the principal component increases. These results are also given in Fig. 2(a) and 2(b).

Effect of COVID 19 on the Performance Measures of a Mortgage Loan

The three-month moratorium allowed by the RBI on EMI repayments seemed to be a relief for those who had been demanding a deferment due to the lockdown. This three-month moratorium will apply to corporate loans, home loans and car loans. Personal loans and credit card bill. EMIs will resume after the moratorium period gets over.

Moratorium provides no relief to our EMIs and credit card's outstanding payments cycle. For example, if we have to pay Rs. 100,000 as an EMI for a home loan or credit card bill on April 5 and we chose not to pay, then we will be charged an applicable monthly interest rate. If we choose not to pay our EMIs and credit card minimum outstanding balance between 1st March 2020 to 31st May 2020 and pay the accumulated interest before the given deadline, then we won't be treated as a defaulter and non-payments of the aforementioned payments won't impact our credit score.

The effect of *COVID 19 on the performance measures of a mortgage loan* such as the monthly instalment, interest component and principal component in jth instalment, and the outstanding amount of principal against an availed loan are obtained numerically. These results are tabulated in the Tables 4 and 5.

From Table 4, it is observed that if three instalments are waived *due to contagious disease COVID 19,* then the loan is closed at the 123rd instalment, and no loss for the borrower. From Table 5, it is observed that if the interest only paid for three instalments *due to contagious disease COVID 19,* then the loan will be closed in the 123rd instalment, but the borrower has to pay extra interest Rs. 54655.02 for the lockdown period 3 months.

Table 1: Annuity (Vs) Deposits

No. of Deposits	*R is Deposited at the End of Each Conversion Period*		*R is Deposited at the Beginning of Each Conversion Period*	
n	*The Total Worth of an Annuity after n Deposits x(n)*	*Present Value of an Annuity*	*The Total Worth of an Annuity after n Deposits x(n)*	*Present Value of an Annuity*
0	0	0	0	0
1	10000.00	9942.33	10058.00	10000.00
2	20058.00	19827.34	20174.34	19942.33
3	30174.34	29655.33	30349.35	29827.34
4	40349.35	39426.66	40583.37	39655.33
5	50583.37	49141.64	50876.76	49426.66
:	:	:	:	:
:	:	:	:	:
:	:	:	:	:
58	687136.40	491324.40	691121.80	494174.10
59	701121.80	498433.50	705188.30	501324.40
60	715188.30	505501.60	719336.40	508433.50

Annuity R = Rs. 10000 Number of deposits n = 0,1,2,3,……60
Annual Interest Rate = 7% Monthly Interest Rate i = 0.0058

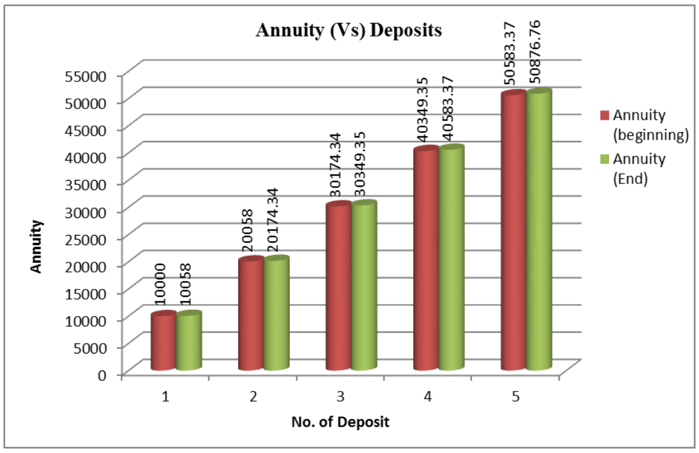

Fig. 1: Annuity (Vs) Deposits (End and Beginning)

Table 2: Interest (Vs) Principal (Vs) Outstanding

Instalment	*The Monthly Instalment*	*The Interest Component in jth Instalment*	*Principal Component in jth Instalment*	*The Remaining Amount of Principal after the Payment at the End of the Month*
0				2000000.00
1	17931.87	11600.00	6331.87	1993668.13
2	17931.87	11563.27	6368.59	1987299.54
3	17931.87	11526.34	6405.53	1980894.01
4	17931.87	11489.19	6442.68	1974451.32
5	17931.87	11451.82	6480.06	1967971.23
:	:	:	:	:
:	:	:	:	:
10	17931.87	11261.70	6670.17	1935002.78
:	:	:	:	:
:	:	:	:	:
15	17931.87	11066.01	6865.87	1901067.10
:	:	:	:	:
:	:	:	:	:
20	17931.87	10864.58	7067.30	1866135.79
:	:	:	:	:
:	:	:	:	:
25	17931.87	10657.23	7274.64	1830179.66
:	:	:	:	:
:	:	:	:	:
:	:	:	:	:
:	:	:	:	:
179	17931.87	206.21	17725.66	17828.46
180	17931.87	103.41	17828.46	0
Total	3227736.60	1227736.60	2000000.00	

Loan Amount: L = Rs. 20,00,000
Number of EMIs : n = 180
Annual Interest Rate: 7%
Monthly Interest Rate: i = 0.0058
Monthly EMI Payment: x(n)
Total Loan Amount Payable: Rs. 3227736.60
Total Interest Payable: Rs. 1227736.60

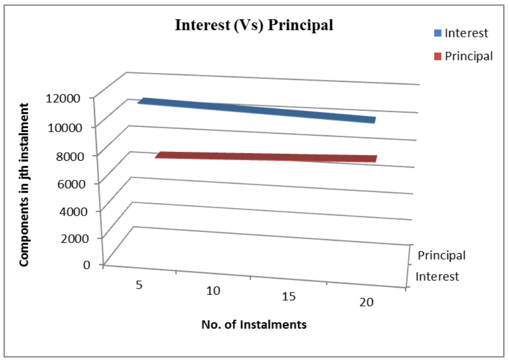

Fig. 2(a): **Interest (Vs) Principal at jth Instalment**

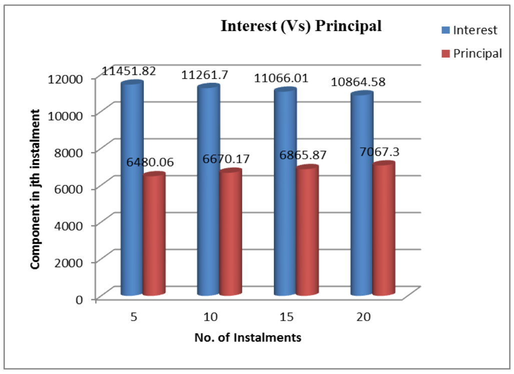

Fig. 2(b): **Interest (Vs) Principal at jth Instalment**

Table 3: **Without the Effect of COVID 19 on a Loan**

Instalment	*The Monthly Instalment*	*The Interest Component in jth Instalment*	*Principal Component in jth Instalment*	*The Remaining Amount of Principal after the Payment at the End of the Month*
0				3000000.00
1	37227.62	21300.00	15927.62	2984072.38
2	37227.62	21186.92	16040.70	2968031.68
3	37227.62	21073.03	16154.59	2951577.09
4	37227.62	20985.33	16269.28	2935607.81
5	37227.62	20842.82	16384.80	2919223.01
:	:	:	:	:
:	:	:	:	:
10	37227.62	20252.84	16974.78	2835537.42
:	:	:	:	:
:	:	:	:	:
15	37227.62	19641.62	17586.00	2748838.51
:	:	:	:	:
:	:	:	:	:
20	37227.62	19008.38	18219.24	2659017.77
:	:	:	:	:
:	:	:	:	:
25	37227.62	18352.35	18875.27	2565962.78
26	37227.62	19417.06	17810.56	2489112.03
:	:	:	:	:
:	:	:	:	:
:	:	:	:	:
119	37227.62	524.30	36703.32	37000.44
120	37227.62	262.70	37000.44	0
Total	4467314.40	1467314.40	3000000.00	

Loan Amount: L = Rs. 30,00,000
Number of EMIs: n = 120
Annual Interest Rate: 8.5%
Monthly Interest Rate: i = 0.0071
Monthly EMI Payment: x(n)
Total Loan Amount Payable: Rs. 4467314.40
Total Interest Payable: Rs. 1467314.40

Table 4: Effect of COVID 19 on a Loan
(Three Instalments Not Paid)

Instalment	*The Monthly Instalment*	*The Interest Component in jth Instalment*	*Principal Component in jth Instalment*	*The Remaining Amount of Principal after the Payment at the End of the Month*
0				3000000.00
1	37227.62	21300.00	15927.62	2984072.38
2	37227.62	21186.92	16040.70	2968031.68
3	37227.62	21073.03	16154.59	2951577.09
4	37227.62	20985.33	16269.28	2935607.81
5	37227.62	20842.82	16384.80	2919223.01
:	:	:	:	:
:	:	:	:	:
10	37227.62	20252.84	16974.78	2835537.42
:	:	:	:	:
:	:	:	:	:
15	37227.62	19641.62	17586.00	2748838.51
:	:	:	:	:
:	:	:	:	:
20	37227.62	19008.38	18219.24	2659017.77
:	:	:	:	:
:	:	:	:	:
25	37227.62	18352.35	18875.27	2565962.78
26	Not Paid due to CORONA 19			2565962.78
27	Not Paid due to CORONA 19			2565962.78
28	Not Paid due to CORONA 19			2565962.78
29	37227.62	18218.34	19009.28	2546953.51
:	:	:	:	:
:	:	:	:	:
:	:	:	:	:
122	37227.62	524.30	36703.32	37000.44
123	37227.62	262.70	37000.44	0
Total	4467314.40	1467314.40	3000000.00	

Loan Amount: L = Rs. 30,00,000
Number of EMIs: n = 120
Annual Interest Rate: 8.5%
Monthly Interest Rate: i = 0.0071
Monthly EMI Payment: x(n)
Total Loan Amount Payable: Rs. 4467314.40
Total Interest Payable: Rs. 1467314.40

Table 5: Effect of COVID 19 on a Loan
(Paid only Interest for the Lockdown Period Three Instalments)

Instalment	*The Monthly Instalment*	*The Interest Component in jth Instalment*	*Principal Component in jth Instalment*	*The Remaining Amount of Principal after the Payment at the End of the Month*
0				3000000.00
1	37227.62	21300.00	15927.62	2984072.38
2	37227.62	21186.92	16040.70	2968031.68
3	37227.62	21073.03	16154.59	2951577.09
4	37227.62	20985.33	16269.28	2935607.81
5	37227.62	20842.82	16384.80	2919223.01
:	:	:	:	:
:	:	:	:	:
10	37227.62	20252.84	16974.78	2835537.42
:	:	:	:	:
:	:	:	:	:
15	37227.62	19641.62	17586.00	2748838.51
:	:	:	:	:
:	:	:	:	:
20	37227.62	19008.38	18219.24	2659017.77
:	:	:	:	:
:	:	:	:	:
25	37227.62	18352.35	18875.27	2565962.78
26	-	18218.34	Not Paid due to CORONA 19	2565962.78
27	-	18218.34		2565962.78
28	-	18218.34		2565962.78
29	37227.62	18218.34	19009.28	2546953.51
:	:	:	:	:
:	:	:	:	:
:	:	:	:	:
122	37227.62	524.30	36703.32	37000.44
123	37227.62	262.70	37000.44	0
Total	4467314.40	1521969.42	3000000.00	

Loan Amount: L = Rs. 30,00,000
Number of EMIs: n = 120
Annual Interest Rate: 8.5%
Monthly Interest Rate: i = 0.0071
Monthly EMI Payment: x(n)
Total Loan Amount Payable: Rs. 4467314.40
Total Interest Payable: Rs. 1521969.42

Conclusion and Future Enhancement

The Annuity and Mortgage Loan in Finance are analyzed through the techniques of linear difference equations. The expressions of the present and future values of annuity are derived. The characteristics of the mortgage loan and types of amortized loans are discussed. Various performance measures of Annuity and Mortgage loan are derived. The impact of the contagious virus COVID 19 on mortgage loan is analyzed with various scenarios. Numerical illustrations are also provided to validate the discussion.

The reliability analysis, forecasting and trend analysis can be performed to the enhancement of this research work. Moreover, the performance measures evaluated in this proposed work are highly useful, and some other various quantitative measures such as various interest rates, topup of loans, changes in tenures, preclosure analysis, concepts of part payments and conversion of rate of interest can be performed which form the future scope pertaining to the enhancement of this research work. An attempt can be made on stochastic and simulation analysis.

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