

Long Memory of NSE Indices

C. V. R. S. Vijaya Kumar and Ashutosh Verma

Associate Professors, Indian Institute of Forest Management (IIFM),
Nehru Nagar, Bhopal

Abstract

Long range memory in share indices show temporal dependence between observations spaced by long intervals of time and has distinct non-periodic cycles. This paper examines the presence of long memory of various indices of National Stock Exchange (NSE). The data consists of closing values of indices over different periods of time. The tests applied to examine long memory are Hurst exponent, Manderbolt-Hurst exponent, Lo's rescaled-range analysis and Geweke and Porter-Hudak (GPH) test. The results of the estimated Hurst exponent, Manderbolt-Hurst exponent and GPH test show that invariably all NSE indices series have long memory. However, the results of Lo's rescaled-range analysis indicate the absence of long memory for all indices.

Keywords:

Long memory, rescaled range analysis, fractional dimension, Hurst exponent, GPH test, NSE indices.

JEL Classification: G11, G14, C58, D53.

1. Introduction

Fama (1970) formulated the efficient market hypothesis which states that in an efficient market the prices of securities reflect their intrinsic worth and it is not possible for any investor to earn super normal profits. According to the weak form of efficient market hypothesis, the current prices reflect all the historical information and it is not possible to predict the future prices by looking at the past prices. Thus, the weak form refutes the utility of technical analysis and is based on the premise that markets do not have either short term or long term memory. However, Manderbolt (1971) found that the information in the market cannot be fully arbitrated away and that there will be significant autocorrelations in the series if the asset returns have long memory. If the returns of the financial assets are long-range dependent, then the series show significant autocorrelations separated by long intervals of time. Long memory in stock markets arises on account of arrival of news as these markets are complex arena wherein different investors react to the same news at different intervals of time. As there is time lag in the reaction of investors to the same news, some reacting immediately and others reacting with an interval, there is dependence in the prices even over longer periods of time which results in fractal data. Wang (1994) found that traders who were fully informed placed smaller orders spread over a period of time in order to ensure that their information is not leaked to the market. Fractal time series show long range dependence and are qualitatively self similar and show similar behavior when measured across different time scales. Such series are characterized by both fractional dimension and self similarity due to which they show stochasticity in part and identical character in whole. The hypothesis of fractal analysis examines the relationship between past and future prices. In a series characterized

by long range memory, temporal dependence will persist between observations spaced by long intervals of time and will have distinct non-periodic cycles. The traditional models failed to capture the long memory process. This led to the development and building of models which examined the non-linear patterns and long memory behavior of the share prices. Long memory process is mathematically expressed in terms of autocorrelation. The presence of long memory has implications about the theoretical and empirical linear asset pricing models. One important question that presence of long memory raises is the sensitivity of investment horizon to consumption and diversification portfolio. Black Scholes model which is a derivative pricing model based on martingale methods will be unreliable in case of presence of long memory as continuous time stochastic process employed in these models is inconsistent with long memory. The presence of long memory raises questions about capital asset pricing model and arbitrage pricing theory, as these theories do not take into account this type of dependence.

A number of studies have been conducted to examine the long memory of developed countries, the literature on long term dependence or otherwise of emerging markets is scant. This paper examines the long memory of various indices of the National Stock Exchange (NSE) and applies a number of tests which have evolved over a period of time to test the presence of long memory.

Review of Literature

This paper mostly reviews the studies conducted after the year 2001. The earliest studies have used Hurst exponent to examine the long memory, but over a period of time different tests have evolved which have tried to overcome the limitations of the Hurst exponent. Cheng and Lai (1995) used modified R/S analysis and fractional differencing and indicated the presence of short memory process. Howe et al. (1999), Henry (2002) and Tolvi (2003) did not find evidence of long memory in the Australian stock market. Barkoulas et al. (2000) used the semi-parametric test and found the Greek Stock market to have long memory. Lobato and Velasco (2000) found the long term memory for both the volume and volatility for the stocks included in the Dow Jones Industrial Index. Ding et al. (2001) applied the ARCH type models and Monte-Carlo simulation to both the absolute and squared returns and found that long memory process was due to ARCH type models. Gang Ma (2003) decomposed the

daily turnover of the stocks included in the Dow Jones industrial Index into non-linear deterministic trend and a random error term and found the error term is a long memory process. Ho et al. (2004), Lillo et al. (2004), Jiang et al. (2007) also did not find the presence of long memory. Cajueiro and Benjamin (2005) investigated the presence of long range memory in the BOVESPA index of the Brazilian stock market using six different methods and found the presence of long memory in the series. Soofi et al. (2006) used spectral regression to examine the exchange rate of twelve Asian currencies vi-a-vis dollar and found that only the Chinese currency did not have long memory. Gil-Alana (2006) examined the usage of fractional representation to describe the behavior of a financial time series. Lu et al. (2008) examined the long memory process in the Shenzhen Stock Exchange (SSE) of China using the rescaled long range test and KPSS test along with the estimation using the LP regression. The findings indicate that SSE's component index has strong presence of long memory and suggested that the market is not weak form efficient.

In Indian context, Golaknath (2001) examined the Nifty daily return data using the R/S test and the variance ratio test. The findings are conflicting and the results of R/S test found the presence of long memory in the Nifty series. The results of variance ratio test concluded the mean reverting behavior of the series and absence of either the long or short memory. Golaknath and Reddy (2002) examined the rupee dollar exchange rates and the results of their tests were contradictory. While one test showed presence of long memory, the other indicated absence of long memory. Kumar (2006) used the ARIFMA-GARCH procedures and other estimating methods to volume series of individual share prices and found the presence of long memory in volume data. Verma (2008) examined the long memory of the return series of sixty companies whose shares are traded on the Bombay Stock Exchange (BSE) using the Lo's R/S test and found that there is no long range dependence in the Indian stock market. This paper examines the presence of long memory of NSE series using various tests and contributes to the existing literature on long memory of Indian stock markets.

Data

Data set consists of the daily closing values of the following indices:

- CNX 100 Index is a diversified index accounting for thirty five sectors of the economy. It represents

about 73.3% free float market capitalisation as on September 30, 2009.

- CNX Nifty Junior Index comprises of the next rung of most liquid stocks after those forming part of S&P CNX Nifty.
- CNX Midcap Index consists of the mid-cap segment of securities and is a benchmark for the movement of midcap stocks in the market. All companies in this index must have a minimum track record of three years of operations and should have positive net worth.

- S & P CNX 500 Index is a very broad based index and companies included in it are disaggregated into seventy two industry indices and it represents more than 90% of total market capitalization.
- S&P CNX Nifty Index is a well diversified 50 stock index accounting for twenty-two sectors of the economy and is computed using market capitalization weighted method. It represents more than 50% of the total traded volume on NSE.

The data has been obtained from the website of NSE.

Table I. Details of Indices

<i>Index</i>	<i>No. of Observations(N)</i>	<i>Official Starting Date</i>	<i>Data available from</i>	<i>Period Covered</i>
CNX100	1318	01-02-2003	01-02-2003	02-01-2003 03-31-2008
CNX Nifty Jr	3128	04-10-1995	04-11-1996	10-04-1995 -03-31-2008
CNX Midcap	1818	01-01-2001	18-07-2001	01-01-2001 -03-31-2008
S&P CNX 500	2212	07-06-1999	07-07-1999	06-07-1999 -03-31-2008
S&P CNX Nifty	4229	03-08-1990	26-11-1997	08-03-1990 -03-31-2008

The period covered, the number of observations and the official date from which the index has been computed are given in Table I. The sample period varies from January, 1990 to March, 2008. The individual indices are of different sizes and are transformed into continuously compounded return series based on $r_t = \ln(P_t/P_{t-1})$, where r_t is the

logarithmic return at time t , P_t represents the closing value on day t and P_{t-1} is the closing value on day $t-1$.

Tools for Data Analysis

Different tests have been applied to examine the presence or otherwise of long memory in the various NSE return series.

Results and Discussion

Table II. Descriptive Statistics

<i>Index</i>	<i>Mean</i>	<i>Median</i>	<i>St. Dev.</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>Jaq-Bera Statistic</i>
CNX100	0.0011	0.0024	0.0159	-0.9899	10.26	3115.39 (0.000)
CNX Nifty Jr	0.0005	0.0014	0.0190	-0.6403	7.58	2949.64 (0.000)
CNX Midcap	0.0009	0.0027	0.0161	-1.3335	11.24	5683.73 (0.000)
S&P CNX 500	0.0007	0.0021	0.0167	-0.8218	8.01	2568.32 (0.000)
S&P CNX Nifty	0.0006	0.0010	0.0177	-0.1783	8.45	5265.57 (0.000)

The descriptive statistics are given in Table II and indicates that all the sample returns appear extremely non-normal. Sample indices returns are significantly leptokurtic since they display negative skewness and large values of kurtosis. The statistics for Jarque-Bera test suggest that

the null hypothesis of normality should be rejected at the 5% level. Thus, sample indices returns show significant departures from the normal distribution. Under the assumption of market efficiency the financial events are normally distributed, however many studies proved that

the probability distribution of stock returns observed are deviating from general normal distribution having flatter tails and higher peak, which is the phenomena of leptokurtic distributions. This phenomenon arises due

to the heterogeneous behavior of trading zones. The flat tailed and leptokurtic distributions normally tend to show the presence of long memory (Skjeltop 2000). The results show that all the indices are having flat tails and are leptokurtic, thereby indicating presence of long memory.

Table III. ADF and Philips–Peron test Statistic (p-values)

Indices ↓	Test	Lags →	90	120	150	180	210	300
CNX100	ADF	(L,I)	0.00	0.00	0.00	0.00	0.00	0.00
		(L,T&I)	0.00	0.00	0.00	0.00	0.00	0.00
	PP	(L,I)	0.00	0.00	0.00	0.00	0.00	0.00
		(L,T&I)	0.00	0.00	0.00	0.00	0.00	0.00
CNX Nifty Jr	ADF	(L,I)	0.00	0.00	0.00	0.00	0.00	0.00
		(L,T&I)	0.00	0.00	0.00	0.00	0.00	0.00
	PP	(L,I)	0.00	0.00	0.00	0.00	0.00	0.00
		(L,T&I)	0.00	0.00	0.00	0.00	0.00	0.00
CNX Midcap	ADF	(L,I)	0.00	0.00	0.00	0.00	0.00	0.00
		(L,T&I)	0.00	0.00	0.00	0.00	0.00	0.00
	PP	(L,I)	0.00	0.00	0.00	0.00	0.00	0.00
		(L,T&I)	0.00	0.00	0.00	0.00	0.00	0.00
S&P CNX 500	ADF	(L,I)	0.00	0.00	0.00	0.00	0.00	0.00
		(L,T&I)	0.00	0.00	0.00	0.00	0.00	0.00
	PP	(L,I)	0.00	0.00	0.00	0.00	0.00	0.00
		(L,T&I)	0.00	0.00	0.00	0.00	0.00	0.00
S&P CNX	ADF	(L,I)	0.00	0.00	0.00	0.00	0.00	0.00
		(L,T&I)	0.00	0.00	0.00	0.00	0.00	0.00
	PP	(L,I)	0.00	0.00	0.00	0.00	0.00	0.00
		(L,T&I)	0.00	0.00	0.00	0.00	0.00	0.00

Table IV. KPSS test Statistic

Indices ↓	Lags →	90	120	150	180	210	300
CNX 100	L, I	0.07	0.07	0.09	0.10	0.12	0.19
	L,T&I	0.04	0.05	0.05	0.06	0.08	0.13
CNX NIFTY Jr	L, I	0.10	0.10	0.10	0.10	0.11	0.10
	L,T&I	0.06	0.06	0.06	0.06	0.06	0.06
CNX midcap	L, I	0.14	0.14	0.14	0.14	0.15	0.19
	L,T&I	0.11	0.11	0.11	0.11	0.15	0.16
S&P CNX 500	L, I	0.15	0.16	0.16	0.16	0.16	0.17
	L,T&I	0.07	0.08	0.08	0.08	0.09	0.09
S&P CNX NIFTY	L, I	0.13	0.03	0.04	0.13	0.02	0.02
	L,T&I	0.14	0.05	0.07	0.13	0.04	0.04

(Critical values for level & constant: 1% (0.74), 5% (0.46), 10% (0.35))

(Critical values for level at trend and intercept: 1% (0.22), 5% (0.15), 10% (0.12))

Testing the stationarity of sample stock returns is important as non-stationarity in mean will cause an upward bias in the estimated Hurst exponent (Klemes, 1974). The stationarity tests, like Augmented Dickey-Fuller (ADF), Phillips-Peron (PP), KPSS tests are estimated with an intercept at level, level and trend for various lag lengths. The null hypothesis of the ADF and PP tests is that a time series contains a unit root, $I(1)$ process against $I(0)$ as an alternative. At 1% and 5% level the p-values as contained in Table III show strong evidence towards alternative hypothesis, indicating that the sample indices are stationary. In KPSS test, the null hypothesis is that a time series contains unit root $I(0)$ and the results as contained in Table-IV infer that the sample indices are stationary.

Long Memory Process

Table V. Estimated Hurst Exponent

CNX 100	0.553
CNX MIDCAP	0.803
CNX NIFTY JUNIOR	0.537
S&P CNX 500	0.537
S&P CNX NIFTY	0.465

Hurst exponent is a very useful tool for analysis of financial time series and the results of the estimated Hurst exponent for various series are presented in Table V. As far as the interpretation of results is concerned, if, $H=0.5$, the financial time series is random in nature with the process being white noise. If the value of H lies between 0 and 0.5, then it shows mean reverting behavior. If the value is between 0.5 and 1, it shows persistence behavior and indicates the presence of long memory. In anti-persistent behavior, a series which has been up in the previous period is more likely to be down in the next period. If the series has persistent behavior, then it is likely to continue its behavior of the past and trends can be easily identified in such a series. This persistence in behavior is fractional Brownian motion and its strength depends upon the value of Hurst exponent. If the value is high and closer to 1.00, higher will be the strength of the persistent behavior. The estimated value of Hurst exponent for all the series except S&P CNX Nifty is above 0.5 and there is absence of random walk as none of them has a value of 0.5. The value of S&P CNX Nifty is 0.4651 which shows anti-persistent behavior and indicates that the series has mean reverting pattern. All the other series have a value

of above 0.5 which is indicative of persistence behavior and the presence of long memory. CNX Midcap has the strongest trend as its value (0.8031) is much closer to 1.00 as compared to other series.

Table VI. Menderbolt-Hurst Exponent

CNX 100	1.52
CNX Midcap	1.34
CNX Nifty Junior	1.11
S&P CNX 500	1.33
S&P CNX Nifty	1.17

Menderbolt R/S Test: Mandelbrot (1972) used the rescaled range analysis and improved the classical test developed by Hurst. His test is popularly known as rescaled long range test and is independent of the assumption of the series being normal. It measures the long range dependence in financial series and specially identifies the dependence in non-periodic cycles. The R/S statistic is calculated by first finding out the range of the maximum and minimum value by which a random stochastic variable deviated from its mean value. This value is then divided by the standard deviation to find out the computed statistic value. If the computed R/S value is small it indicates mean reversion but if it is large, it indicates the persistence of the trend and the presence of long memory. The results of Menderbolt-Hurst exponent are presented in Table VI.

An analysis of the results indicate that the R/S statistic of all the series are more than 1.00, with CNX 100 having the highest value (1.5240) and the CNX Nifty Junior having the lowest value (1.1149). Thus, based on the results of this test it can be concluded that all the series are showing persistent behavior and thereby the presence of long memory.

Lo's R/S Test: Lo (1991) showed that the Hurst exponent and Mandelbrot R/S statistic were influenced to a large extent by short range dependence and heteroskedasticity. Lo, while modifying the statistic has introduced a Newey West Correction to take care of the short range dependence. The modified test is able to derive a variance of the time series with a consistent estimate and is able to identify both the periodic and non-periodic cycles. The R/S statistic is calculated at lags of 30, 60, 90, 120, 150, 180, 210, 300 days and the results are presented in Table VII. The null hypothesis of no long memory is tested

against the alternative hypothesis of having long memory at 90% and 95% level of significance. The results indicate that the null hypothesis cannot be rejected and therefore it can be concluded that the series are not having long memory. The results of this test are totally different from the results of the other two tests. The reason can be that the classical R/S

analysis and Hurst exponent are biased towards presence of long memory quite often. Lo's (1991) approach overcomes this inefficiency of classical R/S method and also is robust to short term dependence. Therefore, in terms of the power of the tests, it is superior to the other two tests as it overcomes the limitations of those two tests.

Table VII. Results for Lo's modified rescaled range statistic

Series/Lags	30	60	90	120	150	180	210	300
CNX100	1.387	1.334	1.346	1.365	1.351	1.360	1.376	1.367
CNX Nifty Junior	1.458	1.409	1.393	1.405	1.416	1.431	1.471	1.656
CNX Midcap	1.538	1.540	1.553	1.596	1.576	1.581	1.616	1.629
S&P CNX 500	1.561	1.590	1.623	1.620	1.619	1.636	1.657	1.726
S&P CNX Nifty	1.001	0.985	0.984	1.047	1.132	1.225	1.343	1.686

(Critical values of Lo's at 90% (0.86, 1.75), 95% (0.81, 1.86), level of significance)

GPH Test: A well known method for analyzing long memory is semi-parametric log-periodogram regression (LP regression) by (Geweke and Porter-Hudake, 1983; Robinson, 1995). The log-periodogram estimator (Geweke and Porter-Hudak, 1983) (GPH) is an estimation method of the memory parameter with observed series. It evaluates 'd' with explicit specification of the short memory by using semi-parametric method, which calculates a spectral regression estimator based on a regression of the ordinates of the log spectral density on the trigonometric function. In this test, it is necessary to set 'v' which is the number of periodogram ordinates, which is function of the sample size T and is to be used in the regression. In financial studies 'v' is generally set to T^{0.5} which has been adopted in this study. The GPH test parameter d (-0.5, 0.5) governs the memory of the time series. For $0 < d < 0.5$, the series exhibits long memory, i.e., its autocorrelations are positive and the series decays at a hyperbolic rate. For $-0.5 < d < 0$, the series exhibits negative long memory. If $0.5 \leq d < 1$ the process is mean-reverting with transitory memory, i.e. any random shock has only a temporary influence on the series (Hassler, Marmol and Velasco (2006)). The long memory is examined using this test without autoregressive and moving average parameters. The results of GPH (d) test are presented in Table VIII. The indices CNX 100 and CNX Nifty Junior are exhibiting positive long memory, while the remaining three are showing the negative long memory.

Table VIII. GPH (d)

CNX100	0.084
CNX Nifty Junior	0.155
CNX Midcap	-0.04
S&P CNX 500	-0.07
S&P CNX Nifty	-0.08

Conclusion

In terms of implications of the findings, the appropriate trading strategies are influenced to a large extent by the presence or absence of long memory. If the stochastic long memory is present in the series, it means there will be autocorrelations between returns spaced over time and this will enable the market participants to devise strategies to earn excessive profits. In this study, a number of tests have been applied including the Lo's R/S test which overcomes the limitations of Hurst test. The estimated Hurst exponent shows that all series except S&P CNX Nifty have long memory whereas the improved GPH estimate which overcomes the limitations of Hurst exponent indicate that all series show the presence of either positive or negative long memory. The Lo's R/S show absence of long memory, in view of the conflicting findings from the tests, it is difficult to come to any definite conclusions and further research is necessary in this area to study the fractal behavior of Indian stock markets.

References

1. Anis, A. A. and Lloyd, E. H. (1976). The expected value of the adjusted rescaled hurst range of independent normal summands, *Biometrika*, 63, 111-116.
2. Barkoulas, J. T., Baum, C. F. and Travlos, N. (2000). Long memory in the Greek stock market, *Applied Financial Economics*, 10, 177-184.
3. Bollerslev, T. and Wright, J. H. (2000). Semiparametric estimation of long-memory volatility dependencies: The role of high-frequency data, *Journal of Econometrics*, 98, 81-106.
4. Cajueiro, D. O. and Benjamin, T. M. (2005). Possible causes of long range dependence in the Brazilian stock market, *Physica A : Statistical Mechanics and its Applications*, 345, pp. 15-26.
5. Cheung, Y W and Lai, K. S. (1995). A search for long memory in international stock market returns, *Journal of International Money and Finance*, 14(4), 597-615.
6. Ding, Z., Clive, W. J. and Robert, F. E. (2001). A long memory property of stock market returns and a new model, *Journal of Empirical Finance*, 1(1), 83-106.
7. Fama, E.F (1970). Efficient Capital Markets: A review of theory and empirical work, *Journal of Finance*, 25, 383-417.
8. Gang, M. (2003). The anatomy of daily stock turnover', www.papers.ssrn.com/sol3/papers.cfm?abstractid=546722
9. Geweke, J. and Porter-Hudak, S. (1983). The estimation and application of long-memory time-series Models, *Journal of Time Series Analysis*, 4, 221-237.
10. Gil-Alana, L.A. (2006). Fractional integration in daily stock market indexes, *Review of Financial Economics*, 15, 28-48.
11. Goalknath, C. (2001). Long memory and Indian stock market – An empirical evidence, Paper presented at UTIICM Conference held at Mumbai.
12. Goalknath, C. and Reddy Y.V. (2002). Long memory in rupee-dollar exchange rate – An empirical study. Paper presented at UTIICM Conference held at Mumbai.
13. Hassler U., Marmol, F., and Velasco, C.. (2006) Residual log-periodogram inference for long-run relationships, *Journal of Econometrics*, 13(1), 165-207.
14. Henry, O T (2002). Long memory in stock returns: some international evidence, *Applied Financial Economics*, 12(10), 725-729.
15. Ho, D., Lee, C. and Wang C. (2004). Scaling characteristics in the Taiwan stock market, *Physica A* 25, 448-460.
16. Howe, J. S, Martin, D. W. and Wood, B. G. (1999). Much ado about nothing: long-term memory in Pacific rim equity markets, *International Review of Financial Analysis*, 8(2), 139-151.
17. Jiang, J., Ma, K., Cai, X. (2007). Non-linear characteristics and long-range correlations in Asian stock markets, *Physica A*, 378, 399- 407
18. Klemes, V. (1974). The Hurst Phenomenon: A Puzzle?, *Water Resources Research*, 10(4), 675-687.
19. Kwiatkowski, D., Phillips, P. C. B., Schmidt P. C. B. and Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics*. 54, 159-178.
20. Kumar, Alok (2006). Long memory in stock trading volume: Evidence from Indian stock market, Working Paper, Indira Gandhi Institute of Development Research, Mumbai.
21. Lillo, F., Farmer, J. D. (2004). The long memory of the efficient market, arXiv: cond-mat/0311053, 2, 178-185.
22. Lo, A. W. (1991). Long-term memory in stock market prices, *Econometrica*. 59, 1279-1313.
23. Lobato, I. N. and Velasco, C. (2000). Long memory in stock-trading volume, *Journal of Business and Economics Statistics*, 18(4), 410-427.
24. Lu, S. Q., Ito, T and Voges, K. (2008). An analysis of long memory in the SSE's component index, *Journal of Economics, Banking and Finance*, 2 (1), 337-346.
25. Mandelbrot, B. B. (1971). When can price be arbitrated efficiently? A limit to the validity of the random walk and martingale models, *Review of Economics and Statistics*, 225-236.

26. Robinson, P. M. (1995). Log-periodogram regression for time-series with long range dependence, *Annals of Statistics*, 23, 1048-1072.
27. Skjeltop, J. A. (2000) Scaling in the Norwegian stock market, *Physica A*, 283, 486-528.
28. Soofi, A.S., Wang, S and Zhang, Y (2006). Testing for long memory in the Asian foreign exchange rates, *Journal of Systems Science and Complexity*, 19(2), 234-247.
29. Tolvi, J (2003). Long memory and outliers in stock market returns, *Applied Financial Economics*, 13, 495-502.
30. Verma, A. (2008). Long memory of the Indian stock market, *The ICFAI Univesirty Journal of Financial Economics*, 3 Sept., 74-83.

