Heuristic Selection of Portfolio Based on Coefficient of Optimism

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Abstract

The mean-variance method developed by Markowitz (1959) was aimed at obtaining optimizing portfolios. But selection of portfolio in the real world mostly deviates from this optimal criterion. In this paper we have considered this issue from an altogether different aspect and developed means for aiming at nearly optimum portfolio. We considered the risk taking propensity as the main driving force and presented a heuristic method to reach the near to the optimal state. For doing so, we have introduced the coefficient of optimism in the decision making process and simultaneously considered conditional optimum portfolio and corresponding heuristic portfolio. In the extreme situations three different human value systems can be considered as optimistic, pessimistic and risk planner. To examine the closeness between the heuristic and optimum portfolios we have carried out empirical analysis covering ten years data of fifteen companies from Nifty (2000-09). Regarding the choice of companies we have adopted random selection technique. From empirical study we have found that for moderate values of the coefficient of optimism a heuristic investor's decision nearly coincides with the corresponding optimum portfolio. However, for extreme situations i.e. optimistic and pessimistic situations heuristic portfolio differs from optimum portfolio.

Keywords:
Expected return, risk, optimum portfolio, heuristic portfolio, coefficient of optimism.

JEL Classification: C1, G1

1. Introduction

The portfolio optimization problem aims to find out the optimal investment strategy of an investor. In other words, the investor looks for an optimal decision on how many shares of which security he/she should invest to maximize his/her expected return or minimize his/her risk. Markowitz (1959) was the first person who had shown that through mean – variance analysis an investor can get optimum portfolio. According to Markowitz model, we need large information to calculate an efficient portfolio. Sharpe (1964) worked on the same problem and developed Capital Asset Pricing Model (CAPM) which can be used for multiple securities to construct an efficient portfolio and can be used with relatively less amount of information. Inspired by their seminal works many researchers have been doing work on optimum choice of the portfolio. Amongst them Merton (1969, 1971, 1973, and 1988) is the lead researcher who discussed the continuous time portfolio optimization problem in the utility framework. Brown (1978) studied the problem when an investor has only two options - a single risky asset and an asset with a constant and known rate of return. He considered Optimal Bayes Portfolio and Certainty Equivalence Strategy to calculate the efficiency of the asset. He found that the optimal Bayes portfolio invests less in risky securities.

While addressing the issue whether the systematic risk plays any role to choose a portfolio of international equities, Das and Uppal (2004) developed two models- one that incorporates systemic risk and the other that ignores it. The models of asset returns help to capture the jumps, which reflects the returns on international equities, occurring at the same time across countries. They also showed how an investor would choose an optimal portfolio when returns have these features. They tested their method to determine the weights for a portfolio allocated over a risk less asset.

However, they have got the result that "for investors with low risk aversion who desire levered positions, the cost of ignoring systemic risk is larger, and in the case of a highly levered portfolio, there is a positive probability of losing one's entire wealth if there is a large negative systemic shock". So to analyse a portfolio, the systematic risk plays an important role.

On the other hand, Jones (1999) showed that for empirical analysis of classical mean-variance problem, regression method is a simple tool to portfolio analysis. By using GRS F- test (formulated by Gibbons, Ross, and Shanken (1989)) of portfolio efficiency he showed that a test of the restriction, the weights of the 'tangency' portfolio equal the weights of the test portfolio. He also proposed the implementation of GRS F-test for portfolio efficiency using linear restrictions on a single linear OLS regression. He analyzed the weights of an international efficient portfolio. He used monthly data from Morgan Stanley Capital International (MSCI) for the 20-years for the equity markets of 11 developed countries.

Over the years, many authors use various techniques to find out an optimum portfolio that can help an investor to take the optimum decision. Some of them gave emphasis on the number of the assets in the portfolio; others assigned importance on the weights of the assets of the portfolio. Recently works are on the suitability of heuristic approach for solving this complex problem. In this paper we have considered two types of portfolios – optimum portfolio and heuristic portfolio and determined weight of the portfolio, i.e. rate of investment in each asset of the portfolio. Calculation of assets' weight is the most crucial task of the investors. If investor can correctly predict the proportion of amount to be invested in each assets of the portfolio, he/she can take right decision to earn maximum utility. But in real world in an efficient market, it is very difficult to predict assets' weights accurately. The problem of arriving at weights of the assets in the portfolio has been discussed from various angles by different authors.

Okhrin and Schmid (2008) proposed a multivariate shrinkage estimator to calculate the optimal portfolio weights. They used Monte Carlo simulation technique to calculate the estimated shrinkage weight of the portfolio. Roy, Mitra and Panja Chowdhury (2010) also proposed assets' weights by considering human value systems. They worked on the human value systems to find out the impact of the value systems on the choice of portfolio.

Though optimum portfolio is conceptually the best choice yet investors opt for conditional portfolios given their own value systems. This calls for detailed study on effects on value systems on the optimum choice of the portfolio. In this paper we have tried to examine those effects. Earlier Roy, Mitra and Panja Chowdhury (2010) considered two human value systems in terms of propensity to take risk. These are optimistic investor and pessimistic investor. But these are two extreme states of an investor's value. In this paper we would like to generalize this concept in terms of dependency on coefficient of optimism of the investor. We would like to compare this heuristic formulation with the optimum portfolio.

Research Design

In portfolio theory expected return and risk play vital role in determining the choice set. The formula, we have used to calculate return is given below:

R_{ij} = (Closing price of the ith security for the jth period – Opening price of the ith security for the jth period) / (Opening price of the ith security for the jth period) (1)

where, R_{ij} = Daily return of the ith security, i= 1,2,3,…, n for the jth period j = 1,2,3,………..,k.

The risk of a security is the standard deviation of the observed returns and is given by the formula:

$$\sigma_i = \sqrt{\sum_{j=1}^{k}\left(R_{ij} - \mu_i\right)^2} \qquad (2)$$

where, σ_i = Standard Deviation of the 'i'th security

μ_i = Average return of the ith security.

Since securities are interrelated in nature because of some commonly affecting factors we also consider covariance between securities as measure of their interrelationship. In portfolio optimization theory, we get two methods through which a portfolio can be optimized. These are (I) maximization of expected return from the portfolio subject to a certain level of risk and (II) minimization of portfolio risk subject to a certain level of expected return from the portfolio. In our paper we try to make a balance between these two methods to introduce the role of value system. In fact, we have minimized the portfolio risk subject to a minimum return constraint under a given coefficient of optimism. A mathematical programming framework with minimum return constraint expressed in terms of coefficient of optimism (α), is given below:

$$\text{Min } \underset{\sim}{w}' \sum \underset{\sim}{w}$$

$$\text{Subject to } \underset{\sim}{w}' \underset{\sim}{R} \geq \alpha\mu_{\max} + (1-\alpha)\mu_{\min}$$

where, $\mu_{\min}$ = Minimum return of the securities in the portfolio and

$\mu_{\max}$ = Maximum return of the securities in the portfolio.

α = Coefficient of optimism of the investor ($0 \leq \alpha \leq 1$),

$\sum$ = Dispersion matrix of the securities,

$\underset{\sim}{w}$ = Weight vector, and

$\underset{\sim}{R}$ = Expected return vector.

Obviously the weight vector will be a function of α and we will get different optimum solutions for different choices of the coefficient of optimism, α. Following the decision theoretic framework the value of α lies between 0 and 1. The weights of a pessimistic investor is represented by α = 0 and the weights of an optimistic investor is represented by α = 1. For weights of a risk planner the value of α is to be taken as 1/2. Conceptually, one may increase the value of α step by step to get a plethora of risk based optimum solutions. In each step one can get optimum weights of the securities for a given α value. With these weights, the expected return and risk of the optimum portfolio can be worked out. The risk and return of the optimum portfolio for a given α have been calculated by using the following equations respectively.

$$R_P^O(\alpha) = \sum_{i=1}^{n} w_i^O(\alpha) R_i \tag{3}$$

$$\sigma_p^o = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i^O(\alpha) w_j^O(\alpha) \sigma_i \sigma_j r_{ij}} \tag{4}$$

where, $R_p^o(\alpha)$ = Expected return of the optimum portfolio for coefficient of optimism as

w_i^o (α) = Optimum weight of the 'i'th security for coefficient of optimism as α,

n = Number of the securities in the portfolio,

$\sigma_p^o(\alpha)$ = Risk of the optimum portfolio for coefficient of optimism as α,

r_{ij} = Correlation coefficient between 'i'th and 'j'th securities.

$i,j = 1,2,3, \ldots, n$

Equations (3) and (4) are used to calculate the expected return and risk of optimum portfolio for different choices of α.

In the next stage, we propose a heuristic solution to get an alternative choice of portfolio based on the same propensity to take risk. For this purpose, we have considered two three of investor viz. optimistic investor, pessimistic investor and risk planner. Since Roy, Mitra and Panja Chowdhury (2010) have considered optimistic portfolio and pessimistic portfolio in their study and calculated their closeness from the optimum portfolio we propose to strike a balance between these two polar opposite states to describe all possible situations and present a general heuristic portfolio with α as the coefficient of optimism.

In case of heuristic optimistic portfolio, we have assumed that an optimistic investor gives more emphasis on expected return not on the expected risk. More is the return, chance of selection of the concerned security by the optimistic investor is more. We have considered security weights to be directly proportional to expected return of the security expect for a change in location to ensure non-negativity in the system. Thus, for heuristic optimistic solution, the formula that has been used for calculating the weights of the securities of the portfolio is

$$w_{io} = \frac{(\mu_i - \mu_{\min})}{\sum_{i=1}^{n} (\mu_i - \mu_{\min})} \tag{5}$$

where, w_{io} = Heuristic optimistic weight of the 'i'th security

μ_i = Expected return of 'i'th security,

$\mu_{\min}$ = Minimum return of the security in the portfolio and

n = Number of securities in the portfolio.

A pessimistic investor has been assumed to give more importance on risk of the portfolio ignoring the expected return part and more is the risk of a security less is its chance of selection. Thus, in case of heuristic pessimistic portfolio, the security weights have been calculated as inversely proportional to standard deviation of the security. Following formula has been used for calculating the weights of the securities of the heuristic pessimistic portfolio.

$$W_{ip} = \frac{1/\sigma_i}{\sum_{i=1}^{n} 1/\sigma_i} \tag{6}$$

where, W_{ip} = Heuristic pessimistic weight of the 'i'th security.

In case the investor is a risk planner, we have assumed that he/she will give importance on both risk and return. Here we have considered both optimistic weight and pessimistic weights to calculate the weights of the securities. As risk planner investor considers both return and risk to select portfolio, geometric mean of pessimistic and optimistic weights has been considered.

$$w_H(1/2) = \frac{\sqrt{w_i^{HO} w_i^{HP}}}{\sum_{i=1}^{n} \sqrt{w_i^{HO} w_i^{HP}}} \tag{7}$$

where, $W_{H(1/2)}$ = Heuristic weight of the 'i'th security for a risk planner.

We would like to observe the closeness between the optimum portfolios and the heuristic portfolios with an effect of coefficient of optimism. Whether coefficient of optimism has any effect on choice of a heuristic portfolio? To find out the answer of the question, we have considered the following weight equation (5) of the securities in the heuristic portfolio. We assume that a general investor gives emphasis both on the return and risk. Weights of optimistic investor and pessimistic investor have been considered together with the value of coefficient of optimism to calculate the weight of the heuristic portfolio. The formula is given below:

$$w_H(\alpha) = \frac{w_{io}^{\alpha} w_{ip}^{(1-\alpha)}}{\sum_{i=1}^{n} w_{io}^{\alpha} w_{ip}^{(1-\alpha)}} \tag{8}$$

where, $w_H(\alpha)$ weight of the ith security in the heuristic portfolio with coefficient of

optimism as α,

w_{io} = Heuristic optimistic weight of 'i'th security and

w_{ip} = Heuristic pessimistic weight of the 'i'th security.

Given the above weight function one may calculate the expected return and risk of the general heuristic portfolio with the following formula.

$$R_H = \frac{\sum_{i=1}^{n} w_{io}^{\alpha} w_{ip}^{(1-\alpha)} R_i}{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{io}^{\alpha} w_{ip}^{(1-\alpha)}} \tag{9}$$

$$\sigma_H = \frac{1}{\sum w_{io}^{\alpha} w_{ip}^{(1-\alpha)}} \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} (w_{io}^{\alpha} w_{ip}^{(1-\alpha)})(w_{jo}^{\alpha} w_{jp}^{(1-\alpha)})\sigma_i \sigma_j r_{ij}} \tag{10}$$

where, R_H Expected return of the general heuristic portfolio and

σ_H = Standard deviation of the general heuristic portfolio.

It is not possible to analytically examine the closeness of the heuristic portfolio with the optimum portfolio, empirical analysis is the only way out to undertake a comparative study.

Analysis and Interpretation

For empirical study, we have considered daily security wise data of fifteen companies listed in Nifty for ten years (2000 to 2009). Data have been collected from www.nseindia.com. Random selection technique has been considered to select fifteen companies out of fifty companies in the Nifty. To select the companies 11th June, 2010 trading day has been considered. Jindal Steel, Ranbaxy, Reliance, HDFC, Tata Motors, Hindalco, Wipro, Ambuja Cement, Siemens, Tata Steel, ACC, Reliance Infrastructure, Axis Bank, BPCL and Sun Pharmaceutical. – these fifteen companies have been selected for empirical purpose based on simple random sampling.

Average return and risk for the securities so selected have been calculated by applying equations (1) and (2). For nine different values of α coefficient optimum weights of the fifteen securities have been calculated. These optimum weights are given in table 1:

Table 1. Weights of the optimum portfolios of fifteen selected securities under different α coefficients

Securities	*α values*								
	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
JINDALSTEL	0	0	0	0	0	0	0	0.04870561	0
RANBAXY	0.1936191	0.1936190	0.1936186	0.1936176	0.1936158	0.1935900	0.1936190	0.07933459	0
RELIANCE	0.06280846	0.06280902	0.06281166	0.06281766	0.06282900	0.06289579	0.06280901	0.07960539	0
HDFC	0.1424088	0.1424080	0.1424066	0.1424035	0.1423969	0.1423651	0.1424096	0.4121370	1
TATAMO-TORS	0.005869277	0.005868693	0.005866985	0.005863447	0.005856850	0.005814507	0.005869393	0	0
HINDALCO	0.05411821	0.05411855	0.5411938	0.05412130	0.05412621	0.05418588	0.05411801	0.01009264	0
WIPRO	0	0	0	0	0	0	0	0	0
AMBUJA-CEM	0.1068151	0.1068155	0.1068160	0.1068164	0.1068158	0.1068162	0.1068148	0	0
SIEMENS	0.07751241	0.07751266	0.07751307	0.07751375	0.07751504	0.07754704	0.07751214	0.05838301	0
TATASTEEL	0	0	0	0	0	0	0	0	0
ACC	0.02198061	0.02197995	0.02197871	0.02197696	0.02197645	0.02202873	0.02198111	0	0
RELINFRA	0	0	0	0	0	0	0	0	0
AXISBANK	0.4030778	0.04030851	0.04030944	0.04031073	0.04031108	0.04028933	0.04030694	0	0
BPCL	0.1195407	0.1195405	0.1195397	0.1195376	0.1195332	0.1195004	0.1195407	0.06761105	0
SUNPHAR-MA	0.1750195	0.1750197	0.1750200	0.1750210	0.1750237	0.1749671	0.1750193	0.2441307	0

These optimum weights under different α coefficients have been used to calculate the expected return and risk of the optimum portfolios. In this manner we have 9 optimum portfolios to be considered. Equations (3) and (4) have been used to calculate the expected risk and return of the respective optimum portfolios. In the following table the expected return and risk of the optimum securities under different values of coefficient of optimism are given in table 2:

Table 2. Expected return and risk of the fifteen selected securities under different α coefficients

α coefficient	*Expected Return*	*Expected Risk*
0	–0.0032105	0.024384
0.125	–0.00114	0.015233
0.250	–0.00169801	0.025905757
0.375	–0.001142747	0.015233
0.500	–0.0011427	0.015233
0.625	–0.00114272	0.015232856
0.750	–0.00114273	0.0152332
0.875	–0.00014375	0.017076738
1.000	0.00065	0.027169

In the next phase we have considered the two heuristic portfolios for optimistic investor and pessimistic investor. The weights of the heuristic portfolios have been calculated by using equations (5) and (6) respectively. Table 3 represents the weights of these two heuristic portfolios.

Table 3. **Weights of two heuristic portfolios taken into consideration**

Securities	*Optimistic*	*Pessimistic*
JINDALSTEL	0.090668	0.053313
RANBAXY	0.066351	0.074502
RELIANCE	0.077863	0.076569
HDFC	0.101525	0.075374
TATAMOTORS	0.059956	0.065455
HINDALCO	0.072906	0.065483
WIPRO	0.064353	0.059075
AMBUJACEM	0.051962	0.077734
SIEMENS	0.073386	0.067679
TATASTEEL	0.056439	0.064709
ACC	0.061715	0.071019
RELINFRA	0.064433	0.059468
AXISBANK	0	0.054746
BPCL	0.071308	0.064254
SUNPHARMA	0.087136	0.07062

After calculating the weights of the optimistic and pessimistic investors we have calculated the weights of the selected securities of the general heuristic portfolio. For this purpose equation (8) has been used. The weights of the securities of the general heuristic portfolios under different coefficient of optimism values are given in Table 4:

Table 4. Weights of the heuristic portfolios of fifteen selected securities under different α coefficients

Securities	*α values*								
	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
JINDALS-TEL	0.053313	0.060000264	0.063783791	0.067757147	0.071926093	0.076296261	0.08087313	0.085661989	0.090668
RANBAXY	0.074502	0.077333969	0.075824542	0.074291121	0.072736197	0.071162256	0.069571766	0.067967182	0.066351
RELIANCE	0.076569	0.080808152	0.080555369	0.080245638	0.079879428	0.079457315	0.078979983	0.078448221	0.077863
HDFC	0.075374	0.082391597	0.085070973	0.087774326	0.090498248	0.093239143	0.095993226	0.098756526	0.101525
TATAMO-TORS	0.065455	0.068182275	0.067086828	0.065961521	0.0648083	0.063629136	0.062426019	0.061200949	0.059956
HINDALCO	0.06548	0.069895679	0.070470803	0.070999572	0.071480702	0.071912985	0.072295296	0.072626593	0.072906
WIPRO	0.059075	0.062884237	0.063229096	0.063530135	0.063786556	0.06399764	0.06416275	0.064281336	0.0643533
AMBUJA-CEM	0.077734	0.077846268	0.073637952	0.06960705	0.065749329	0.062060441	0.058535939	0.055171303	0.051962
SIEMENS	0.067679	0.072001455	0.07235453	0.072657059	0.072908215	0.073107267	0.073253577	0.073346609	0.073386
TATASTEEL	0.064709	0.066993697	0.065514936	0.064022749	0.062519413	0.061007171	0.059488237	0.057964783	0.056439
ACC	0.071019	0.073492666	0.071837395	0.070168916	0.068489743	0.066802349	0.065109166	0.063412583	0.061715
RELINFRA	0.059468	0.063259956	0.063564051	0.063823683	0.064038144	0.064206801	0.064329109	0.064404611	0.064433
AXISBANK	0.054746	0	0	0	0	0	0	0	0
BPCL	0.064254	0.068556297	0.069092621	0.069583073	0.070026449	0.070421623	0.070767553	0.071063276	0.071308
SUNPHAR-MA	0.07062	0.076353488	0.077977116	0.07957801	0.081153183	0.082699611	0.08421425	0.08569404	0.087136

With the weights given in Table 4 we have calculated the expected return and risk of the general heuristic portfolio. Equations (9) and (10) have been used to find out the expected return and risk of the general heuristic portfolios respectively. Expected return and risk of the general heuristic portfolios under different values of coefficient of optimism is show in the following table 5.

Table 5. **Expected return and risk of heuristic portfolios under different α coefficients**

α coefficient	*Expected Return*	*Expected Risk*
0	-0.00147818	0.016463595
0.125	-0.001214085	0.016412405
0.250	-0.00119441	0.016414064
0.375	-0.001174638	0.01641861
0.500	-0.001154781	0.016426197
0.625	-0.001134838	0.016436991
0.750	-0.001114814	0.016451174
0.875	-0.001094711	0.016468941
1.000	-0.001074536	0.016490524

Once we have obtained the expected return and risk of the general heuristic portfolios under different values of coefficient of optimism, we have considered City Block Distance to measure the closeness of the heuristic portfolio with the optimum portfolio. City Block Distance is a standard measure of distance to calculate the similarities or dissimilarities between two points. Since we would like to find out the similarities or dissimilarities between the optimum portfolios and the heuristic portfolios, this City Block Distance can be applied on a two dimensional set up with the expected return and risk of the optimum portfolios and the heuristic portfolios. City Block Distance between the optimum portfolios and the heuristic portfolios under different coefficient of optimism values are given in Table 6:

Table 6. **City Block Distances between optimum portfolios and heuristic portfolios under different α coefficients**

α Coefficient	*City Block Distance values*
0	0.009652725
0.125	0.00125349
0.250	0.009995293
0.375	0.001217501
0.500	0.001205278
0.625	0.001212017
0.750	0.00124589
0.875	0.001557958
1.000	0.012403012

These City Block Distances between the optimum portfolios and heuristic portfolios are presented in a graph (see figure 1). In the said graph we have presented the values of coefficient of optimism (α coefficient) along the horizontal axis and the values of City Block Distance along the vertical axis.

Form Figure 1 it is clear that when the value of the coefficient of optimism (α coefficient) lies between 0 or 0.25 the decision of the heuristic investor does not match with the decision of the optimum investor. It means that for $0 \leq \alpha \leq 0.25$, there is less similarities between the optimum solution and the heuristic solution. The decision of the heuristic investor again does not match when the value of α coefficient is nearly 1. For $0.375 \leq \alpha \leq 0.875$, the heuristic portfolio is very close to the optimum portfolio. So, there is a marked similarity between the optimum portfolio and the heuristic portfolio when the value of coefficient of optimism is in between 0.375 to 0.875. But the calculation involved is considerably less.

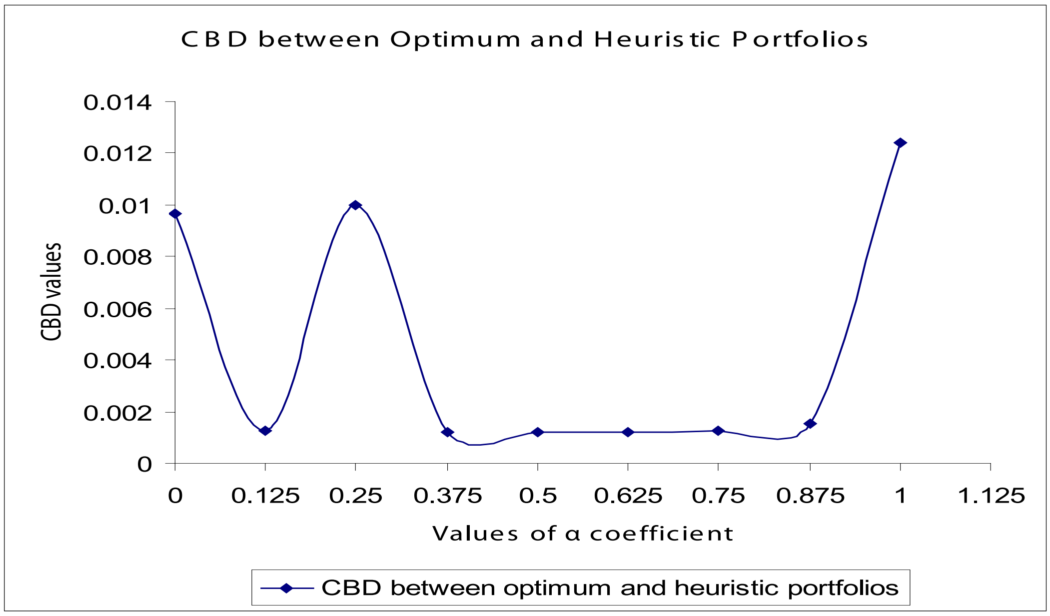

Figure 1

Conclusion

After thorough empirical analysis along with comparative studies, it has been noted that heuristically one can get a result very close to the optimum result. The investor has to be aware of his/her coefficient of optimism while making investment in any security. It has been observed that the decision of the pessimistic investor does not match with the optimum solution because when the value of coefficient of optimism α is equal to 0 (represents the weight of the pessimistic investor) the distance between the optimum solution and the heuristic solution is very large. When the value of the coefficient of optimism is 1 then also the decision of the optimistic investor does not match with the optimum decision. But when we have considered the value of coefficient of optimism in between 0.375 to 0.875 covering the risk planner's decision, the heuristic solution is very close to the optimum decision. So the moderate risk taker may reach nearly optimum portfolio if they considered the heuristic approach. And mostly an investor falls in this moderate range. He/ she can easily arrive at near optimum solution heuristically.

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