

Thermal Effect Over Fibre Laminated Composite Plate

Raj Kumar^{1*}, Yogesh Shrivastava² and Dalvir Singh³

¹Assistant Professor, Department of Mechanical Engineering, Galgotias College of Engineering and Technology, Greater Noida, Uttar Pradesh, India.

Email: raj.kumar@galgotiacollege.edu

²Assistant Professor, Department of Mechanical Engineering, Galgotias College of Engineering and Technology, Greater Noida, Uttar Pradesh, India.

³Assistant Professor, Department of Mechanical Engineering, Galgotias College of Engineering and Technology, Greater Noida, Uttar Pradesh, India.

*Corresponding Author

Abstract: In the present work, four layers of laminated fiber composite plates have been taken to make a Micro-mechanical model that rests without foundation. Moreover, the thermally induced static bending response has been investigated in the presence of small random variation in the material variables. The variables have been accounted for transverse shear strain using higher-order shear deformation theory (HSDT) for temperature-dependent (TD) and temperature-independent (TID) material properties. The investigation has been carried out using MATLAB 10b, whereas graphs have been plotted using Origin Pro8. From the investigation it has been perceived that strict control of these random parameters is required in order to obtain desired reliability of laminated composite plate.

Keywords: Deformation, HSDT, Laminated composite, static bending, TD, TID.

I. INTRODUCTION

There is a lot of literature on static bending of laminated composite plates. Few of the literature reviewed here are mentioned as below:

A. M. Zenkour [1] has provided analytical solution for bending of cross-ply laminated plates under thermo-mechanical loading, J. Yang, K. M. Lieu, and S. Kitipornchai [2] has given stochastic Analysis of Compositionally Graded Plates with System Randomness under Static Loading, Achchhe Lal and B. N. Singh [3] has studied effect of random system properties on bending response of thermo-mechanically loaded laminated composite plates, C. A. Shankara and N. G. R. Iyenger [4] provided a C^0 element method for buckling of laminated composite plates. They have employed the closed-form solution

and employed the first-order perturbation technique (FOPT) to handle randomness in the material properties using classical plate theory (CPT), J N Reddy [5] provided theory of mechanics of Laminated Composite Plates, R. M. Jones [6] has given mechanics of Composite materials, B. N. Singh, A. Lal and R. Kumar [7] provided bending response analysis of laminated composite plates on nonlinear elastic foundation with uncertain system properties. They also used the micromechanical model to find out the effects on buckling, vibration and bending response of laminated composite plates, Achchhe Lal, B. N. Singh and Rajesh Kumar [8] provided stochastic nonlinear bending response of laminated composite plates with system randomness under lateral pressure and thermal loading,

From the literature review presented above, it is evident that there is no literature covering the static bending response of laminated composite plates resting without foundations, subject to thermo-mechanical loading involving randomness in material properties to the best of my knowledge. This is the problem studied and analyzed in this thesis report.

In the present work, thermal effects over laminated fiber composite plates are investigated and analyzed for linear deformation of the system. The necessity of analyzing linear behavior of laminated composite plates with thermo-mechanical loading is important not only for their application in aerospace and other important engineering applications but also from the point of view of the interest they stimulate the classical problems in bending. Moreover, all engineering structures analysis or classical method of structures analysis, the system parameters are taken to be deterministic and mean value of system parameters are used in the analysis. This gives only the mean response and misses the deviation caused by randomness in system properties.

II. PROBLEM FORMULATION

For an accurate study of structural, the random variation in the lamina material properties should be incorporated in the analysis. Otherwise the predicted response may differ significantly from the observed values and the structures may not be safe. This may be appropriately handled by modeling these properties as basic random variables (RVs).

We consider a rectangular laminated composite plate of length a , width b , and total thickness h , as shown below. Let $(\bar{u}, \bar{v}, \bar{w})$ be the displacement parallel to the (X, Y, Z) respectively. The coordinate of thickness 'Z' of the top and bottom surfaces of k_{th}

layer are denoted by Z_{k-1} and Z_k respectively. The K_{th} layer fiber is oriented with angle θ_k to the X-axes.

The assumptions made in the micromechanical model:

- The deformation of the plate is very small compared to the dimensions of the sides.
- Transverse normal stress is negligible.
- The transverse shear satisfies the traction free surface conditions.
- The lateral load 'w' of any lamina is given by displacement of the midplane.

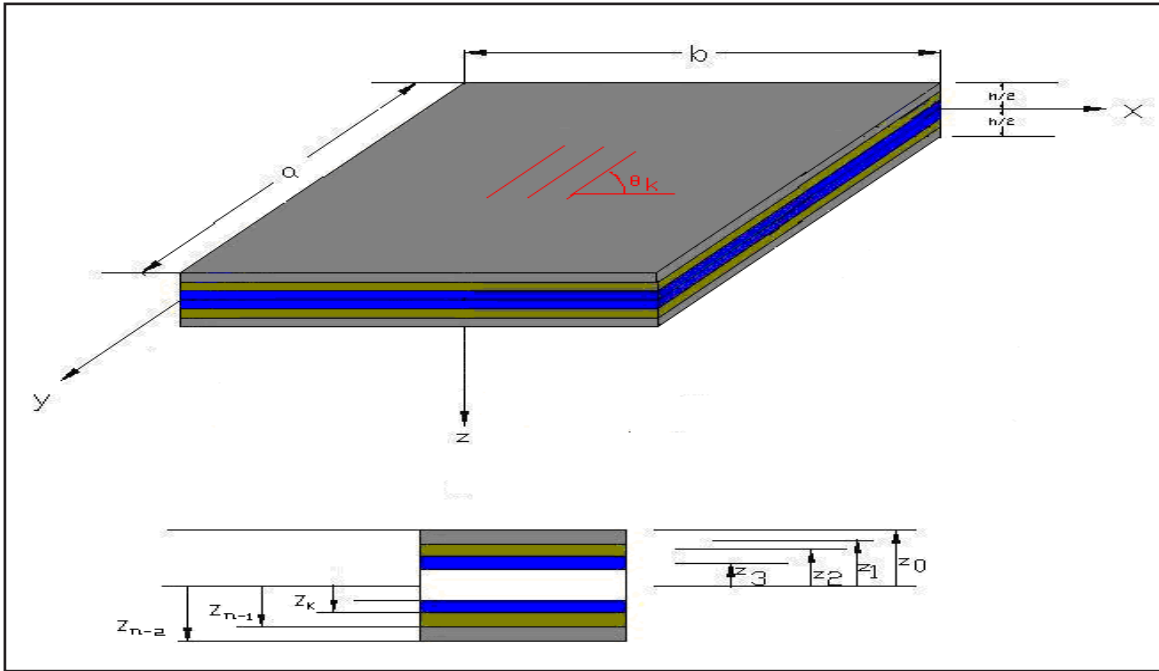


Fig. 1: Geometry of a Laminated Composite Plate Resting Without Foundation

A. Thermal Buckling Analysis

Using the finite element model,

$$U_{TH} = \sum_{e=1}^{NE} U_{TH}^{(e)} = \frac{1}{2} \sum_{e=1}^{NE} \{\Lambda\}^T \lambda [K_g]^{(e)} \{\Lambda\}^{(e)} dA = \frac{1}{2} \lambda \{q\}^T [K_g] \{q\} \quad (1)$$

Where λ and $[K_g]$ are defined as the thermal buckling load parameters and the global geometric stiffness matrix respectively.

B. Governing Equation

The equation governing composite laminate in static bending can be derived using Variational Principle [4], which is generalization of the principle of virtual displacement. For the bending analysis, the minimization of first variation of total potential energy $\Pi(U_{SE} + U_{TH} - W)$ with respect to displacement vector is made which is given as [3].

$$\delta(U_{SE} + U_{TH} - W) = 0 \quad (2)$$

$$[K] \{W\} = \{F\}, \quad (3)$$

With $\{F\} = \{P_M\} + \{P_T\}$, where $[K]$, $\{W\}$, $\{P_M\}$ and $\{P_T\}$ are global linear stiffness matrix, response vector, mechanical and thermal force vector, respectively.

III. SOLUTION APPROACH: PERTURBATION TECHNIQUE

The lamina material properties are treated as independent random variables (RVs) as [9].

The governing equation (3) thus can be written as:

$$[K_{ij}^R] \{W_i^R\} = \{F_i\} \quad (4)$$

where $[K_{ij}^R]$, $\{W_i^R\}$ and $\{F_i\}$ are the random stiffness matrix, the random response vector and the deterministic forcing

vector. Here $[K_{sij}^R]$ are the functions of a set of primary RVs $[b_i^R]$ and $[W_i^R]$ is unknown.

Our aim in present study is to find the second-order statistics of $[W_i^R]$ when the second-order statistics of primary RVs $[b_i^R]$ are known. Any random variable can be expressed as the sum of its mean and the zero mean random variables which have been expressed.

The operating random variables in the present case are defined as:

$$b_i^R = b_i^d + b_i^r; K_{sij}^R = K_{sij}^d + K_{sij}^r; W_i^R = W_i^d + W_i^r \quad (5)$$

We can express the above relations in the form:

$$b_i^R = b_i^d + \epsilon b_i^r; K_{sij}^R = K_{sij}^d + \epsilon K_{sij}^r; W_i^R = W_i^d + \epsilon W_i^r \quad (6)$$

where ϵ is a scaling parameter

We consider a problem where the zero-mean random variation is very small as compared to the mean part of random variables. i.e., $RV^d \ll RV^r$

By Taylor expansion series neglecting the second and higher-order terms,

$$[K_{sij}^d + \epsilon K_{sij}^r] \{W_i^d + \epsilon W_i^r\} = \{F_i\}; \quad (7)$$

By equating the terms of same order, we get the zeroth order perturbation equation and first order perturbation equation as follows [4, 12];

Zeroth order perturbation equation

$$(\epsilon^0): [K_{sij}^d] \{W_i^d\} = \{F_i\} \quad (8)$$

First order perturbation equation

$$(\epsilon^1): [K_{sij}^d] \{W_i^r\} + [K_{sij}^r] \{W_i^d\} = \{F_i\} \quad (9)$$

Using Taylor's expansion series the system matrix and response vector can be expressed as:

$$[K_{sij}^r] = \sum_r \frac{\partial K_{sij}^d}{\partial b_i^R} b_i^r, [W_i^r] = \sum_r \frac{\partial W_i^d}{\partial b_i^R} b_i^r \quad (10)$$

Substituting Eq. (23) in Eq. (22) and equating the coefficients of b_i^r . For each I , we get:

$$[K_{sij}^d] \left\{ \frac{\partial W_i^d}{\partial b_i^R} \right\} + \left[\frac{\partial K_{sij}^d}{\partial b_i^R} \right] \{W_i^d\} = 0, I = 1, 2 \quad (11)$$

Using Eq. (11) we can solve the only unknown $\left\{ \frac{\partial W_i^d}{\partial b_i^R} \right\}$, for each I . So the sensitivity matrix of eigenvectors [11, 13] can be found out.

Total deflection and its variance can thus be written as:

$$W = W^d + \left\{ \frac{\partial W_i^d}{\partial b_i^R} \right\} b_i^r \text{ and } \text{var}(W) = E \left[\sum_r \frac{\partial W_i^d}{\partial b_i^R} b_i^r \right]^2 \quad (12)$$

Where $E[\]$ is the expectation. The variance can be written as:

$$\text{Var}(W) = \sum_r \sum_r \text{diag} \left[\frac{\partial W_i^d}{\partial b_i^R} \left(\frac{\partial W_i^d}{\partial b_i^R} \right)^T \right] E(b_i^r, b_i^r) \quad (13)$$

IV. RESULTS AND DISCUSSION

The finite element method approach for the static bending response of the laminated composite plates resting without foundations and subjected to temperature changes with among all random input variables has been illustrated in the present study. The approach has been validated by comparing the results with those available in the literature and Monte Carlo Simulation.

Temperature Independent Material Properties (TID)

$$E_{111} = 0; \quad E_{21} = 0; \quad G_{121} = 0; \quad G_{131} = 0; \quad G_{231} = 0; \quad \alpha_{11} = 0; \quad \alpha_{21} = 0;$$

Temperature-Dependent Material Properties (TD)

$$E_{111} = -0.5*1e-3; \quad E_{21} = -0.2*1e-3; \quad G_{121} = -0.2*1e-3; \\ G_{131} = -0.2*1e-3; \quad G_{231} = -0.2*1e-3;$$

$$\alpha_{11} = 0.5*1e-3; \quad \alpha_{21} = 0.5*1e-3;$$

$$E_1 = (E_{10}*(1 + E_{111}*T)); \quad E_2 = (E_{20}*(1 + E_{21}*T));$$

$$G_{12} = (G_{120}*(1 + G_{121}*T)); \quad G_{13} = (G_{130}*(1 + G_{131}*T));$$

$$G_{23} = (G_{230}*(1 + G_{231}*T)); \quad \alpha_1 = \alpha_{110}*(1 + \alpha_{11}*T); \quad \alpha_2 = \alpha_{210}*(1 + \alpha_{21}*T);$$

The boundary conditions for simply supported, clamped and combination of both used for the present analysis are given as:

All edges simply-supported (SSSS):

$$v = w = \rho_y = \zeta_y = 0, \text{ at } x = 0, a; \quad u = w = \rho_x = \zeta_x = 0 \text{ at } y = 0, b$$

All edges clamped (CCCC):

$$u = v = w = \zeta_x = \zeta_y = \rho_x = \rho_y = 0, \text{ at } x = 0, a \quad \text{and } y = 0, b;$$

Two opposite edges clamped and the other two simply supported (CSCS):

$$u = v = w = \zeta_x = \zeta_y = \rho_x = \rho_y = 0, \text{ at } x = 0 \quad \text{and } y = 0;$$

$$v = w = \rho_y = \zeta_y = 0, \text{ at } x = a \quad u = w = \rho_x = \zeta_x = 0, \text{ at } y = b;$$

V. CONCLUSION

It is concluded that deformation of laminated composite plates resting without foundation using higher-order shear deformation theory (HSDT) is studied in the present analysis. Further the C^0 finite element method (FEM) and mean-centered first-order perturbation technique (FOPT) is employed to determine the mean and standard deviation of transverse central deflection of fiber laminated composite plates loading mechanically with a linearly varying transverse temperature distribution across the thickness. In order to assess the effects of temperature on the bending behavior of shear deformable laminated plates, a theoretical analysis is developed based on a micromechanical model. The material properties are considered to be dependent on temperature, which is given explicitly in terms of the fiber and matrix properties and the fiber volume ratio. The results were taken in *MATLAB 10b*.

Notations:

A_{ij}, B_{ij} , etc	: Laminate stiffnesses
BB	: Strain-displacement matrix
a, b	: Plate length and breadth
b_j	: Basic random system properties
E_{11}, E_{22}	: Longitudinal and Transverse elastic moduli
G_{12}, G_{13}, G_{23}	: Shear moduli
h	: Thickness of the plate
KI	: Linear bending stiffness matrix
Kg	: Thermal geometric stiffness matrix
M, m	: Mass and inertia matrices
NE, NL	: Number of elements, number of layers in the laminated plate
Nx, Ny, Nxy	: In-plane thermal buckling loads
NN	: Number of nodes per element
φ_i	: Shape function of i th node
Q_{ij}	: Reduced elastic material constants
$\{\Lambda\}^{(e)}$: Vector of unknown displacements, displacement vector of e th element
U ,	: Strain energy due to bending
u, v, w	: Displacements of a point on the mid plane of plate
u, v, w	: Displacement of a point (x, y, z)
$\{\sigma\}, \{\epsilon\}$: Stress vector, Strain vector
y, x	: Rotations of normal to mid plane about the x and y axis respectively
x, y, k	: Two slopes and angle of fiber orientation wrt x -axis for k th layer
x, y, z	: Cartesian coordinates
$Var()$: Variance
RVs	: Random variables
ΔT	: Rise in temperatures
ΔC	: Rise in moisture percentage

α_1, α_2	: Thermal expansion coefficients along x and y direction
β_1, β_2	: Coefficients of hygroscopic expansion
k_1, k_2	: Winkler and Pasternak elastic foundations

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