International Journal of Knowledge Based Computer Systems
6 (1), June 2018, 23-29
http://www.publishingindia.com/ijkbcs/

Recent Developments in Mathematical Analysis of Queues for Machine Repair Problem: A Short Survey

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Abstract: The present paper is based on survey of machine repair problem (MRP). We have reviewed machine repair problems studied by many researchers and given some basic performance characteristic of queuing models. A bunch of work has been previously done on this front from past since it is realistic phenomena of many service systems. The current investigation presents a brief review of the major works done on machine repair problem (MRP) in past years. The bibliography consists of research articles which were published in journals of repute during the period 1994-2015. While determining the performance of any machining system, the main aim of decision makers or entrepreneurs is to get the maximum profit and reduce the inconvenience due to delay in service to the users; these objectives can be achieved by proper scheduling of the system. The main aim of this survey paper is to analyze various problems related to machine repair problem. We have tried to collect important results in the theory of machine repair problem and hope this will help other researchers to study such kind of problems.

Keywords: Machine Repair Problem (MRP), Queuing models, Busy cycle, Performance indices, Standbys.

I. Introduction

With the rapid advances in science and technology, machining systems come to our rescue in today's competitive environment as the machines are integral part of day-to-day and industrial activities. The computer systems, production systems, communication systems, transportation systems, flexible manufacturing systems are a few examples that can be cited for industrial real time systems. The performance of any machining system is highly affected by the factors such as variability of processing times, occurrence of random failures, randomness of repair times, etc. The variability in a machining environment arises from unreliable equipments, unpredictable yield, glitches in human performance and numerous other sources. When a machining system whose components are working at close to full capacity is subjected to the stress of such variability and are likely to fail; the resulting waiting times can become very long compared with expected actual processing times. Thus for the improvement of throughput and performance of any machining system, queuing models can play vital role; even small reduction in capacity loss may yield significant financial benefits or savings. For this reason, the system analyst puts great efforts in the reduction of blocking and delay due to disturbances such as unavailability of spare parts, machine breakdown, switching failure, set up time, etc.

Mathematical modelling of a real world problem is the process by which the problem is interpreted and represented in mathematical terms. In this paper we have analyzed the work done by different researchers on Machine Repair Problem. The machine repair problem with spares in different frameworks has been an area of interest for many researchers since long; here we will give a brief overview of some important contributions in past years.

Ke, Lee and Liou [1] have studied the machine repair problem consisting of M operating machines with S spare machines and R servers (repairmen), who leave for a vacation of random length when there are no failed machines queuing up for repair in the repair facility. At the end of the vacation the servers return to the repair facility and operate one of three vacation policies: single vacation, multiple vacations, and hybrid single/multiple vacation. Maheshwari and Ali [2] considered a machine repair problem with spares, balking and reneging. The Chapman-Kolmogorov equations are formulated for steady state, which have been solved recursively. The queue size distribution for the system having N operating units along with the mixed spares of which S_1 are warm and S_2 are cold is established using product type technique. Raj and Chandrasekar [3] have studied the machine repair problem with a single service station subject to breakdown under the steady-state conditions. Jain and Preeti [4] have investigated a machine repair model with standbys, working vacation and server breakdown. As soon as an operating unit fails, it is immediately replaced by a standby unit for the smooth running of the production. When there is no failed unit in the system, the server goes on vacation; meanwhile, the server performs some works and is said to be on working vacation. El-Damcese and Shama [5] investigated reliability and availability of a repairable system with degradation facility. They have considered two types of repair. The first is due to failed state, the second is due to degraded state. Sharma [6] investigates the machine repair problem consisting of M operating machines with S spare machine and R servers where machines have two failure modes and server are subjected to breakdown under-steady state conditions. Liou [7] investigated the M/M/1 warm-standby machine repair problem with multiple vacations and working breakdowns. He compared the searching results of the PSO algorithm with those of Genetic algorithm (GA) and Exhaustive Search Method (ESM) to ensure the superior searching quality of the PSO algorithm. Jain [8] has developed the diffusion approximation model for (m, M) general machine repair problems with spare part support. The spare units may also fail with rates different from operating units. The repairmen switch to the faster rate to reduce a backlog of down units in the case of a busy repair facility. Goswami and Vijaya Laxmi [9] analyzed a controllable discrete-time machine repair problem with L operating machines and two technicians. The number of working servers can be adjusted depending on the number of failed machines in the system one at a time at machine's failure or at service completion epochs. Wang et al. [10] studied the M/M/1 machine repair problem with working vacation in which the server works with different repair rates rather than completely terminating the repair during a vacation period. They assumed that the server begins the working vacation when the system is empty. Wang et al. [11] investigated a multiple-vacation M/M/1 warm-standby machine repair problem with an unreliable repairman. The particle swarm optimization (PSO) algorithm is implemented to determine the optimal number of warm standbys S^* and the service rate μ^* as well as vacation rate ν^* simultaneously at the optimal maximum profit. They compared the searching results of the PSO algorithm with those of exhaustive search method to ensure the searching quality of the PSO algorithm. Yue et al. [12] analyzed a machine repairable system with warm spares and two repairmen. The first repairman never takes vacations. The second repairman leaves for a vacation of random length when the number of the failed units is less than N. At the end of a vacation period, this repairman returns back if there are N or more failed units accumulated in the system. Otherwise this repairman goes for another vacation. Yang and Chiang [13] dealt with a machine repair problem in which M identical operating machines are maintained by one server (repairman). The server is subject to breakdown and operates a threshold recovery policy. Wu and Ke [14] considered a machine repair problem with M operating machines and S standbys, in which R repairmen are responsible for supervising these machines and operate a (V, R) vacation policy. With such policy, if the number of the failed machines is reduced to $R - V$ ($R > V$) (there exists V idle repairmen) at a service completion, these V idle servers will together take a synchronous vacation (or leave for other secondary job). Upon returning from the vacation, they do not take a vacation again and remain idle until the first arriving failed machine arrives. Ke and Wu [15] studied a machine repair problem with homogeneous machines and standbys available. They model a machine repair problem in which some servers take multiple vacations. Armstrong [16] extends the model which assumes that repairs are made only after machine failure, by incorporating preventive repair for ageing machines. Along with deriving a maintenance policy with a single critical age, they have introduced the idea of a double critical age policy. Albright [17] examined the structure of optimal maintenance and repair policies for an exponential repair model. Gupta [18] derived relationship between arrival point probabilities and measures of performance for related queueing systems. Lee et. al. [19] emphasized on the machine repair problem. They have developed two approximate solutions that help to obtain the solution of machine repair problem with a simple desk calculator. They considered the elementary return boundary (ERB) and the instantaneous return boundary (IRB) and obtained approximate solutions to the probability distribution of the number of inoperative machines for each of the boundary policies. Jain et al. [20] analyzed N-Policy for machine repair queuing system with standbys, degraded failure, Bernoulli feedback and set up. Time dependent solution for M/M/R machining system with mixed standbys, switching failures, and balking, reneging and additional removable repairmen was given by Jain et al. [21]. They considered the switching failure of standbys and provision of additional repairmen according to threshold policy. Singh et al. [22] applied diffusion approximation approach and used elementary return boundaries to obtain probability density function for the queue size for machine repair problem with mixed standbys, discouragement and additional removable repairmen. Jain et al. [23] investigated a time-shared machine repair problem with mixed spares under N-policy. Shekhar et al. [24] considered repairable redundant system with switching failure and geometric reneging.

II. Queue Theoritic Indices For MRP

Queueing models allow a number of useful performance measures to be determined. We would like to discuss some basic performance characteristics of queueing models as follows:

Queue Size Distribution

When the nature of probability distributions of the arrival and service patterns is given, we can find the probability distribution of queue length, which allows us to calculate various measures of effectiveness such as system size, queue size, etc. The *system size* is defined as the expected number of the units in the system and *queue size* is the expected number of the units in the queue.

Waiting Time Distribution

By using probability distribution of waiting time of units we can find the time spent by a unit in waiting before service which is known as *waiting time*. Thus we can find the expected waiting time per unit in the system and expected waiting time per unit in the queue. The time-interval between request for service by a unit and completion of service is defined as response time or sojourn time.

Busy Period Distribution

The busy period distribution is important from server's point of view. The *busy period* is defined as the interval between the arrival of a unit to an empty system and the first epoch there after the system becomes empty again. The fraction of time during which the server remains idle in the service facility is recognized as *idle period.* A busy period and the idle period following it together constitute a *busy cycle.*

There are several other performance measures which are important with the application point of view such as throughput, probability of the server being idle and busy, blocking probability of the server, mean delay, server utilization, expected number of operating units in the system, expected number of standby units in the system, average balking and reneging rate, etc.. It is worthwhile to provide some performance indices for some basic machine repair problems (MRPs). Following notations have been used to describe the model for concerned MRPs:

- M, Y (S) : Total number of operating, cold (warm) standby units
- λ (α) : Failure rate of operating (warm) standby units
- μ : Repair rate of the repairman
- b : Exponentially distributed breakdown rate of the repairman
- g : Exponentially distributed service rate of the server
- q : Probability of switching failure
- 1-b : Balking probability
- r : Reneging parameter

Now we give some performance characteristics for some MRPs as:

A. M/M/R MRP with Mixed Spares

The state transition diagram for the model is shown in Fig. 1 and the state dependent failure and repair rates are

$$\lambda_n = \begin{cases} M\lambda + S\alpha, & 0 \le n < Y \\ M\lambda + (S+Y-n)\alpha, & Y \le n < Y+S \\ (M+Y+S-n)\lambda, & Y+S \le n < M+Y+S \end{cases}$$

and

$$\mu_n = \begin{cases} n\mu, & 0 < n \le R \\ R\mu, & R < n \le M+Y+S \end{cases}$$

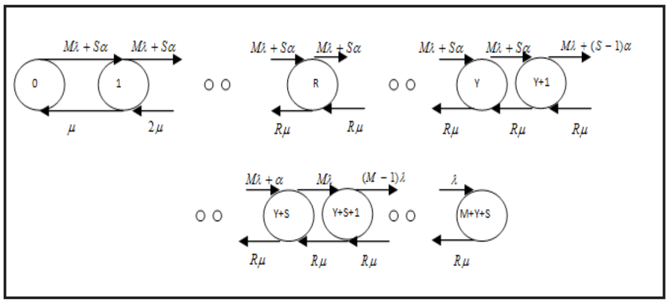

Fig. 1: State Transition Diagram for M/M/R MRP with Mixed Spares for the Case R ≤ Y

Queue Size Distribution

Using the failure and repair rates the steady state equations governing the model are constructed and the product form solution is obtained as

$$P_n = \begin{cases} \dfrac{(M\lambda + S\alpha)^n}{n!\,\mu^n} P_0, & 0 \le n \le R \\[2ex] \dfrac{(M\lambda + S\alpha)^n}{R!\,\mu^n R^{n-R}} P_0, & R+1 \le n \le Y \\[2ex] \prod_{i=Y+2}^{n} [M\lambda + (Y+S-i+1)\alpha] \dfrac{(M\lambda + S\alpha)^{Y+1}}{R!\,\mu^n R^{n-R}} P_0, & Y+1 \le n \le Y+S \\[2ex] \prod_{i=Y+2}^{n-(Y+S+1)} (M-j)\lambda \prod_{i=Y+2}^{Y+S} [M\lambda + (Y+S-i+1)\alpha] \dfrac{(M\lambda + S\alpha)^{Y+1}}{R!\,\mu^n R^{n-R}} P_0, & Y+S+1 \le n \le M+Y+S \end{cases} \quad \ldots(1)$$

where P_0 can be determined using the normalizing condition

$$\sum_{n=0}^{M+Y+S} P_n = 1 \quad \ldots(2)$$

Queue Size Distribution for Special Cases

The results for some classical models can be deduced by setting appropriate parameters as

(1) M/M/R MRP with cold standbys

If we substitute S=0, $\alpha = 0$ in eq. (1), we get the results for the M/M/R MRP with cold standbys (cf. Gross and Harris, 2004).

(2) M/M/1 MRP with mixed standbys

By taking R=1, eq. (1) can be used to obtain the solution for this model.

(3) M/M/1 MRP with warm standbys

For this model, we substitute R=1, Y=0 in eq. (1) and obtain the solution.

Performance Indices

The probabilities obtained in eq. (1) can be used to find explicit expressions for various system characteristics as

- Expected number of failed units in the system is

$$E\{N_s\} = \sum_{n=1}^{M+Y+S} nP_n \qquad \ldots(3)$$

- Expected number of unused cold and warm standbys respectively, are given as

$$E(UCS) = \sum_{n=0}^{Y} (Y-n)P_n \qquad \ldots(4)$$

and

$$E(UWS) = S\sum_{n=0}^{Y} P_n + \sum_{n=Y+1}^{Y+S} (Y+S-n)P_n \qquad \ldots(5)$$

- Expected number of operating units is obtained using

$$E(O) = M - \sum_{n=Y+S+1}^{M+Y+S} (n-\overline{Y+S})P_n \qquad \ldots(6)$$

- Expected number of idle repairmen is

$$E(I) = \sum_{n=0}^{R-1} (R-n)P_n \qquad \ldots(7)$$

- Expected number of busy repairmen is

$$E(B) = R - E(I) \qquad \ldots(8)$$

- Throughput of the system is given as

$$E(T) = \mu\sum_{n=1}^{R} nP_n + R\mu \sum_{n=R}^{M+Y+S} P_n \qquad \ldots(9)$$

- Average delay time is

$$\tau = \frac{E(N_s)}{E(T)} \qquad \ldots(10)$$

B. M/M/1 MRP with Server Breakdown

In steady state, we define

$P_{0,n} =$ Prob. {the server is in busy state and there are n failed units in the system}

$P_{1,n} =$ Prob. {the server is in breakdown state and there are n failed units in the system}

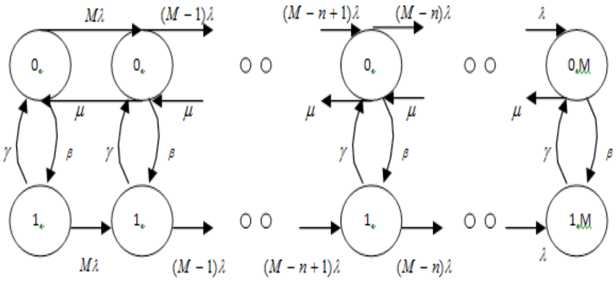

Fig. 2: State Transition Diagram for M/M/1 MRP with Server Breakdown

The failure and repair rates are

$\lambda_n = (M-n)\lambda, \quad 0 \le n < M$ and $\mu_n = \mu$

The steady state equations governing the model can be constructed using state transition diagram (Fig. 2). The generating function technique can be used to find the probabilities for the concerned model. Using these probabilities, the expressions for various performance characteristics can be obtained as follows:

- Expected number of failed units in the queue is

$$E\{N_q\} = \sum_{n=1}^{M} (n-1)P_{0,n} + \sum_{n=1}^{M} P_{1,n} \qquad \ldots(11)$$

- Expected number of failed units in the system is given as

$$E\{N_s\} = \sum_{m=0}^{1}\sum_{n=0}^{M} nP_{m,n} \qquad \ldots(12)$$

- The expected waiting time in the queue is

$$E(W_q) = \frac{E(N_q)}{\lambda_{eff}} \qquad \ldots(13)$$

where $\lambda_{eff} = \sum_{m=0}^{1}\sum_{n=0}^{M} (M-n)\lambda P_{m,n}$

- Throughput of the system is given as

$$E(T) = \mu\sum_{n=1}^{M} P_{0,n} \qquad \ldots(14)$$

- Average delay time is obtained using

$$\tau = \frac{E(N_s)}{E(T)} \qquad \ldots(15)$$

- Probability that the server is in busy state

$$P(B) = \sum_{n=0}^{M} P_{0,n} \qquad \ldots(16)$$

- Probability that the server is in breakdown state

$$P(D) = \sum_{n=0}^{M} P_{1,n} \qquad \ldots(17)$$

C. M/M/R MRP with Spares, Balking, Reneging and Standby Switching Failure

The state dependent failure and repair rates for the case R<S are

$$\lambda(n)=\begin{cases} M\lambda+(S-i)\alpha & ;\ 0\le n\le R-1 \\ [M\lambda+(S-i)\alpha]b & ;\ R\le n\le S \\ (M+S-n)\lambda b & ;\ S+1\le n< M+S-1 \end{cases}$$

$$\mu(n)=\begin{cases} n\mu & ;\ 1\le n\le R \\ R\mu & ;\ R+1\le n\le M+S \end{cases}$$

In particular if we take switching failure probability q=0, joining probability b=1 and reneging parameter r=0, the above model reduces to the model (I) which is discussed earlier in the present section.

The probabilities for the concerned model can be obtained by using suitable analytical or numerical technique; however the results from numerical technique can be obtained quite easily. With the help of these probabilities we can find various performance characteristics as:

- Expected number of failed units in the system

$$E(N_s)=\sum_{n=1}^{M+S} nP_n \qquad \ldots(18)$$

- Expected number of operating units in the system

$$E(O)=M-\sum_{n=S+1}^{M+S}(n-S)P_n \qquad \ldots(19)$$

- Expected number of spare units in the system is given by

$$E(S)=\sum_{n=0}^{S}(S-n)P_n \qquad \ldots(20)$$

- Expected number of idle repairmen in the system is

$$E(I)=\sum_{n=0}^{R-1}(R-n)P_n \qquad \ldots(21)$$

- Expected number of busy repairmen is

$$E(B)=R-E(I) \qquad \ldots(22)$$

- Average switching failure rate is given by

$$ASF=\sum_{n=1}^{S} M\lambda qP_{n-1} \qquad \ldots(23)$$

- Average balking rate is

$$ABR=\sum_{n=R}^{S}[M\lambda+(S-n)\alpha](1-b)P_n + \sum_{n=S+1}^{M+S}\left[(M+S-n)\lambda(1-b)\right]P_n \qquad \ldots(24)$$

- Average reneging rate of failed units is

$$ARR(t)=\sum_{n=R+1}^{M+S}(n-R)\alpha P_n \qquad \ldots(25)$$

- The average rate of failed units loosed due to balking/reneging is

$$ALR=ABR+ARR \qquad \ldots(26)$$

- Machine availability i. e. the fraction of the total time that the machines are working

$$M.A.=1-\frac{E(N_s)}{M+S} \qquad \ldots(27)$$

- Operative utilization i. e. the fraction of busy repairmen

$$O.U.=\frac{E(B)}{R} \qquad \ldots(28)$$

III. Comparision of Models

In this section, we will compare models given by different researchers and derive conclusions. Ke, Lee and Liou[1] and Sharma [6] both have investigated the machine repair problem consisting of M operating machines with S spare machine and R servers. [1] Developed a cost model to obtain the optimal values of the number of spares and the number of servers while maintaining a minimum specified level of system availability while [6] presented a model which generalizes the perfect M/M/R machine repair model with spares and two modes of failures. Generalised solutions for the non-perfect M/M/R machine repair problem with two modes of failure is obtained for R =1. Sharma [6] has considered that each server is subject to breakdown even if no failed machine is in the system. Maheshwari and Ali [2] and Yue, Yue and Qi [12] both analyzed machine repair problem with spares. [2] investigated a machine repair model with balking reneging, spares and additional repairmen have been developed. The machining system under study comprises of warm and cold standby together with a repair facility having both permanent as well as additional repairmen. The provision of multiple standby support and additional repair facility may help the system manager in providing consistent production up to a desired grade of demand in particular when number of failed machines increases. The expressions for several system performance measures and cost function are derived explicitly which can be further used to determine the optimal combination of standbys and repairmen and might be aid system designer to determine appropriate system components at optimum cost subject to availability constraints. On the other side [12] developed a cost model to determine the optimum N while the system availability is maintained at a certain level. They have also investigated sensitivity analysis. Raj and Chandrasekar [3] and Jain and Preeti [4] investigated machine repair model subject to breakdown. The main difference in their investigation is that [3] developed a queueing model with working server breakdown and busy period. Which require a sequence of state of repair before service is restored while [4] established a cost function to maximize the gain. The sensitivity analysis is also carried out to examine the effect of different parameters on various system characteristics. Wang, Liou and Wang [11] and Liou [7] both investigated a multiple-vacation M/M/1 warm-standby machine repair problem. The main difference between both the models is that Liou [7] investigated the profit optimization of the M/M/1 MRP with warm standbys, multiple vacations, and working breakdowns while Wang, Liou and Wang [11] investigated a multiple-vacation M/M/1 warm-standby machine repair problem with an unreliable repairman. They construct the total expected profit per unit time and formulate an optimization problem to find the maximum profit. The particle swarm optimization (PSO) algorithm is implemented to determine the optimal number of warm standbys S^* and the service rate μ^* as well as vacation rate ν^* simultaneously at the optimal maximum profit and compared the searching results of the PSO algorithm with those of exhaustive search method to ensure the searching quality of the PSO algorithm. On other side [7] firstly established the steady-state equations and applied matrix-analytic method to derive the steady-state probabilities. Next, set up the expected profit function per unit time to determine the joint optimal values of (S, μ_w, μ_d) at maximum profit and employed the PSO algorithm to search for the optimal maximum solutions $F(S^*, \mu^*_w, \mu^*_d)$ and justified the solution quality by using the GA and ESM. Finally, comparison the search results of the PSO algorithm with those of the GA and ESM has been done. We can observe that the PSO algorithm has superior search characteristics and can acquire the optimal solutions within a reasonable time. In future research, one can utilize the PSO algorithm to solve the optimization problem appeared in other queueing system.

Goswami and Vijaya Laxmi [9] and Yang and Chiang [13] both analyzed L and M identical operating machines respectively. [9] developed the total expected cost function per machine per unit time and obtain the optimal operating policy and the optimal service rate at minimum cost using quadratic fit search method and simulated annealing method. The method of analysis used by Goswami and Vijaya Laxmi [9] can be applied to bulk-arrival and bulk service Geo/Geo/1 models under different constraints of service patterns. On the other side Yang and Chiang [13] developed a cost model to determine the optimal threshold value, optimal service rate and optimal repair rate. They applied the particle swarm optimization algorithm to solve this optimization problem. Ke and Wu [14,15] have discussed synchronous single vacation and synchronous multiple vacation. In 2012, they presented a paper in which they developed system performances by matrix decomposition and sub-matrix inverse technique. A cost model is given by them to determine the optimal vacation policy including related rate. In 2014, they solved the steady-state probabilities in terms of matrix forms and the obtained the system performance measures. They have provided algorithmic procedures to deal with the optimization problem of discrete/continuous decision variables. Jain et al. [21,22] in 2007 established expressions for various performance measures in terms of transient probabilities of system states. Singh et al. [23] provided expressions for performance measure useful for improving the grade of service.

IV. Conclusion

In this survey paper, we have presented the review on machine repair problems which have attracted researchers and inspired to build a model similar to such problems to find the solution. The complexity of a machining system leads to the necessity of the creation of new and more complicated models based on queueing theory and to develop new approaches to analyze them. Taking into account some complex situations of real time machining systems several authors have investigated queueing models of machining systems which provide the quantitative prediction of the system performance. Various congestion aspects of machining systems have been studied via queue theoretic approach which is helpful to understand and quantify the effect of variability which is essential feature of any system. We have reviewed different research papers done by researchers in different frameworks. We have cited the machine repair models with the combination of different concepts.

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