A COMPARATIVE ANALYSIS OF DIFFERENT MEASUREMENT SCALE AND NORMALIZATION METHOD PERFORMANCES IN ELECTRE METHOD

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Abstract— There are several preference measurement scale and normalization method in the literature. In this paper, we compare these possibilities in ELECTRE method, and compares results. In the absence of any other standards, the priorities provided by the original ELECTRE (involve: Linear scale, Vector normalization and Entropy weights) was used as the benchmark. The results indicate that linear measurement scale and vector normalization method is not the only or best scale and normalization method in ELECTRE method. In other words, the ranking for some measurement scale (i.e. Linear, Power, Logarithmic and Root Square measurement scales) and normalization method (i.e. non-linear normalization method) are effectively same, as the benchmark.

Keywords— ELECTRE, Measurement Scale, Normalization Method

I. INTRODUCTION

Multi Attribute Decision Making (MADM) is an important component of modern decision science. The theory and methods of MADM have been extensively applied to the fields of engineering project, economy, management and military affairs, such as investment decision making, venture capital project evaluation, facility location, bidding, maintenance services, military system efficiency evaluation, development ranking of industrial sector, comprehensive evaluation of economic performance, etc. (Azadfallah, 2015). Multi attribute decision making (MADM) models are selector models that are used for evaluating, ranking and selecting the most appropriate alternative from among several alternatives (Alinezhad, 2011).

ELECTRE is a multi-attribute decision making method for ranking multiple alternatives based on some criteria. This method is a very effective assessment solution that equips the decision activities with quantitative and qualitative features (amiri et al., 2008). In this paper, we aim to demonstrate that the different preference measurement scale and normalization method in ELECTRE has a strong influence on the final priorities and the fact that, which the preference measurement scale and normalization method is more appropriate for replacement in the original ELECTRE method to obtain the same results. The paper is organized as follows. In section 2, ELECTRE, section 3, measurement scale and normalization method and section 4, literature is reviewed. Numerical example is provided in section 5. The paper is concluded in section 6.

II. ELECTRE

ELECTRE uses the concept of an **"outranking relationship"**. The outranking relationship of $A_k \rightarrow A_l$ says that even though two alternatives k and l do not dominate each other mathematically, the DM accepts the risk of regarding A_k as almost surely better than A_l. the dominated alternatives defined by the outranking relationship can be eliminated. This method consists of a pair wise comparison of alternatives based on the degree to which evaluations of the alternatives and the preference weights confirm or contradict the pair wise dominance relationships between alternatives. It examines both the degree to which the preference weights are in agreement with pair wise dominance relationships and the degree to which weighted evaluations differ from each other. These stages are based on a "concordance and discordance" set, hence this method is also called concordance analysis. The ELECTRE method takes the following steps (Hwang and yoon, 1981):

Step1. Calculate the normalized decision matrix. Each normalized value rij of the normalized decision matrix R can be calculated as:

$$r_{ij} = X_{ij} / \sqrt{\sum_{i=1}^{m} X_{ij}^{2}} \quad (1)$$

Step2. Calculate the weighted normalized decision matrix. This matrix can be calculated by multiplying each column of matrix R with its associated weight Wj. Therefore, the weighted normalized decision matrix V is equal to:

$$V = RW \quad (2)$$

Step3. Determine the concordance and discordance set. The concordance set C_{kl} of A_k and A_l is composed of all criteria for which A_k is preferred to A_l . In other words,

$$C_{kl} = \{j \mid X_{kj} \geq X_{lj}\} \quad (3)$$

The complementary subset is called the discordance set, which is:

$$D_{kl} = \{j \mid X_{kj} < X_{lj}\} \quad (4)$$

Step4. Calculate the concordance matrix. The relative value of the concordance set is measured by means of the concordance index. The concordance index is equal to the sum of the weights associated with those criteria which are contained in the concordance set. Therefore, the concordance index C_{kl} between A_k and A_l is defined as:

$$Ckl = \sum j\ \varepsilon ckl\ Wj \ / \sum nj=1\ Wj \quad (5)$$

For the normalized weight set:

$$Ckl = \sum j\ \varepsilon ckl\ Wj \quad (6)$$

The concordance index reflects the relative importance of A_k with respect to A_l. obviously, $0 \leq C_{kl} \leq 1$. A higher value of C_{kl} indicates that A_k is preferred to A_l as far as the concordance criteria are concerned.

Step5. Calculate the discordance matrix. A second index, called the discordance index, has to be defined:

$$d_{kl} = \max_{j\ \varepsilon Dkl} \left| V_{kj} - V_{lj} \right| / \max_{j\ \varepsilon J} \left| V_{kj} - V_{lj} \right| \quad (7)$$

It is clear that $0 \leq d_{kl} \leq 1$. A higher value of d_{kl} implies that, for the discordance criteria, A_k is less favorable than A_l, and a lower value of d_{kl}, A_k is favorable to A_l.

Step6. Determine the concordance dominance matrix. This matrix can be calculated with the aid of a threshold value for the concordance index. A_k will only have a chance of dominating A_l, if its corresponding concordance index C_{kl} exceeds at least a certain threshold value C', i.e.,

$$Ckl \geq C' \quad (8)$$

This threshold value can be determined, for example, as the average concordance index, i.e.,

$$C' = \sum\nolimits^{m}_{\substack{k=1\\k\neq l}} \sum\nolimits^{m}_{\substack{l=1\\l\neq k}} C_{kl} / m\,(m-1) \quad (9)$$

On the basis of the threshold value, a Boolean matrix F can be constructed, the elements of which are defined as:

$$f_{kl} = 1 \text{ , if } \quad C_{kl} \geq C'$$

(10)

$$f_{kl} = 0 \text{ , if } \quad C_{kl} < C'$$

Then each element of 1 on the matrix F represents a dominance of one alternative with respect to another one.

Step7. Determine the discordance dominance matrix. This matrix is constructed in a way analogous to the F matrix on the basis of a threshold value d' to the discordance indices. The elements of g_{kl} of the discordance dominance matrix G are calculated as:

$$d' = \sum_{\substack{k=1 \\ k\neq l}}^{m} \sum_{\substack{l=1 \\ l\neq k}}^{m} d_{kl} / m(m-1)$$

$$g_{kl} = 1 \text{ , if } \quad d_{kl} \leq d'$$

$$g_{kl} = 0 \text{ , if } \quad d_{kl} > d'$$

(11)

Also the unit elements in the G matrix represent the dominance relationships between any two alternatives.

Step8. Determine the aggregate dominance matrix. The next step is to calculate the intersection of the concordance dominance matrix F and discordance dominance matrix G. the resulting matrix, called the aggregate dominance matrix E, and is defined by means of its typical elements e_{kl} as:

$$e_{kl} = f_{kl}.g_{kl}$$

(12)

Step9. Eliminate the less favorable alternatives. The aggregate dominance matrix E gives the partial – preference ordering of the alternatives. If $e_{kl} = 1$, then A_k is preferred to A_l for both the concordance and discordance criteria, but A_k still has the chance of being dominated by the other alternatives. Hence, the condition that A_k is not dominated by ELECTRE procedure is:

$e_{kl} = 1$, for at least one l, $l = 1,2,\dots,m$, $k \neq l$

$e_{ik} = 0$, for all i , $i = 1,2,\dots,m$, $i \neq k, i \neq l$.

III. MEASUREMENT SCALE AND NORMALIZATION METHOD

Brief description presented as follow:

A. Measurement scale (For qualitative attributes)

Measurement scale, in the broadest sense, is defined as the assignment of numerals to objects or events according to rules (Stevens, 1946). The rule of assignment can be any consistent rule. The only rule not allowed would be random assignment, for randomness amounts in effect to a non rule (Luce, 1997).when introducing AHP; saaty advocated the use of an additive scale ranging from 1-9. He defended the scale by providing evidence from a variety of sources (Ramanathan, 2001). **In the absence of any defined scale (for qualitative attribute) for ELECTRE, above scales, in this survey was used as the benchmark.** Theoretically there is no reason to be restricted to these numbers. Therefore other scales (table 1) have been proposed (Ishizaka et al., 2011).

Table 1 Different scales for comparing two alternatives

Scale	Definition	Parameters
Linear (Saaty,1977)	c = a. x	; x = 1,2,…,9 a > 0
Power (Harker and Vargas,1987)	$C = x^a$	a >1 ; x = 1,2,…,9
Geometric (Lootsma , 1989)	C = a ^ x-1	a >1 ; x = 1,2,…,9
Logarithmic (Ishizaka , balkenborg et al. , 2006)	C = Log a(x + 1)	a >1 ; x = 1,2,…,9
Root square (Harker and Vargas,1987)	$\sqrt[a]{X}$ C =	a >1 ; x = 1,2,…,9
Inverse linear (Ma and Zheng , 1991)	C = 9 / (10 – x)	x = 1,2,…,9
Balanced (salo and Hämäläien , 1997)	C = w / (1 – w)	W = 0.5 , 0.55, 0.6,…, 0.9

Source: Ishizaka et al. (2011, p. 4)

Notes: In this paper a = 1 for the linear scale and a = 2 for all other scales are used.

B. Normalization method

Normalization is the mapping of empirical attribute values (measured on different scales) to the scale [0, 1] (Pavl, 2000). Normalization helps to convert all the attribute values into no dimensional, i.e. comparable quantities (Zavadskas et al., 2010). In the present investigation, the vector, weitendorf linear, jüttler - körth, nonlinear and logarithmic methods were used (table 2).

Table 2 Normalization method

Normalization method	Preferable max a i j	Preferable min a i j	Notes
1 Vector (Van delft and Nijkamp,1977)	$b_{ij} = a_{ij} / \sqrt{\sum_{i=1}^{m} a_{ij}^2}$	$b_{ij} = 1 - a_{ij} / \sqrt{\sum_{i=1}^{m} a_{ij}^2}$	The ratio of the values remains constant for this type of normalization in the interval [0, 1].
2 weitendorfs(1976) linear	$b_{ij} = a_{ij} - \min_i a_{ij} / \max_i a_{ij} - \min_i a_{ij}$	$b_{ij} = \max_i a_{ij} - a_{ij} / \max_i a_{ij} - \min_i a_{ij}$	The calculated values are dependent on the size of the interval [max i a $_{ij}$; mini a $_{ij}$].
3 jüttler – körth (1969)	$b_{ij} = 1 - \lvert \max_i a_{ij} - a_{ij} \rvert / \max_i a_{ij}$	$b_{ij} = 1 - \lvert \min_i a_{ij} - a_{ij} \rvert / \min_i a_{ij}$	The application of this type of normalization is limited to the interval [0, 1].
4 Peldschus et al (1983) non linear	$b_{ij} = (a_{ij} / \max_i a_{ij})^2$	$b_{ij} = (\min_i a_{ij} / a_{ij})^3$	The values are diminished more than when using other methods.
5 New Logarithmic	$b_{ij} = \ln(a_{ij}) / \ln(\prod_{i=1}^{n} a_{ij})$	$b_{ij} = (1 - \ln(a_{ij}) / \ln(\prod_{i=1}^{n} a_{ij})) / n-1$	The sum of normalized criterion values is always equal to 1.

Source: Zavadskas and Turskis (2008, p.305)

IV. LITERATURE REVIEW

ELECTRE methods have been widely considered as an effective and efficient decision aid method with successful applications in areas such as: agriculture and forest management, energy sector, environmental and water management, financing, military, project selection and transportation (Shofade, 2011). In Handbook of multi criteria analysis (2010), Zopounidis and Pardalos has dealt with some aspects related to new developments in ELECTRE method. These aspects may be briefly defined as follows: 1. methodological developments (i.e. Pure-inference based approach …), 2. Improvements and new approaches (i.e. Bi-polar outranking based procedure …), 3. Axiomatic and meaningfulness analysis (i.e. Axiomatic analysis …), 4. Other aspects (i.e. the relative importance of criteria …). Meanwhile, there is no any reference that deals with the ELECTRE method in the conditions that both measurement scale and normalization method is changed. Therefore, it is the aim of this paper.

V. NUMERICAL EXAMPLE

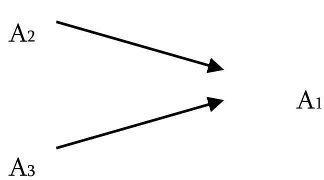
In this section, we work out a numerical example to illustrate priorities in different conditions. Let us consider the following decision matrix of 3 alternatives with 3 attributes. The decision matrix after assigning numerical values (by using the Linear measurement scale formula in table 1) to qualitative attributes is:

Table 4 Decision matrix

A \ C	C_1^*	C_2^*	C_3^*
A_1	1	9	7
A_2	7	5	9
A_3	9	7	3

Benefit type criteria *.

There are some methods of determination of the weights of the criteria. In this paper, we consider weights obtained by Entropy method; Wj= (.671, .080, .249). The original ELECTRE (in other words: ELECTRE with Linear measurement scale, Vector normalization method and Entropy weights); results:

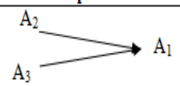
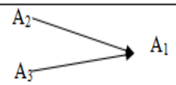
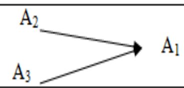
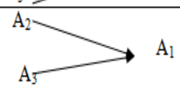


Is used as the <u>**standard**</u> (Benchmark), for evaluating the other situations in this context.

A. Testing the different measurement scale in ELECTRE method

In the present investigation, the different measurement scale proposed in the literature used is: Linear, Power, Geometric, Logarithmic, Root Square, Inverse Linear and Balanced. And converted from the judgments in table 3. In order to show the impacts of different measurement scale on results in ELECTRE, of course, with no intention to describe the whole procedure, we shall only point to the final results (table 4).

Table 4 Different measurement scale / Vector normalization / Entropy weights

Measurement scale	priorities
Linear W_j=(.671, .080, .249)	
Power W_j=(.538, .137 .325)	
Geometric W_j=(.374, .278, .348)	—
Logarithmic Wj=(.793, .041, .167)	
Root Square Wj=(.733, .060, .207)	
Inverse Linear Wj=(.384, .277, .339)	—
Balanced Wj=(.426, .237, .338)	—

Notes: the criteria weights (W_j) are changed in according to different measurement scale.

Finding:

1. As can be seen from table 4, the Linear, Power, Logarithmic and Root Square measurement scales, exhibit the same priorities in comparison with standards.

2. Another important point to observe that in this illustrative example, the priorities of the existing alternatives may muddled with scale changes

B. Testing the different normalization method in ELECTRE method

The different normalization method that used is: Vector, Weitendorf Linear, Juttler-korth, Non Linear and Logarithmic. Again, with no intention to describe the whole procedure, a comparison of the test results is given in table 5.

Table 5 Linear measurement scale / different normalization method / Entropy weights

Normalization method	Priorities
Vector	A_2, A_3 → A_1
Weitendorf Linear	A_2, A_3 → A_1
Juttler-korth	A_2, A_3 → A_1
Non Linear	A_2, A_3 → A_1
Logarithmic	A_2, A_3 → A_1

Notes: the criteria weights are constant for all of the normalization method (Wj= (.671, .080, .249).

Findings:

1. As can be seen from table 5, the all of normalization method exhibit the same priorities in comparison with standards.

Generally, it is remarkable to observe that in this illustrative example, the scale to be used has a significant effect on the outcomes. But, the normalization method has not impacts in final results.

C. Testing the different measurement scale, normalization method and weights in ELECTRE method

Different methods (measurement scale and normalization method) for deriving priorities in the same framework (ELECTRE) used. Again, with no intention to describe the whole procedure, a comparison of the test results is given in table 6 (i.e. Detail calculation for Power scale, Weitendorf Linear normalization method and Entropy weights are presented in Appendix).

Table 6 Different measurement scale, normalization method and weights

Sca. / Nor.	Linear	Power	Geometric	Logarithmic	Root Square	Inverse Linear	Balanced
Vector	A_2, A_3 → A_1	A_2, A_3 → A_1	———	A_2, A_3 → A_1	A_2, A_3 → A_1	———	———
Weitendorf Linear	A_2, A_3 → A_1	A_2, A_3 → A_1	———	A_2, A_3 → A_1	A_2, A_3 → A_1	———	———
Juttler-korth	A_2, A_3 → A_1	A_2, A_3 → A_1	———	A_2, A_3 → A_1	A_2, A_3 → A_1	———	———
Non Linear	A_2, A_3 → A_1	A_2, A_3 → A_1	A_2 → A_1	A_2, A_3 → A_1	A_2, A_3 → A_1	A_2 → A_1	A_2 → A_1
Logarithmic	A_2, A_3 → A_1	A_2, A_3 → A_1	A_2 → A_1	A_2, A_3 → A_1	A_2, A_3 → A_1	———	A_2 → A_1

Notes: The criteria weights are changed in according to different measurement scale.

Finding:

1. In 20 possibilities out of 35 cases, the priority of alternatives are in correct standard priorities.

2. The Linear, Power, Logarithmic and Root Square measurement scales and all of the normalization method priorities are the same as was determined with standards.

3. The Inverse Linear measurement scale and all of the normalization method results, except Non Linear normalization method, are the incorrect.

4. The Non Linear normalization method and all of the measurement scale results are in the closeness results as was determined with standards.

Generally, the same results of the original ELECTRE method can be obtained from the modified ELECTRE (with different measurement scale and normalization method) as displayed in table 6.

VI. CONCLUDING REMARKS

After analyzing the impacts of the measurement scale (Linear, Power, Geometric, Logarithmic, Root Square, Inverse Linear and Balanced) and normalization method (Vector, Weitendorf Linear, Juttler-korth, Non Linear and Logarithmic) on the priorities of ELECTRE, in illustrative example, we made the following conclusions:

1. The type of measurement scales applied could determine the final priorities. It has been demonstrated that the Linear, Power, Logarithmic and Root Square measurement scales priorities for all normalization method are the same as was determined with standards. 2. The normalization method to be used has a significant effect on the final priorities. So that, The Non Linear normalization method (with all of the measurement scale) results are in the closeness results as was determined with standards.

The above observation indicates that linear measurement scale and Vector normalization method is not the only or best scale and normalization methods. Other measurement scale and normalization method might be reasonable in some cases. In general, however, one example seems not enough to conclude, but this study an experimentally proved that some measurement scale and normalization method are more effective than others. Since, the finding in this paper can help the ELECTRE decision makers select suitable measurement scale and normalization method, for replacement with original ELECTRE assumptions and obtained the same results.

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