INFLUENCE OF THE DIFFERENT MEASUREMENT SCALE AND NORMALIZATION METHOD ON RESULTS IN TOPSIS

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Abstract—The present study presents a comparative analysis of different measurement scale (i.e. linear, power, and so on) and normalization method (i.e. vector, linear, and so on.) in TOPSIS, by testing them against conventional TOPSIS assumption (in the absence of any other standards, the priorities provided by the conventional TOPSIS was used as the benchmark). In this study, the following two questions are considered: 1. Does a decision priorities changed when using different measurement scale and normalization method? and 2. Does the same results of the conventional TOPSIS, be obtained from the modified TOPSIS (in other words: TOPSIS with different measurement scale and normalization method)? It is shown that, the different measurement scale and normalization method can be lead to different priorities (rank and preference intensities). Also, the modified TOPSIS priorities (TOPSIS with geometric measurement scale and logarithmic normalization method), the only or best combination that is roughly equivalent priorities as compared with conventional TOPSIS. So, we suggest (after the more experimental research in future) the modified TOPSIS method as the alternative solution.

Keywords—TOPSIS, Modified TOPSIS, Measurement Scale, Normalization Method

1. INTRODUCTION

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), one of the major MADM techniques, ranks the alternatives according to their distance from the ideal and the negative-ideal solution (Roghanian et al., 2010). So, TOPSIS selects the closest to the ideal solution and farthest from negative-ideal solution (Tayeb et al., 2007).

Topsis can rank a finite number of feasible alternatives in order of preference according to the features of each attribute of every alternative and select a suitable alternative that conforms to the decision maker's ideal. The basic concept of topsis technique is that the selected alternative will have the shortest Euclidean distance from the ideal solution and the farthest Euclidean distance from the anti-ideal solution (Lin et al., 2008). The ideal solution is the solution that maximizes the benefit attributes and minimizes the cost attributes, whereas the negative ideal solution is the solution that maximizes the cost attributes and minimizes the benefit attributes. In short, the ideal solution consists of all best attribute values, whereas the negative ideal solution is composed of all worst attribute values. The optimal alternative is the one, which has the shortest distance from the ideal solution and the farthest distance from the negative ideal solution (Yue, 2013).

In this paper, we aim to show that the different measurement scale and normalization method in topsis has a strong influence on the final priorities and that, which the measurement scale and normalization method is more proper for replacement in the original topsis model to get the same results. The paper is organized as follow: in section 2, topsis; section3, measurement scale and normalization method; and section 4, literature is reviewed. Numerical examples are provided in section 5. The paper is concluded in section 6.

2. TOPSIS

Topsis assumes that we have m alternatives (options) and n attributes/criteria and we have the score of each option with respect to each criterion. Let x_{ij} score of option i with respect to criterion j. we have a matrix $X = (x_{ij})_{m.n}$ matrix. Let J be the set of benefit attributes or criteria (more is better). Let J' be the set of negative attributes or criteria(less is better). The idea of Topsis can be expressed in a series of steps (Tayeb et al., 2007):

Step 1: Obtain performance data for n alternatives over k criteria. Raw measurements are usually standardized; converting raw measures x_{ij} into standardized measures s_{ij}. Construct normalized decision matrix. This step transforms various attribute dimensions into non-dimensional attributes, which allows comparisons across criteria. Normalize scores or data as follows:

$$r_{ij} = X_{ij} / \sqrt{\sum_i X_{ij}^2} \text{ for } i = 1, \ldots, m; j = 1, \ldots, n.$$

Step 2: develop a set of importance weights wk, for each of the criteria. The basis for these weights can be anything, but usually, is ad hoc reflective of relative importance. Scale is not an issue if standardizing was accomplished in step 1. Construct the weighted normalized decision matrix. Assume we have a set of weights for each criteria wj for j = 1, …, n. multiplies each column of the normalized decision matrix by its associated weight. An element of the new matrix is:

$$V_{ij} = w_j \, r_{ij}$$

Step 3: determine the ideal and negative ideal solutions.

Ideal solutions:

$$A^* = \{v_1^*, \ldots, v_n^*\}, \text{ where}$$
$$V_j^* = \{\max_i (v_{ij}) \text{ if } j \in J; \min_i (v_{ij}) \text{ if } j \in J'\}$$

Negative ideal solutions:

$$A' = \{v_1', \ldots, v_n'\}, \text{ where}$$

$$V_j^* = \{\min_i (v_{ij}) \text{ if } j \in J; \max_i (v_{ij}) \text{ if } j \in J'\}$$

Step 4: Calculate the separation measures for each alternative. The separation from the ideal alternative is:

$$S_i^* = [\sum_j (v_j^* - v_{ij})^2]^{1/2} \quad i = 1, \ldots, m.$$

Similarly, the separation from the negative ideal alternative is:

$$S_i' = [\sum_j (v_j' - v_{ij})^2]^{1/2} \quad i = 1, \ldots, m.$$

Step 5: calculate the relative closeness to the ideal solution C_i^*:

$$C_i^* = S_i' / S_i^* + S_i' \quad 0<C_i^*<1$$

Step 6: Rank order alternatives by maximizing the ratio in step 5. Select the option with C_i^* closest to 1.

Here, we are focusing on steps 1, and with changed original topsis assumptions (scale and normalization method) will compare results.

3. MEASUREMENT SCALE AND NORMALIZATION METHOD

Brief description presented as follow:

3.1. Measurement scale (For qualitative attributes)

Measurement scale, in the broadest sense, is defined as the assignment of numerals to objects or events according to rules (Stevens, 1946). The rule of assignment can be any consistent rule. The only rule not allowed would be random assignment, for randomness amounts in effect to a non rule (Luce, 1997).when introducing AHP; saaty advocated the use of an additive scale ranging from 1-9. He defended the scale by providing evidence from a variety of sources (Ramanathan, 2001). In the absence of any defined scale (for qualitative attribute) for TOPSIS, above scales, in this study was used as the benchmark. Theoretically there is no reason to be restricted to these numbers. Therefore other scales (table 1) have been proposed (Ishizaka et al., 2011).

3.2. Normalization method

Normalization is the mapping of empirical attribute values (measured on different scales) to the scale [0,1](Pavl , 2000).normalization helps to convert all the attribute values into no dimensional , i.e. comparable quantities(Zavadskas et al. , 2010).in the present investigation , the vector , weitendorf linear , jüttler - körth , nonlinear and logarithmic methods were used(table 2).

Table 1 Different scales for comparing two alternatives

Scale	Definition	parameters
Linear (Saaty,1977)	c = a. x	a > 0 ; x = 1,2,…,9
Power (Harker and Vargas,1987)	$C = x^a$	a >1 ; x = 1,2,…,9
Geometric (Lootsma , 1989)	C = a ^ x-1	a >1 ; x = 1,2,…,9
Logarithmic (Ishizaka , balkenborg et al. , 2006)	C = Log a(x + 1)	a >1 ; x = 1,2,…,9
Root square (Harker and Vargas,1987)	$C = \sqrt[a]{X}$	a >1 ; x = 1,2,…,9
Inverse linear (Ma and Zheng , 1991)	C = 9 / (10 – x)	x = 1,2,…,9
Balanced (salo and Hämäläien , 1997)	C = w / (1 – w)	W = 0.5 , 0.55, 0.6,…, 0.9

Source: Ishizaka et al. (2011, p. 4)

Notes: In this paper a = 1 for the linear scale and a = 2 for all other scales are used.

4. LITERATURE REVIEW

In recent years, topsis has been successfully adopted in various fields, e.g., location analysis, construction processes, human resources management, transportation, product design, manufacturing, water management, and quality control, and demonstrated with satisfactory results (Lin et al., 2008). However, there is not any reference that deals with topsis in the conditions that different both measurement scale and normalization method are present.

But, there are several limited examples where modified topsis is used. For instance: different distance [Euclidean and city block] (Hwang and Yoon, 1981), weighted Euclidean distance (Deng et al., 2000), different weighting [centroid, regression, equal] and different distance [linear, Euclidean, tchebycheff] (Olson, 2004), compared the vector and linear normalization method results (Zavadeskas et al., 2006), optimized ideal reference point (Ren et al., 2007), weighting based on Hessian matrix (Tayeb et al., 2007), and calculating progress or regress by malmquist index by way of topsis method (Irvani et al., 2009), etc. That seems the performance of different methods (scale and normalization methods) investigated in a few studies. In this paper, we are focusing on the different measurement scale (Linear, power, geometric, logarithmic, root square, inverse linear and balanced) and normalization methods (Saaty, vector, Weitendorf's linear, juttler – korth, non linear and logarithmic) in topsis and compares results.

5. EXPERIMENT

In this section, we work out a numerical example to illustrate priorities (rank and preference intensities) in different condition. Let us consider the following decision matrix of 4 alternatives with 6 attributes. The decision matrix after assigning numerical values (table 3) to qualitative attributes is (table 4): There are some methods of determination of the weights of the criteria. In this study we consider weights got by entropy method.

$$W_j = (.136, .166, .366, .104, .161, .067)$$

The original topsis (topsis method with linear measurement scale (1-9), vector normalization method and entropy weighted) results (A_1 (.716) > A_3 (.347) > A_4 (.330) > A_2 (.250)); is used as the benchmark for the evaluating of the other situation in this context (for a detail calculation [for geometric measurement scale and juttler-korth normalization method - table 7], see Appendix).

Table 2 Normalization methods

Normalization method	Preferable max a i j	Preferable min a i j	Notes
1 Vector (Van delft and Nijkamp,1977)	b ij $\sqrt{\sum_{i=1}^{m} aij^{2}}$ = a ij /	b ij = $\sqrt{\sum_{i=1}^{m} aij^{2}}$ 1 - a ij /	The ratio of the values remains constant for this type of normalization in the interval [0, 1].
2 weitendorfs(1976) linear	b $_{ij}$ = a $_{ij}$ – min $_i$ a $_{ij}$ / max $_i$ a $_{ij}$ – min$_i$ a $_{ij}$	b $_{ij}$ = max $_i$ a $_{ij}$ - a $_{ij}$ / max $_i$ a $_{ij}$ – min$_i$ a $_{ij}$	The calculated values are dependent on the size of the interval [max i a $_{ij}$; mini a ij].
3 jüttler – körth (1969)	ax i a ij - a ij / max i a ij |	b ij=1-| min i a ij - a ij / min i a ij |	The application of this type of normalization is limited to the interval [0, 1].
4 Peldschus et al (1983) non linear	b ij = (a ij /max i a ij)^2	b ij = (min i a ij /a ij)^3	The values are diminished more than when using other methods.
5 New Logarithmic	b ij = Ln(a ij) / $\prod_{i=1}^{n} aij)$ Ln(	b ij = (1 - Ln(a ij) / / n-1) $\prod_{i=1}^{n} aij)$ Ln(	The sum of normalized criterion values is always equal to 1.

Source: Zavadskas and Turskis (2008, p.305)

Table 3 (from table 1, c = a.x; a=1, x= 1, 2,…,9)

Linear Scale values	1	2	3	4	5	6	7	8	9

5.1. Testing the different measurement scales in topsis method

In the present investigation, the different measurement scale proposed in the literature used is: Linear, power, geometric, logarithmic, root square, inverse linear and balanced. To show the impacts of different measurement scale on results in topsis, with no intention to describe the whole procedure, we will only point to the final results (table 5). The findings as noted earlier, the linear measurement scale results (with vector normalization method) are the same as standard results.

Table4 Decision matrix

Cri. / Alt.	C_1	C_2	C_3	C_4	C_5	C_6
A_1	5	1	9	3	5	7
A_2	1	9	1	7	1	3
A_3	5	7	3	3	3	9
A_4	7	5	1	9	7	5

·. Note, that all attributes are the benefit criteria.

Table 5- Different scale results (with vector normalization method and entropy weighted)

Scale	The relative closeness to the ideal solution	Rank the preference order
Linear(saaty)(**L**)	C_1=.716 C_2= .250 C_3=.347 C_4=.330	**A1 > A3 > A4 > A2**
Power(**P**)	C1= .672 C2= .260 C3= .246 C4=.345	A1 > A4 > A2 > A3
Geometric(**G**)	C1=.599 C2=.275 C3=.159 C4=.358	A1 > A4 > A2 > A3
Logarithmic(**LG**)	C1=.732 C2=.252 C3=.496 C4=.340	**A1 > A3 > A4 > A2**
Root square(**RS**)	C1=.730 C2=.254 C3=.444 C4=.337	**A1 > A3 > A4 > A2**
Inverse linear(**IL**)	C1=.644 C2=.258 C3=.170 C4=.283	A1 > A4 > A2 > A3
Balanced(**B**)	C1=.655 C2=.277 C3=.198 C4=.305	A1 > A4 > A2 > A3

Notes: the criteria weights are constant for all measurement scale.

Table 6: Different normalization methods result (with linear scale and entropy weighted)

Normalization method	The relative closeness to the ideal solution	Rank the preference order
Vector(**V**)	C_1=.716 C_2= .250 C_3=.347 C_4=.330	**A1 > A3 > A4 > A2**
weitendorf □s Linear(**WL**)	C1= .653 C2= .295 C3= .384 C4=.398	A1 > A4 > A3 > A2
Juttler – Korth(**JK**)	C1=.666 C2=.292 C3=.382 C4=.383	A1 > A4 > A3 > A2
Non linear(**NL**)	C1=.636 C2=.289 C3=.272 C4=.378	A1 > A4 > A2 > A3
Logarithmic(**LG**)	C1=.790 C2=.201 C3=.535 C4=.287	**A1 > A3 > A4 > A2**

Notes:

1. The criteria weights are constant for all normalization methods.
2. The vector normalization method result is the same as the standard result.

As can be seen from table 5, the logarithmic and root square method show the same ranking with different intensities, in comparison with benchmark. Another important to observe that in this illustrative example, the rank of the alternatives may change with scale changed (rank reversal phenomena).From table 5, A_1 ranking is the best for all scales.

5.2. Testing the different normalization methods in topsis method

The different normalization method that used is: Saaty, vector, weitendorf s linear, Juttler – korth, non linear and logarithmic. Again, with no intention to describe the whole procedure, a comparison of the test results is given in the table 6. **Finding:** From table 6, the logarithmic normalization methods show the same ranking with different intensities, in comparison with benchmark. Another important point to observe is that, A_1 ranking is best for different normalization method. It is remarkable to observe that in this illustrative example, the rank of the alternatives may change with scale changed (rank reversal phenomena). In this section, **to answer first questions** can be said: the priorities (rank and preference intensities) may change when using different measurement scales or normalization methods.

5.3. Testing the different measurement scales and normalization methods in topsis method

Different methods (scale and normalization method) for deriving priorities in the same framework (topsis) were used. Again, with no intention to describe the whole procedure, a comparison of the test results is given in table 7. **Finding:** In all possibilities (35 cases) A_1 ranking is best (with different intensities). The geometric measurement scale and vector normalization method results are in the closeness results as was determined with benchmark rank and intensities). In 13 possibilities out of 35 cases, the ranking of alternatives are in correct benchmark ranking (with different preference intensities).

Table 7: results of different measurement scale, different normalization method and different weights

Normalization Method	Measurement scale			
	L(saaty)	P	G	LG
V	**A1 > A3 > A4 > A2**	A1 > A4 > A2= A3	A1 > A4 > A2 > A3	**A1 > A3 > A4 > A2**
	(.716 .347 .330 .250)	(.679 .348 .237 .237)	(.504 .411 .292 .284)	(.693 .527 388 .290)
WL	A1 > A4 > A3 > A2	A1 > A4 > A2 = A3	A1 > A4 > A2 > A3	**A1 > A3 > A4 > A2**

	(.653 .398 .384 .295)	(.635 .388 .267 .267)	(.494 .419 .294 .289)	(.644 553 .455 .328)
GK	A1 > A4 > A3 > A2	A1 > A4 > A2> A3	A1 > A4 > A2> A3	**A1 > A3 > A4 > A2**
	(.666 .383 .382 .292)	(.644 .380 .264 .262)	(.496 .417 .295 .288)	(.649 .557 .442 .328)
NL	A1 > A4> A2 > A3	A1 > A4 > A2> A3	A1 > A4 > A2> A3	**A1 > A3 > A4 > A2**
	(.633 .378 .281 .272)	(.600 .377 .251 .197)	(.468 .414 .282 .270)	(.636 .462 .435 .327)
LG	**A1 > A3 > A4 > A2**	**A1 > A3 > A4 > A2**	**A1 > A3> A4 > A2**	**A1 > A3 > A4 > A2**
	(.790 .535 .287 .201)	(.811 .525 .274 .177)	**(.709 .342 .326 .244)**	(.753 .623 .341 .240)

Table 7.1: results of different measurement scale, different normalization method and different weights

Normalization Method	**Measurement scale**		
	RS	**IL**	**B**
V	**A1 > A3 > A4 > A2**	A1 > A2 = A4 > A3	A1 > A4 > A2 > A3
	(.689 .477 .385 .296)	(.518 .332 .332 .308)	(.579 .321 .311 .280)
WL	**A1 > A3 > A4 > A2**	A1 > A4 > A2> A3	A1 > A4 > A2> A3
	(.638 .508 .462 .334)	(.499 .352 .335 .320)	(.547 .359 .321 .302)
GK	**A1 > A3 > A4 > A2**	A1 > A4 > A2 > A3	A1 > A4 > A2> A3
	(.647 .512 .442 .333)	(.505 .341 .339 .315)	(.556 .340 .325 .295)
NL	A1 > A4 > A3 > A2	A1 > A4 > A2 > A3	A1 > A4 > A2 > A3
	(.637 .437 .432 .331)	(.490 .342 .326 .302)	(.535 .344 .309 .270)
LG	**A1 > A3> A4 > A2**	A1 > A2 > A3 > A4	**A1 > A3 > A4 > A2**
	(.757 .553 .332 .237)	(.639 .285 .279 .277)	(.702 .283 .257 .250)

Notes: the criteria weights are changed in according to different measurement scales

The logarithmic measurement scale and the entire normalization method ranking are the same as was determined with benchmark (with different intensities). The root square measurement scale and all the normalization method ranking, except nonlinear normalization method are the same as was determined with benchmark (with different intensities). In this section, **to answer second questions** can be said: the roughly results of the conventional TOPSIS can be obtained from the modified TOPSIS method (only with geometric measurement scale and logarithmic normalization method) as displayed in table 7.

6. CONCLUDING REMARK

After analyzing the impacts of the measurement scales (Linear, power, geometric, logarithmic, root square, inverse linear and balanced) and normalization methods (Saaty, vector, Weitendorf's linear, juttler – korth, non linear and logarithmic) on the priorities of TOPSIS, we made the following conclusions: The type of measurement scales applied could determine the final priorities (rank and preference intensities). It showed that the logarithmic and root square measurement scales ranking are the same as was determined with benchmark. The type of normalization methods applied could determine the final priorities (rank and preference intensities). It means that the logarithmic normalization method ranking, are the same as was determined with benchmark. The finding in this paper shows that modified TOPSIS priorities, with geometric measurement scale beside logarithmic normalization method ((**A_1 (.709) > A_3 (.342) > A_4 (.326) > A_2 (.244)**) the only or best combination that are roughly equivalent priorities as compared with conventional TOPSIS priorities **(A_1 (.716) > A_3 (.347) > A_4 (.330) > A_2 (.250)).** Generally, the findings of this study show that some methods (measurement scale and normalization methods) are better than others. But, none of the methods is perfectly correct.

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