The Exponentiated Inverted Exponential Distribution

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Abstract: In this paper, we have introduced the new family of distribution called Exponentiated inverted Exponential distribution (EIED). The proposed model is positively skewed and its shape could be decreasing or unimodal (depending on its parameter values). Several statistical properties of the Exponentiated inverted exponential distribution are studied and derived. The method of Maximum Likelihood Estimation (MLE) was proposed in estimating its parameters. An application to real data set is finally presented for illustration.

Keywords: Inverted exponential distribution, Exponentiated inverted, Exponential distribution, Structural properties and Maximum likelihood estimation.

I. Introduction

The exponential distribution occupies an important position in the analysis of data. In probability theory and statistics, the exponential distribution is a family of continuous probability distribution. Historically, the exponential distribution was the first lifetime model for which statistical methods were extensively developed. It describes the time between events in the Poisson process i.e., a process in which events occur continuously and independently at a constant rate. The exponential distribution, because of memory-less property, is used for life testing of the products that do not age with time. There are several electronic devices whose failure rate does not depend upon their age and, therefore, the Exponential distribution is considered. Sukhatme (1937), Lawless (2003) and Ahmed et. al. (2007) has discussed this distribution with applications.

The IE distribution was proposed by Keller and Kamath (1982). It has an inverted bathtub failure rate and it is an important competitive model for the Exponential distribution. It has been identified and discussed by Lin et al (1989) as a lifetime model. If X is a non-negative Exponential random variable, then the distribution of a random variable $Y=\frac{1}{X}$ follows an Inverse Exponential distribution. Hence, if X denotes a random variable, the cumulative density function (cdf) and the probability density function (pdf) of the Inverse Exponential distribution with a scale parameter λ are respectively given by;

$$G(x)=e^{\frac{-\lambda}{x}} \quad ;\ x>0,\lambda>0 \tag{1.1}$$

The pdf of X is:

$$g(x)=\frac{\lambda}{x^2}e^{\frac{-\lambda}{x}} \quad ;x>0,\lambda>0 \tag{1.2}$$

Where $x>0$ the scale parameter $\lambda>0$

II. Exponentiated Inverted Exponential Distribution

The concept of Exponentiated distributions have been studied widely in statistics since 1995 and a number of authors have developed various classes of these distributions, Mudholkar et al (1995), introduced the Exponentiated Weibull distribution and since then, a number of authors have proposed and generalized many standard distributions based on the Exponentiated distributions. The two-parameter Exponentiated Exponential distribution, firstly proposed by Gupta and Kundu (2001). The Exponentiated Gamma, Exponentiated Weibull, Exponentiated Gumbel and Exponentiated Frechet distributions, defined and studied by Nadarajah and Kotz (2006). Flaih et al. (2012) extended the IW distribution to the exponentiated inverted Weibull (EIW) distribution by adding another shape parameter. This study suggested that the EIW distribution can provide a better fit to the real dataset than the IW distribution. The Exponentiated type distribution which can be widely applied in many areas of biology and engineering; see Cordeiro et al (2013) for details. A generalization of the Generalized Inverse Exponential distribution called the Exponentiated Generalized Inverse Exponential distribution has been defined and studied by Oguntunde et al (2014). Rehman and Dar (2015) studied Exponentiated inverse Rayleigh distribution, examined its mathematical properties and estimated the parameter using Bayesian estimation

In this study, we propose a new distribution which is a EIE distribution. We first provide general definitions of the EIE distribution which will subsequently reveal its pdf and cdf.

Definition 1. If X has a lifetime distribution with cdf $F(x)$ the cdf of Exponentiated Inverse Exponential distribution of X can be defined as;

$$F(x)=\left[G(x)\right]^{\alpha} \tag{2.1}$$

Definition 2. If X has a lifetime distribution with pdf $f(x)$ the pdf of Exponentiated Inverse Exponential distribution of X can be defined as;

$$f(x)=\alpha g(x)\left[G(x)\right]^{\alpha-1} \tag{2.2}$$

Theorem 2.1: Let X be a random variable of an IE distribution with pdf $g(x)$.Then $f(x)=\alpha g(x)\left[G(x)\right]^{\alpha-1}$ is a pdf of the EIE distribution with scale parameter λ and shape parameter α .The notation for X with the EIE distribution is denoted as X~EIE(α,λ) . The pdf of X is given by:

$$f(x)=\frac{\alpha\lambda}{x^2}\left\{e^{\frac{-\lambda}{x}}\right\}^{\alpha} \quad ;x>0,\alpha,\lambda>0 \tag{2.3}$$

Proof: By definition 2, substitute (1.1) and (1.2) into (2.2), then the pdf for the EIE distribution can be obtained by:

$$f(x)=\frac{\alpha\lambda}{x^2}e^{\frac{-\lambda}{x}}\left\{e^{\frac{-\lambda}{x}}\right\}^{\alpha-1}$$

$$f(x)=\frac{\alpha\lambda}{x^2}\left\{e^{\frac{-\lambda}{x}}\right\}^{\alpha} \quad ;x>0,\alpha,\lambda>0$$

Theorem 2.2: Let X be a random variable of an IE distribution with cdf $G(x)$.Then $F(x) = [F(x)]^{\alpha}$ is a cdf of the EIE distribution with scale parameter λ and shape parameter α. The distribution function of the EIE distribution takes the following form:

$$F(x)=\left\{e^{\frac{-\lambda}{x}}\right\}^{\alpha} \quad ;x>0,\alpha,\lambda>0 \tag{2.4}$$

Proof: By definition 1, substitute (1.2) into (2.1), then the cdf for the EIE distribution can be obtained by:

$$F(x)=\left\{e^{\frac{-\lambda}{x}}\right\}^{\alpha}$$

The graphs of density function and cumulative distribution function are plotted for different values of parameters α and λ are given in Figure 1 and 2 respectively.

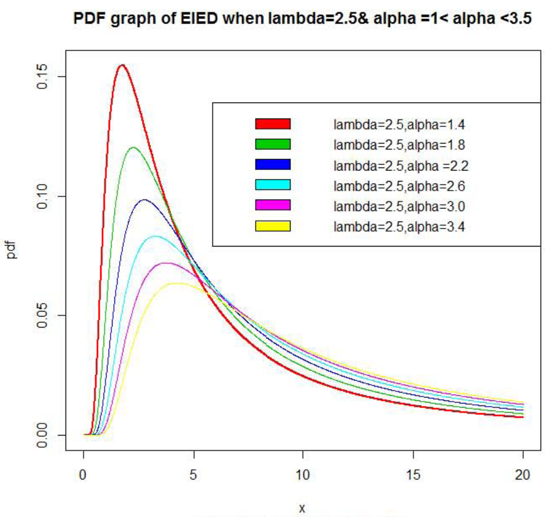


Figure.1 The graph of density function

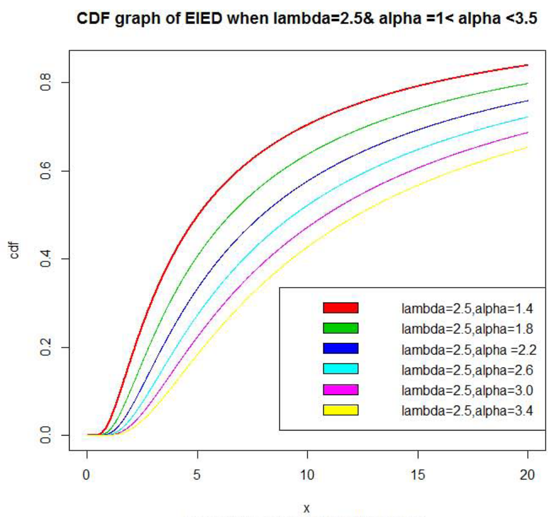


Figure.2 The graph of distribution function

Figure 1 and 2 illustrates some of the possible shapes of EIE distribution for different values of the parameters α and λ. Figure 1 shows that the density function of EIE distribution is unimodal and positively skewed.

Theorem 2.3: Let X be a random variable of the EIE distribution with parameters α and λ

The survival function of the EIE distribution can be written as:

$$S(x)=1-\left\{e^{\frac{-\lambda}{x}}\right\}^{\alpha} \quad ;x>0,\alpha,\lambda>0 \tag{2.5}$$

Proof: By definition, the survival function of the random variable X is given by:

$S(x) = 1 – F(x)$

Using (2.4), the survival function of the EIE distribution can be expressed by:

$$S(x)=1-\left\{e^{\frac{-\lambda}{x}}\right\}^{\alpha} \quad ;x>0,\alpha,\lambda>0$$

Theorem 2.4: Let X be a random variable of the EIE distribution with parameters α and λ

The hazard rate of the EIE distribution takes the form:

$$h(x)=\frac{\alpha\lambda x^{-2}\left\{e^{\frac{-\lambda}{x}}\right\}^{\alpha}}{1-\left\{e^{\frac{-\lambda}{x}}\right\}^{\alpha}} \quad ;x>0,\alpha,\lambda>0 \tag{2.6}$$

Proof: Let X be a continuous random variable with pdf and survival function, $f(x)$ and $S(x)$, respectively, then the hazard rate is defined by:

$$h(x) = \frac{f(x)}{S(x)} \tag{2.7}$$

Substituting (2.3) and (2.5) into (2.7), we obtain:

$$h(x) = \frac{\alpha\lambda x^{-2}\left\{e^{\frac{-\lambda}{x}}\right\}^{\alpha}}{1-\left\{e^{\frac{-\lambda}{x}}\right\}^{\alpha}} \quad ;x>0,\alpha,\lambda>0$$

The plots for the survival and hazard functions are shown in Figure 3 and Figure. 4 respectively;

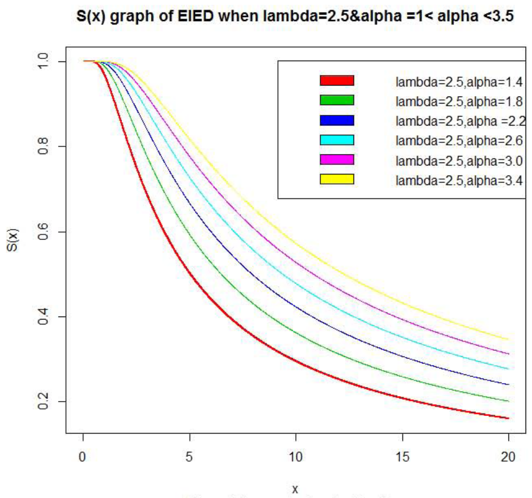


Figure.3 The graph of survival function

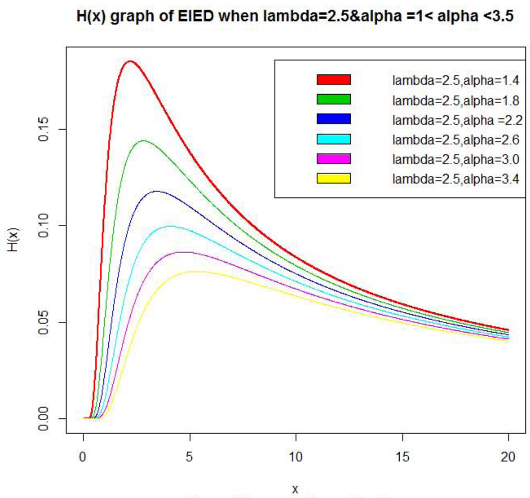


Figure.4 The graph of hazard function

We can infer from Figure 4 that the shape of the hazard rate is unimodal, it increases at the initial stage and later decreases. We can also say that the hazard rate shows an inverted bathtub shape. Also, we observed that the survivals curves show the decreasing rate.

III. Relationship with Other Distributions

Some well-known theoretical distributions can be derived from the proposed EIE distribution. For instance;

For α = 1, Equation (2.3) reduces to give the one-parameter Inverse Exponential distribution with probability density function as:

$$f(x) = \frac{\lambda}{x^2} e^{\frac{-\lambda}{x}} \quad ;x>0,\lambda>0 \tag{3.1}$$

For λ = 1, Equation (2.3) reduces to give the one-parameter Exponentiated standard inverted exponential distribution with probability density function as:

$$f(x) = \frac{\alpha}{x^2}\left\{e^{\frac{-1}{x}}\right\}^{\alpha} \quad ;x>0,\alpha>0 \tag{3.2}$$

For λ = 1 and λ = 1, Equation (2.3) reduces to give the standard inverted exponential distribution with probability density function as:

$$f(x) = \frac{1}{x^2} e^{\frac{-1}{x}} \quad ;x>0 \tag{3.3}$$

If a random variable is such that $Y = 1/X$ and $\alpha = 1$ in Equation (2.3) reduces to give the Exponential distribution with pdf as:

$$f(x) = \lambda e^{-\lambda x} \quad ;x>0,\lambda>0 \tag{3.4}$$

If a random variable is such that Y = $1/X$ and λ = 1, α = 1 in Equation (2.3) reduces to give the standard Exponential distribution with pdf as:

$$f(x) = e^{-x} \quad ;x>0 \tag{3.5}$$

IV. Statistical Properties of the EIE Distribution

This section provides some basic statistical properties of the Exponentiated Inverted Exponential Distribution

A. The r^{th} Moment of the EIE Distribution

Theorem 4.1: If $X \sim \text{EIE}(\alpha,\lambda)$, then rth moment of a continuous random variable X is given as follow:

$$\mu_r = E(X^r) = \alpha^r \lambda^r \Gamma(1-r)$$

Proof: Let X is an absolutely continuous non-negative random variable with pdf $f(x)$, then rth moment of X can be obtained by:

$$\mu_r = E(X^r) = \int_0^{\infty} x^r f(x)dx$$

From the pdf of the EIE distribution in (2.3), then shows that $E(X)$ can be written as:

$$E(X^r) = \int_0^\infty x^{r-2}\alpha\lambda\left(e^{\frac{-\lambda}{x}}\right)^\alpha dx$$

$$E(X^r) = \alpha\lambda\int_0^\infty \frac{1}{x^{1-r-1}}\left(e^{\frac{-\alpha\lambda}{x}}\right)dx$$

After some calculations,

$$\mu_r = E(X^r) = \alpha^r\lambda^r\Gamma(1-r) \quad (4.1)$$

We observe that Equation (4.1) only exists when $r < 1$. The implication is that the first moment, second moment and other higher-order moments does not exist.

B. Harmonic Mean of EIE Distribution

The harmonic mean (H) is given as:

$$\frac{1}{H} = E\left(\frac{1}{X}\right) = \int_0^\infty \frac{1}{x} f(x)dx$$

$$\frac{1}{H} = \int_0^\infty \frac{\alpha\lambda}{x^{2+1}}\left(e^{\frac{-\alpha\lambda}{x}}\right)dx$$

After some calculations,

$$\frac{1}{H} = \frac{1}{\alpha\lambda} \quad (4.2)$$

C. Mode

Consider the density of the EIE distribution given in (2.3), we take the logarithm of (2.3) as follows:

$$\log f(x) = \log(\alpha) + \log(\lambda) - 2\log(x) - \frac{\alpha\lambda}{x} \quad (4.3)$$

Differentiating equation (4.3) with respect to x, we obtain

$$\frac{\partial \log f(x)}{\partial x} = -\frac{2}{x} + \frac{\alpha\lambda}{x^2} \quad (4.4)$$

Now, set equation (4.4) equal to 0 and solve for x, to get

$$x_0 = \frac{\alpha\lambda}{2} \quad (4.5)$$

D. Quantile Function and Median

The Quantile function is given by;

$$Q(u) = F^{-1}(u)$$

Therefore, the corresponding quantile function for the proposed model is given by;

$$Q(u) = -\frac{\alpha\lambda}{\log(u)} \quad (4.6)$$

where U has the uniform U (0,1) distribution. We obtain the median by substituting u=0.5. Hence, the median of the proposed model is given by;

$$F^{-1} = -\frac{\alpha\lambda}{\log(0.5)} \quad (4.7)$$

E. Moment Generating Function

In this sub section we derived the moment generating function of EIE distribution.

Theorem 4.5: Let X have an EIE distribution. Then moment generating function of X denoted by $M_X(t)$ is given by:

$$M_X(t) = \sum_{r=0}^\infty \frac{t^r}{r!}\alpha^r\lambda^r\Gamma(1-r) \quad (4.8)$$

Proof: By definition

$$M_X(t) = E(e^{tx}) = \int_0^\infty e^{tx} f(x)dx$$

Using Taylor series

$$M_X(t) = \int_0^\infty\left(1 + tx + \frac{(tx)^2}{2!} + \cdots\right)f(x)dx$$

$$\Rightarrow \quad M_X(t) = \sum_{r=0}^\infty \frac{t^r}{r!}\int_0^1 x^r f(x)dx$$

$$\Rightarrow \quad M_X(t) = \sum_{i=0}^\infty \frac{t^r}{r!}E(X^r)$$

$$\Rightarrow \quad M_X(t) = \sum_{r=0}^\infty \frac{t^r}{r!}\alpha^r\lambda^r\Gamma(1-r)$$

F. This Completes the Proof

Characteristic Function

In this sub section we derived the Characteristic function of EIE distribution.

Theorem 4.6: Let X have a EIE distribution. Then characteristic function of X denoted by $\phi_X(t)$ is given by:

$$\varphi_X(t) = \sum_{r=0}^\infty \frac{(it)^r}{r!}\alpha^r\lambda^r\Gamma(1-r) \quad (4.9)$$

Proof: By definition

$$\varphi_X(t) = E(e^{itx}) = \int_0^1 e^{itx} f(x)dx$$

Using Taylor series

$$\varphi_X(t) = \int_0^\infty\left(1 + itx + \frac{(itx)^2}{2!} + \cdots\right)f(x)dx$$

$$\Rightarrow \quad \varphi_X(t = \sum_{r=0}^\infty \frac{(it)^r}{r!}\int_0^\infty x^r f(x)dx$$

$$\Rightarrow \quad \varphi_X(t) = \sum_{r=0}^\infty \frac{(it)^r}{r!}E(X^r)$$

$$\Rightarrow \quad \varphi_X(t) = \sum_{r=0}^\infty \frac{(it)^r}{r!}\alpha^r\lambda^r\Gamma(1-r)$$

This completes the proof.

V. Order Statistics

In this section, we derive closed form expressions for the pdfs of the k[th] order statistic of the EIE distribution. In statistics, the k[th] order statistic of a statistical sample is equal to its k[th] smallest value. Together with rank statistics, order statistics are among the most fundamental tools in non-parametric statistics and inference. We know that if $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ denotes the order statistics of a random sample $X_1, X_2, \ldots, X_n$ from a continuous population with cdf $F_X(x)$ and pdf $f_X(x)$, then the pdf of kth order statistics $X_{(k)}$ is given by

$$f_{X\,(k)}(x) = \frac{n!}{(k-1)!(n-k)!} f_X(x)(F_X(x))^{k-1}(1-F_X(x))^{n-k} \tag{5.1}$$

Substituting equation (2.3) and equation (2.4) in equation (5.1), we get pdf of k[th] order statistic given as

$$f_{X\,(k)}(x) = \frac{n!}{(k-1)!(n-k)!}\frac{\alpha\lambda}{x^2}\left(e^{\frac{-\alpha\lambda}{x}}\right)^k\left(1-e^{\frac{-\alpha\lambda}{x}}\right)^{n-k} \tag{5.2}$$

Note that at $\alpha = 1$, (5.2) yields the pdf of the k[th] order statistic of IE distribution.

Therefore, the pdf of the first (smallest) order statistic $X_{(1)}$ is given by

$$f_{X\,(1)}(x) = \frac{n\alpha\lambda}{x^2}\left(e^{\frac{-\alpha\lambda}{x}}\right)\left(1-e^{\frac{-\alpha\lambda}{x}}\right)^{n-1} \tag{5.3}$$

and the pdf of the largest order statistic $X_{(n)}$ is given by

$$f_{X\,(n)}(x) = \frac{n\alpha\lambda}{x^2}\left(e^{\frac{-\alpha\lambda}{x}}\right)^n \tag{5.4}$$

VI. Estimation of Parameters

In a view to estimating the parameters of the EIE distribution, we make use of the method of Maximum Likelihood Estimation (MLE). Let $\underline{x} = (x_1, x_2, \ldots, x_n)$ be a random sample having probability density function (2.3), and then the likelihood function is given by

$$L(\underline{x}) = \alpha^n \lambda^n \prod_{i=1}^{n} \frac{1}{x_i^2}\left\{e^{-\frac{\lambda}{x_i}}\right\}^{\alpha} \tag{6.1}$$

By taking logarithm of (6.1), we find the log likelihood function as

$$\ln L(\underline{x}) = n\ln\alpha + n\ln\lambda - 2\sum_{i=1}^{n}\ln x_i - \sum_{i=1}^{n}\left(\frac{\alpha\lambda}{x_i}\right) \tag{6.2}$$

Taking the partial derivatives of the log-likelihood function in (6.2) with respect to the parameters α and λ yields:

$$\frac{\partial \ln L(\underline{x})}{\partial\alpha} = \frac{n}{\alpha} - \frac{\lambda}{\sum_{i=1}^{n} x_i} = 0 \tag{6.3}$$

$$\frac{\partial \ln L(\underline{x})}{\partial\lambda} = \frac{n}{\lambda} - \frac{\alpha}{\sum_{i=1}^{n} x_i} = 0 \tag{6.4}$$

Setting equations (6.3) and (6.4) to zero and solving the maximum likelihood estimates of the two parameters can be obtained as

$$\hat{\alpha} = \frac{n}{\lambda\sum_{i=1}^{n} x_i^{-1}} \tag{6.5}$$

$$\hat{\lambda} = \frac{n}{\alpha\sum_{i=1}^{n} x_i^{-1}} \tag{6.6}$$

VII. Application

In this section, we use a real data set to show that the EIE distribution can be a better model than the EIR distribution. We consider a data set corresponding to remission times (in months) of a random sample of 128 bladder cancer patients given in Lee and Wang (2003). The data set is given as follows : 0.08, 2.09, 2.73, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.22, 3.52, 4.98, 6.99, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 15.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.10, 1.46, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 3.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.93, 8.65, 12.63 and 22.69. These data are used here only for illustrative purposes. The required numerical evaluations are carried out using the Package of R software.

We have fitted EIR and EIE models to this data. These two distributions are fitted to the subject data using maximum likelihood estimation. The MLEs of the parameters with standard errors in parentheses and the corresponding log-likelihood values, AIC, AICC and BIC are displayed in Table 1.

Table I: MLEs (S.E. In Parentheses) And Criteria For Comparison

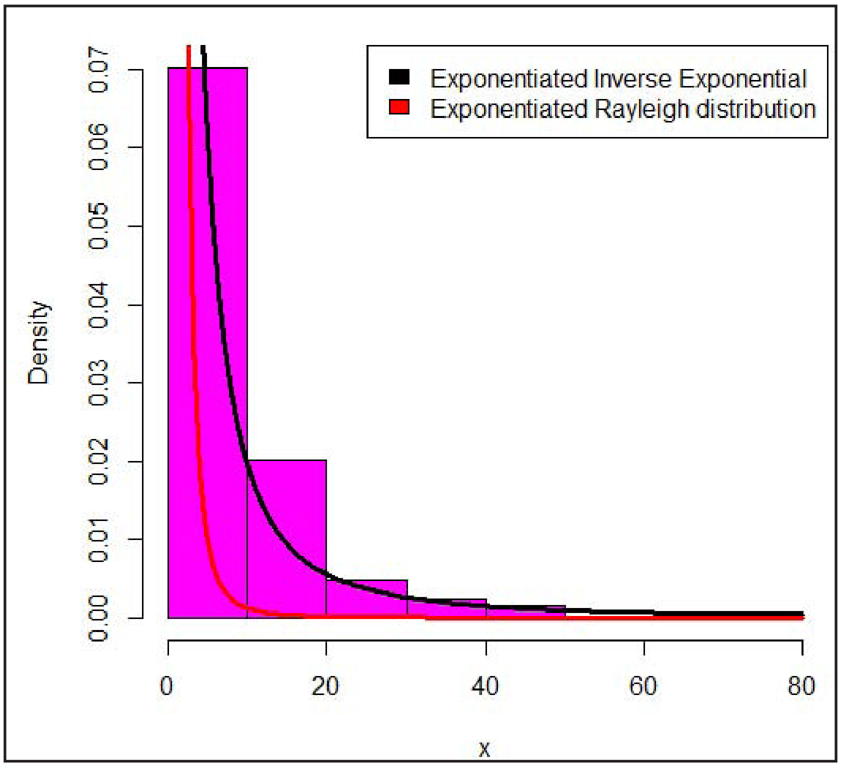
Distribution	Parameter Estimates		-2Log L	AIC	AICC	BIC
	α	λ				
Exponentiated Inverse Exponential (EIE)	1.666803 (117.611)	1.510827 (106.605)	883.4167	887.4167	893.2168	893.1208
Exponentiated Inverse Rayleigh (EIR)	0.8107348 (28.8669)	0.7622493 (27.1406)	1497.54	1501.54	1507.34	1507.244

In order to compare the two distribution models, we consider the criteria like AIC (Akaike information criterion), AICC (corrected Akaike information criterion) and BIC (Bayesian information criterion). The better distribution corresponds to lesser AIC, AICC and BIC values.

$$\mathrm{AIC} = 2k - 2\log L \quad \mathrm{AICC} = \mathrm{AIC} + \frac{2k(k+1)}{(n-k-1)}$$

$$\text{and } \mathrm{BIC} = k\log n - 2\log L$$

where k is the number of parameters in the statistical model, n is the sample size and - 2log$L$ is the maximized value of the log-likelihood function under the considered model. From Table 1, it has been observed that the Exponentiated inverted exponential distribution have the lesser AIC, AICC and BIC values as compared to Exponentiated Inverse Rayleigh (EIR). Hence we can conclude that the Exponentiated inverted exponential distribution leads to a better fit than the Exponentiated Inverse Rayleigh (EIR) distribution.



VIII. Conclusion

In this paper the Exponentiated inverted exponential distribution has been introduced and studied various properties of the distribution. The median, mode, survival function, hazard function and the maximum likelihood estimates of the parameters, have been investigated. The application of the distribution has also been demonstrated with real life data. It has been observed that Exponentiated inverted exponential distribution provides a better fit than the Exponentiated Inverse Rayleigh (EIR) distribution.

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