ISSN.: 2321-6115

The Effect of Scale Ranges on Priorities and Discrimination Level of Alternatives in Analytic Hierarchy Process (AHP)

Mohammad Azadfallah
[1]Researcher, business Studies and Development Office, Saipa Yadak (Saipa After sale services), Islamic Republic of Iran. Email: m.azadfallah@yahoo.com

Abstract: In this paper, we first reviews different measurement scales (Linear, Power, Geometric, Logarithmic, Root square, Inverse linear, and Balanced) adopted in Analytic Hierarchy Process (AHP). then, with reduction of different measurement scale ranges to: left position (i.e., for linear measurement scale: 1-3), middle position (4-6), right position (7-9), left & middle position (1-6), middle & right position (4-9), and perfect ranges (1-9), the effects of different measurement scale on priorities and discrimination level (to discriminate an important alternative from others) of alternatives are investigated. The findings of this paper reveal that first, in 39 possibilities out of 42 cases, the same ranking (A1>A2>A3) with different intensities were obtained, and in 3 possibilities rank reversal are happened. Next, the geometric measurement scale in all ranges and particularly in perfect range have the best performance in discriminating an important alternative than others. Moreover, only the left position and perfect ranges in the most of measurement scales have the best performance in discriminating an important alternative from others.

Keywords: Analytic Hierarchy Process (AHP), Measurement scale, Scale ranges, Discrimination level.

I. Introduction

Multi criteria Decision Making (MCDM) is one of the most widely use decision methodologies in the sciences, business, and engineering worlds (Wang, 2007). The multi-criteria aspect of decision analysis appears because outcomes must be evaluated in terms of several objectives. These are stated in terms of properties, rather desirable or undesirable that determines the decision maker's preferences for the outcomes (Liu and Hai, 2005). MCDM mainly consists of the following two parts: 1. collect decision information. The decision information generally includes the attribute weights and the attribute values. In a MCDM problem, there are generally a finite set of alternatives and a collection of attributes. The attributes are the indices used to measure the given alternatives, and each attribute has its importance, which is to be determined in the process of decision-making. The attribute values are usually the measure values for the alternatives with respect to each attribute, which mainly take the form of real numbers, interval numbers, triangular fuzzy numbers, intuitionistic fuzzy numbers and linguistic variables, etc. 2. Aggregate the decision information through some proper approach and then rank or select more of the alternatives. In other words, MCDM models are used for evaluating, ranking and selecting the most appropriate alternative from among several alternatives (Azadfallah, 2016). In literature, a number of techniques have been developed to deal with evaluating / selecting alternatives. In this paper, AHP that is one of these methods is discussed. AHP is a powerful tool, which is used to deal with complex decision-making problems. Each problem can be broken down into several levels where each level presents a set of criteria or sub-criteria or alternatives. Constituting the hierarchical structure, the pair wise comparison matrices are made for each level in order to generate the final priorities for the alternatives (Mirhedayatian et al., 2011). In other words, Analytic Hierarchy Process (AHP) is a theory of measurement concerned with deriving dominance priorities from paired comparisons of homogeneous elements with respect to a common criterion or attribute (Saaty, 2000). Pair wise comparisons are fundamental in the use of the AHP. The value defined on a scale is used to represent the relative importance between two compared objects in terms of ratio (Ji and Jiang, 2003). In Saaty's AHP, the verbal statements are converted into integers from one to nine. Theoretically, there is no reason to be restricted to these numbers. Therefore, other scales have been proposed (Ishizaka et al., 2011). On the other side, one of the principal purposes of the AHP is to quantify the degree of importance for each alternative and to discriminate an important alternative from others (Sato, 2001). Therefore, the aim of this paper is to show the effect of scale ranges on priorities and discrimination level of alternatives in AHP.

The paper is organized as follow. In section 2; AHP, section 3; measurement scale, and section 4; literature is reviewed. Numerical example is provided in section 5; the paper is concluded in section 6.

II. Analytic Hierarchy Process (AHP)

A brief discussion of AHP is provided in this section. Interested readers may refer to Saaty (2000) for more discussions on the existing approaches. The AHP method uses the pair wise comparisons and eigenvector methods to determine the a_{ij} values and also the criteria weights W_j. in this method; a_{ij} represents the relative value of alternative A_i when it is considered in terms of criterion C_j. in the original AHP method, the a_{ij} values of the decision matrix need to be normalized vertically. That is, the elements of each column in the decision matrix add up to 1. In this way, values with various units of measurement can be transformed into dimensionless ones. If all the criteria are benefit criteria (that is, the higher the score the better the performance is), then according to the original AHP method, the best alternative is the one that satisfies the following expression:

$$P^*_{AHP}=\max_i P_i=\max_i \sum_{j=1}^{n} a_{ij}W_j, \text{ for } i=1, 2, 3\ldots m.$$

From the above formula, it can be seen that the original AHP method uses an additive expression to determine the final priorities of the alternatives in terms of all the criteria simultaneously (Wang, 2007). Generally, the purpose of the AHP is to assist people in organizing their thoughts and judgments to make more effective decisions (Saaty, 2000).

III. Measurement Scales

Measurement is the assignment of numbers to objects or events according to rule. The rule of assignment can be any consistent rule. The only rule not allowed would be random assignment, for randomness amounts in effect to a non-rule (Luce, 1997). According to Stevens's categorization, there are four levels of measurement. The levels, ranging from lowest to highest are nominal, ordinal, interval, and ratio. Each level has all of the meaning of the levels below plus additional meaning (Forman and Selly, 2001). When magnitudes can be identified but information about their order is lacking, one has nominal scales. In this case, numerals are used only as names. If one can order magnitudes but cannot determine the differences between magnitudes, one has ordinal scales. If one knows the differences but cannot determine the ratios between magnitudes, one has interval scales. Finally, if one knows the ratios between magnitudes, one has ratio scales (Masin, 2006).

A commonly used measurement scales in the AHP is the ratio scale (Vachajitpan, 2004). Perhaps the most significant aspect of the AHP is its use of ratio scales (Saaty, 2000). So that, In a judgment matrix, instead of assigning two numbers W_i and W_j (that generally we do not know), as one does with tangibles, and forming the ratio W_i / W_j we assign a single number drawn from the fundamental scale of absolute numbers shown in table 1, to represent the ratio $(W_i / W_j)/1$, (Saaty, 2005). Theoretically, there is no reason to be restricted to these numbers. Therefore, other scales have been proposed (table 2), (Ishizaka et al., 2011).

Table I: The Fundamental Scale of Absolute Numbers

Intensity of Importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective.
2 3	Weak Moderate importance	Experience and judgment Slightly favor one activity over another.
4 5	Moderate plus Strong importance	Experience and judgment Strongly favor one activity over another.
6 7	Strong plus Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice.
8 9	Very, very strong Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation.
Reciprocals of above	If activity I has one of the above non zero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i	A reasonable assumption.
Rationales	Ratio arising from the scale	If consistency were to be forced by obtaining n numerical values to span the matrix.

Ref. Saaty (2005), p. 356.

TABLE II: DIFFERENT MEASUREMENT SCALES

Scale	Definition	Parameters
linear	C=a.x	a>0;x=1,2,…,9
power	$C=x^a$	a>1;x=1,2,…,9
geometric	$C=a^{x-1}$	a>1;x=1,2,…,9
logarithmic	$C=\log_a^{(x+1)}$	a>1;x=1,2,…,9
Root square	$C=\sqrt[a]{x}$	a>1;x=1,2,…,9
Inverse linear	C=9/(10-x)	a>1;x=1,2,…,9
balanced	C=w/(1-w)	W=0.5,0.55,0.6,…,0.9

Ref. Ishizaka et al., (2011), p. 4.

Generally, Evaluating of the effect of different measurement scale range alteration on priorities and discrimination level of alternatives in AHP is the aim of this paper.

IV. Literature Review

According to the view point proposed by Dong et al., (2008), in the AHP, a decision maker first gives linguistic pair wise comparisons, then obtains numerical pair wise comparisons by selecting certain numerical scale to quantify them, and finally derives a priority vector from the numerical pair wise comparisons. So, in Saaty's AHP the verbal statements are converted into integers from one to nine. As noted earlier, theoretically there is no reason to be restricted to these numbers. Therefore, other scales have been proposed (Ishizaka et al., 2011). In the current literature, many studies exist on measurement scale in the AHP. Saaty (2001), Ji and Jiang (2003), Saaty (2004), Vachajitpan (2004), Wedley (2007), Dong et al. (2008), and Cox (2009) could be referred to as an example. In addition, some related work about of the different measurement scales in literature presented. For instance; Triantaphyllou et al. (1994) used two evaluative criteria (1. ranking inversions, and 2. ranking indiscrimination), to examine 78 scales that can be derived from two widely used scales (the original Saaty scale and exponential scales). The findings in this study reveal that there is no single scale that can outperform all the other scales. Webber et al. (1996) reported the results of three related experiments investigating whether differences in the scale used (numerical, verbal, and graphical), format (random, and non-random), and order (top-down, and bottom-up) of paired comparisons, yields significant differences in the AHP models. Poyhonen et al. (1997) performed a comparative study in which subjects were requested to quantify verbal ratio statements by adjusting the heights of visually displayed bars. Subjects were also asked to employ verbal expressions in pair wise comparisons of areas of figures with different shapes. Their results indicate that there are alternative numerical scales (verbal preference statements) which yield more accurate estimates than the usual 1-to-9 scale and reduce the inconsistency of the comparison matrices. Salo and Hamalainen (1997) proposed new balanced scales to improve the sensitivity of the AHP ratio scales. Ishizaka et al. (2011) studied the influence of aggregation and measurement scale on ranking a compromise alternative in AHP> six different measurement scales (linear, power, geometric, root square, inverse linear, and balanced), and the new proposed logarithmic scale are considered for applying the additive and the multiplicative aggregation techniques. Their results indicated that the aggregation method of local priorities and the measurement scale in AHP has a strong influence on the selection of the compromise and therefore on the degree of concordance with the utility theory. Nevertheless, a few experimental studies have dealt with the purpose of this paper. For instance; Sato (2001). This study evaluated the two-measurement scale: 1. linear and 2. Power, based on three aspects: 1. the consistency index, 2. rank reversal and 3. Discrimination sensitivity in AHP. From statistical viewpoint (at %1 significant difference level in X^2-test), the discriminating sensitivity (for largest and second largest element) presented. The result indicated that, the power scale is superior to linear scales. In addition, in Azadfallah (2016), we present a comparative analysis of different measurement scales adopted in Analytic Hierarchy Process (AHP), by testing them versus a problem with a known composite answers. Then experimentally, the impact of the different measurement scale elements alteration from three aspects: 1. the limited scale upper bound (up to 9), 2. Changing the scale parameters (a parameters), and 3. Changing the system numbers (from 1, 3…9; to 2, 4…10) on priorities are investigated. The results are shown that, the linear measurement scale have the best performance in compare to another scales.

It is also worth noting here that, the described idea in this paper is inspired of Birnbaum (1998, p. 24), who argue that, stimuli at the edges of an experimental range are much better identified than stimuli in the middle. Therefore, in this paper, we shall focus on the scale range alteration and their effects on priorities and discriminating level of alternatives in AHP.

V. Numerical Example

In this section, the reduction of the linear measurement scale ranges to: left position (1-3), middle position (4-6), right position (7-9), left & middle position (1-6), middle & right position (4-9), and perfect ranges (1-9), was used as the standard. Therefore, different measurement scale range reductions, is based on linear measurement scales performed. Then, via numerical example, the effect of different measurement scale ranges on priorities and discriminating level of alternatives, are investigated. Moreover, according to the viewpoint proposed by Saaty (2000, p. 455), in the AHP one needs to be careful with criteria measured on the same absolute scale. Criteria measured in dollars are a common example of this. The priority of each criterion must be equal to the sum of the measurements of its alternatives divided by the sum of the measurements of the alternatives with respect to all these criteria. Only then, can one normalize the measurements of the alternatives, weight them by these priorities and add to obtain the relative weights of the alternatives with respect to all these criteria. Since, with assumes that, all of the criteria are expressed in the same unit, a simple 3.3 example (i.e. for linear measurement scale, 1-9) is presented (table 3-8). In addition, a comparison of the different measurement scales (based on the formula in table 2) is given in table 9.

TABLE III: DECISION MATRIX* - LINEAR SCALE, LEFT POSITION (1-3)

Cri. / Alt.	C1	C2	C3	Total
A1	1	3	3	7
A2	2	1	3	6
A3	1	3	1	5
total	4	7	7	18

* The all of the criteria is benefit type (in all tables).

TABLE IV: DECISION MATRIX - LINEAR SCALE, MIDDLE POSITION (4-6)

Cri. / Alt.	C1	C2	C3	total
A1	4	6	6	16
A2	5	4	6	15
A3	4	6	4	14
total	13	16	16	45

TABLE V: DECISION MATRIX - LINEAR SCALE, RIGHT POSITION (7-9)

Cri. / Alt.	C1	C2	C3	Total
A1	7	9	9	25
A2	8	7	9	24
A3	7	9	7	23
total	22	25	25	72

TABLE VI: DECISION MATRIX - LINEAR SCALE, LEFT & MIDDLE POSITION (1-6)

Cri. / Alt.	C1	C2	C3	total
A1	1	4	6	11
A2	2	5	3	10
A3	1	3	5	9
total	4	12	14	30

TABLE VII: Decision Matrix- Linear Scale, Middle & Right Position (4-9)

Cri. / Alt.	C1	C2	C3	total
A1	4	7	9	20
A2	5	8	6	19
A3	4	6	8	18
total	13	21	23	57

TABLE VIII: DECISION MATRIX - LINEAR SCALE, PERFECT RANGES (1-9)

Cri. / Alt.	C1	C2	C3	total
A1	1	5	9	15
A2	2	7	5	14
A3	6	5	2	13
total	9	17	16	42

TABLE IX: DIFFERENT SCALES FOR COMPARING TWO ALTERNATIVES

Scale type	**values**								
linear	1	2	3	4	5	6	7	8	9
power	1	4	9	16	25	36	49	64	81
geometric	1	2	4	8	16	32	64	128	256
Logarithmic	1	1.58	2	2.32	2.58	2.81	3	3.17	3.32
Root square	1	1.41	1.73	2	2.23	2.45	2.65	2.83	3
Asymptotical*	1	0.12	0.24	0.36	0.46	0.55	0.63	0.70	0.76
Inverse linear	1	1.13	1.29	1.5	1.8	2.25	3	4.5	9
Balanced	1	1.22	1.5	1.86	2.33	3	4	5.67	9

Ref. Ishizaka and Labib (2009), p. 209.

In this section, evaluating of the effect of different measurement scale range alteration on priorities and discrimination level of alternatives is considered.

The Effect Of Different Measurement Scale Range Alteration On Priorities

Here, we concentrate our attention on the effect of different measurement scale range alteration on priorities. A comparison of the test results is given in table 11. i.e. for linear measurement scale (table 10, based on table 3):

$$A1=(0.250*0.222)+(0.429*0.389)+(0.429*0.389)=0.389$$

TABLE X: AHP RESULTS FOR LINEAR SCALE, LEFT POSITION (1-3, TABLE3)

Cri. / Alt.	C1 4/18 =.222	C2 7/18 =.389	C3 7/18 =.389	Composite priorities
A1	0.250	0.429	0.429	0.389

A2	0.500	0.143	0.429	0.333
A3	0.250	0.429	0.143	0.278

Notes: According to axiom 3 of AHP, the criteria are assumed to be independent of the alternatives (Wedley, 2001). Here, we violated of this properties.

Discrimination Level of Alternatives (D.L.A.)

As noted earlier, according to the viewpoint proposed by Sato (2001), one of the principal purposes of the AHP is to quantify the degree of importance for each alternative and to discriminate an important alternative from others. Therefore, in this section, the discrimination level of the important alternative from others is considered (table 12). i.e. for linear measurement scale, left position (based on table 11):

$D.L._{1,2}$ = A1-A2 = 0.389-0.333=0.056

$D.L._{1,3}$ = A1-A3 = 0..389-0.278=0.111

Table XI: AHP Results For Linear Measurement Scales In Different Ranges

Scale Range	Priorities (Rank and Intensity)
left position (1-3)	A1 > A2 > A3 .389 .333 .278
middle position (4-6)	A1 > A2 > A3 .356 .333 .311
right position (7-9)	A1 > A2 > A3 .347 .333 .319
left & middle position (1-6)	A1 > A2 > A3 .429 .214 .357
middle & right position (4-9)	A1 > A2 > A3 .351 .333 .316
perfect ranges (1-9)	A1 > A2 > A3 .357 .333 .310

Table XII: A Discrimination Level for Important Alternative from Others

Scale Range	Discrimination Level of Alternatives
left position (1-3)	$D.L._{1,2}$ = A1-A2 = 0.389-0.333=0.056 $D.L._{1,3}$ = A1-A3 = 0.389-0.278=0.111
middle position (4-6)	$D.L._{1,2}$ =0.023 $D.L._{1,3}$ =0.045
right position (7-9)	$D.L._{1,2}$ =0.014 $D.L._{1,3}$ =0.028
left & middle position (1-6)	$D.L._{1,2}$ =0.034 $D.L._{1,3}$ =0.067
middle & right position (4-9)	$D.L._{1,2}$ =0.018 $D.L._{1,3}$ =0.035
perfect ranges (1-9)	$D.L._{1,2}$ =0.024 $D.L._{1,3}$ =0.047

A. Findings:

As seen from the table, the linear measurement scale in different range results are the same priorities (same ranking with slightly different intensities), as displayed in the last column of table 11.

The results (table 12-fir linear measurement scale) indicate that, the discriminating level in both $D.L._{1,2}$ and $D.L._{1,3}$ for left position is larger and inversely, for right position are lesser than others. So, we can say (for left position) that alternative A1 is clearly preferred to A2 and A3, and Vice-versa, we cannot say (for right position) that alternative A1 is clearly preferred to A2 and A3.

A comparison of the test results is given in table 13.

Table XIII: AHP Results For Different Measurement Scales

Range / Scale Type	-	Left Position	Middle Position	Right Position	Left & Middle Position	Middle & Right Position	Perfect Ranges
linear (1-9)	Rank	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3
	Intensity	.389 .333 .278	.356 .333 .311	.347 .333 .319	.367 .333 .300	.351 .333 .316	.357 .333 .310
	D.L.	0.056, 0.111	0.023, 0.045	0.014, 0.028	0.034, 0.067	0.018, 0.035	0.024, 0.047
power (1-81)	Rank	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3
	Intensity	.432 .318 .250	.378 .330 .292	.361 .332 .307	.431 .302 .278	.377 .323 .300	.428 .312 .260
	D.L.	0.114, 0.182	0.048, 0.086	0.029, 0.054	0.129, 0.153	0.054, 0.077	0.116, 0.168
geometric (1-256)	Rank	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3
	Intensity	.409 .318 .273	.409 .318 .273	.409 .318 .273	.488 .262 .250	.488 .262 .250	.674 .202 .123
	D.L.	0.091, 0.136	0.091, 0.136	0.091, 0.136	0.226, 0.238	0.226, 0.238	0.472, 0.551

Range / Scale Type	-	**Left Position**	**Middle Position**	**Right Position**	**Left & Middle Position**	**Middle & Right Position**	**Perfect Ranges**
Logarithmic (1-3.32)	Rank	A1>A2>A3	A1>A2>A3	A1>A2>A3	**A2>A1>A3**	A1>A2>A3	**A2>A3>A1**
	Intensity	.368 .337 .295	.344 .334 .323	.339 .334 .328	.345 .343 .312	.339 .336 .325	.340 .331 .328
	D.L.	0.031, 0.073	0.010, 0.021	0.005, 0.011	0.002, 0.033	0.003, 0.014	0.009, 0.012
Root square (1-3)	Rank	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3	**A2>A1>A3**
	Intensity	.362 .336 .303	.344 .333 .322	.340 .333 .326	.345 .340 .314	.341 .335 .324	.338 .335 .327
	D.L.	0.026, 0.059	0.011, 0.022	0.007, 0.014	0.005, 0.031	0.006, 0.017	0.003, 0.011
Inverse Linear (1-9)	Rank	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3
	Intensity	.348 .332 .320	.357 .330 .313	.400 .314 .286	.364 .323 .313	.446 .282 .272	.515 .259 .226
	D.L.	0.016, 0.028	0.027, 0.044	0.086, 0.114	0.041, 0.051	0.164, 0.174	0.256, 0.289
Balanced (1-9)	Rank	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3	A1>A2>A3
	Intensity	.357 .332 .312	.361 .330 .309	.381 .324 .295	.372 .321 .307	.408 .302 .289	.467 .286 .248
	D.L.	0.025, 0.045	0.031, 0.052	0.057, 0.086	0.051, 0.065	0.106, 0.119	0.181, 0.219

B. Findings:

a. For The Effect of Different Measurement Scale Range Alteration on Priorities

- As seen from the table (table 13), in 39 possibilities out of 42 cases, are the same ranking (A1>A2>A3), with different intensities. Except geometric scale with the left, middle and right position (A1>A2>A3, 0.409>0.318>0.273), and left & middle and middle & right position (A1>A2>A3, 0.488>0.262>0.250), which both of ranking and intensities are the same.
- In 3 possibilities out of 42 cases, rank reversal is happened, and A2 ranking is best (the left & middle position and perfect ranges for logarithmic and only perfect range for root square measurement scale).

b. For Discrimination Level of Alternatives (D.L.)

- For linear measurement scale, the discriminating level in both $D.L._{1,2}$ and $D.L._{1,3}$ for left position is larger and inversely, for right position are lesser than others.
- For power measurement scale, $D.L._{1,2}$ for left & middle position and $D.L._{1,3}$ for left position is superior and inversely, for right position are inferior to others.
- For geometric measurement scale, the discriminating level in both $D.L._{1,2}$ and $D.L._{1,3}$ for perfect ranges is larger and inversely, for the left, middle and right positions are lesser than others.
- For logarithmic measurement scale, the discriminating level in both $D.L._{1,2}$ and $D.L._{1,3}$ for left position is superior and inversely, for $D.L._{1,2}$ in left & middle position and $D.L._{1,3}$ in right position are inferior to others.
- For root square measurement scale, the discriminating level in both $D.L._{1,2}$ and $D.L._{1,3}$ for left position is larger and inversely, for the perfect range are lesser than others.
- For inverse linear measurement scale, the discriminating level in both $D.L._{1,2}$ and $D.L._{1,3}$ for perfect ranges is larger and inversely, for the left positions are smaller than others.
- For balanced measurement scale, the discriminating level in both $D.L._{1,2}$ and $D.L._{1,3}$ for perfect ranges is superior and inversely, for the left position are inferior to others.

In general, as seen from the table 13, the geometric measurement scale in both $D.L._{1,2}$ and $D.L._{1,3}$ for perfect ranges is larger than other scale and positions. And inversely, for $D.L._{1,2}$, the logarithmic measurement scale in left & middle and for $D.L._{1,3}$, the logarithmic measurement scale in right position and root square measurement scale in perfect ranges, is lesser than other scale and positions.

VI. Concluding Remarks

In this paper, we are focusing on the different measurement scale (Linear, Power, Geometric, Logarithmic, Root square, Inverse linear, and Balanced) range alteration (to left position (i.e., for linear measurement scale: 1-3), middle position (4-6), right position (7-9), left & middle position (1-6), middle & right position (4-9), and perfect ranges (1-9)), and their effects on priorities and discrimination level of alternatives in AHP. The result are shown that, in 39 possibilities out of 42 cases, the same ranking (A1>A2>A3) with different intensities were obtained, and in 3 possibilities rank reversal are happened. The geometric measurement scale in all ranges and particularly in perfect range has the best performance in discriminating an important alternative from others in compare to another scale and positions. Furthermore, in all of the measurement scales, the left position (except inverse linear and balanced measurement scales) and perfect range (except linear, logarithmic and root square measurement scales) are the very best performance in discriminating of an important alternative than others. In other words, we can say that an alternative with first rank is clearly preferred to other alternatives. In sum, finding in this paper can help the AHP decision makers to select a suitable scale for best performance in both priority and discrimination level of alternatives.

Reference

[1] M. Azadfallah, "Marketing strategy selection by interval TOPSIS under incomplete data," *XIBA Business Review*, vol. 2, no. 2, pp. 29-38, 2016.

[2] M. H. Birnbaum, "*Measurement, Judgment, and decision making*," Academic Press, 1998.

[3] M. A. A. "Multidimensional scaling as an aid for the Analytic Network and Analytic Hierarchy Process," *Journal of Data Science*, vol. 7, pp. 381-396, 2009.

[4] Y. Dong, Y. Xu, H. Li, and M. Dai, "A comparative study of the numerical scales and the prioritization methods in AHP," *European Journal of Operational Research*, vol. 186, no. 1, pp. 229-242, April, 2008.

[5] E. Forman, and A. Selly, "Decision by objectives," World Scientific Press, 2001..

[6] A. Ishizaka, and A. Labib, "Analytic hierarchy process and expert choice: Benefits and limitations," *OR Insight*, vol. 22, no. 4, pp. 201-220, 2009.

[7] A. Ishizaka, D. Balkenborg, and T. Kaplan, "Influence of aggregation and measurement scale on ranking a compromise alternative in AHP," *The Journal of the Operational Research Society*, vol. 62, no. 4, pp. 700-710, 2011.

[8] P. Ji, and R. Jiang, "Scale transitivity in the AHP," *The Journal of the Operational Research Society*, pp. 54, no. 8, pp. 896-905, 2003.

[9] F. H. F. Liu, and H. L. Hai, The voting Analytic Hierarchy Process method for selecting supplier," *International Journal Production Economics*, vol. 97, no. 3, pp. 308-317, 2005.

[10] R. D. Luce, "Quantification and symmetry," *British Journal of Psychology*, vol. 88, no. 3, pp. 395-398, 1997.

[11] S. C. Masin, "Norman Robert Campbell, A neglected forerunner in sensory measurement," *Proceedings of the Twenty-Second Annual Meeting of the International Society for Psychophysics*, UK, vol. 22, pp. 217-222, 2006.

[12] S. M. Mirhedayatian, M. Jafarian, and R. F. Saen, "A multi-objective slack based measure of efficiency model for weight derivation in the Analytic Hierarchy Process," *Journal of Applied Sciences*, vol. 11, no. 19, pp. 3338-3350, 2011.

[13] M. A. Poyhonen, R. P. Hämäläinen, and A. A. Salo, "An experiment on the numerical modeling of verbal ratio statements," *Journal of Multi Criteria Decision Analysis*, vol. 6, no. 1, pp. 1-10, 1997.

[14] T. l. Saaty, "*Fundamentals of decision making and priority theory*", RWS Publication, vol. 6, 2000.

[15] T. L. Saaty, T. L. "Deriving the AHP 1-9 scale from first principles," *Proceedings of the 6th ISAHP 2001 Bern, Switzerland*, pp. 397-401, August, 2001.

[16] T. L. Saaty, "Scales from measurement not measurement from scales", *MCDM 2004, Whistler, B.C., Canada*, pp. 6-11, August, 2004.

[17] Saaty T. L. (2005). The Analytic Hierarchy and Analytic Network Processes for the measurement of intangible criteria and for decision making", in Multi Criteria Decision Analysis: state of the art survey (Figueira et al., Eds), Kluwer academic publisher, 345-406.

[18] A. Salo, and R. Hamalainen, "On the measurement of preferences in the Analytic hierarchy Process," *Journal of Multi Criteria Decision Analysis*, vol. 6, no. 6, pp. 309-319, 1997.

[19] Y. Sato, "The impact on scaling on the pair wise comparison of the Analytic Hierarchy Process", *International Symposium on the Analytic Hierarchy Process, Berns Switzerland,* pp. 421-430, August, 2001.

[20] E. Triantaphyllou, F. A. Lootsma, P. M. Pardalos, S. H. Mann, "On the evaluation and application of different scales for quantifying pair wise comparisons in fuzzy sets," *Journal of Multi Criteria Decision Analysis*, vol. 3, no. 3, pp. 133-155, 1994.

[21] P. Vachajitpan, "Measurements scales and derivation of priorities in pair wise and group decision making", *MCDM 2004, Whistler, B.C., Canada*, pp. 1-6, August, 2004.

[22] X. Wang, "Study of ranking irregularities when evaluating alternatives by using some ELECTRE methods and a proposed new MCDM method based on regret and rejoicing", *MSc. Thesis, Louisiana State University, USA*, 2007.

[23] S. Webber, and F. Eisenfuhr, "The sensitivity of the Analytic Hierarchy Process to alternative scale and cue presentations," *European Journal of Operational Research*, vol. 96, no. 2, pp. 351-362, 1996.

[24] W. C. Wedley, "AHP answers to problems with known composite values," *International Symposium on the Analytic Hierarchy Process,* Berns Switzerland, pp. 551-560, August, 2001.

[25] W. C. Wedley, "AHP/ANP- Where is natural zero?," *International Symposium on the Analytic Hierarchy Process,* VinaDel Mar, Chile, pp. 1-15, August, 2007.