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Study of Solutions to Occurrences of Random Variables using PCM Analogies over 'Three Cups' Problem

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Abstract – In this report an attempt was taken to prove that we can have multiple solutions for any type of problem. This may be related to our daily work or life. This report does not provide any scientific discovery. In this a simple situation was taken and solved by three different logics. Mathematics, Physics and Chemistry were used to solve this real life problem. So it is called 'THE ANALOGIES IN PCM'. The purpose of this document was to prove that the situation can be handled in a different way. The process of solution depends upon how we look in to the problem and solve it.

Keywords: Enantiomers, Diastereomers, Homomers, Stereochemistry

I. The Problem

The problem is a real life situation. You are given three cups. Out of those three cups which are placed on a table, two of them are facing upwards and one is facing downwards. You are asked to flip orientation of the cups. But there is a constraint. You are permitted to flip the orientation of any two cups at a time in one move. Prove that you cannot have all three cups facing upwards after any number of moves.

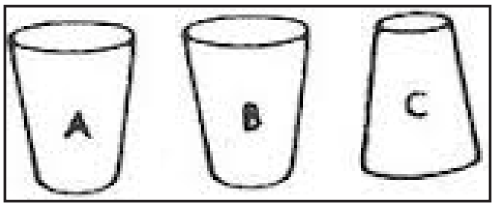

Fig. 1

1. Example

We had three cups, say cup A, cup B and cup C. In the initial scenario, say cup A and cup B are facing upwards and cup C is facing downwards with its bottom at the top (Fig. 1). So in one move cup A and cup B can be flipped and can be made to face downwards. So after the end of one valid move we have all three cups A, B and C facing downwards. But the question is that we need to prove that after any number of moves considered valid we cannot have all three cups facing upwards.

II. Mathematics Solution

I have tried to solve this problem by using the trivial concept of multiplication. Let us say we assign a value of +1 to all those cups which face upwards and a value of -1 to all those cups which face downwards. In the initial scenario two cups were facing upwards and one downwards. So we had two +1s and one -1. If we multiply the values which we have assigned to the three cups then we get $(+1) \times (+1) \times (-1) = (-1)$ (product). But we realize here that when we make a move considered valid we flip the configuration and we do that to a couple of cups. So when we do so, we do not change the value obtained by multiplying the numbers associated with the three cups. This is because during multiplication we convert a (+1) to a (–1) or a (–1) to a (+1) and that too twice. By changing the sign once we change the value of the product. But by changing the sign twice (which we are constrained to do as we are supposed to flip the configurations of a couple of cups) we do not change the value of the product. Now here is some food for thought. We must try to think why we cannot have all the three cups facing upwards……….

If you got that then it is quite great.

It is because; if all three cups were facing upwards then our product would be +1.

From the above discussion it is quite clear that we cannot obtain a +1 from a -1.

-1 shall always remain -1 subject to the constraints given above in the question.

Thus it is mathematically proven that there cannot be a situation in which there could be all the three cups facing upwards.

What is even better to conclude?

- At some point of time it is possible to have all the three cups facing downwards. (-1) x (-1) x (-1) = -1
- It is impossible to have one cup facing upwards and two cups facing downwards after any number of valid moves. (+1) x (-1) x (-1) = +1

III. Physics Solution

In physics, the term 'relative' carries a lot of meaning. We can solve this problem by calculating the relative changes between two subsequent moves. However, the solution also has some mathematical sense in it.

So here we go. Let us first assign the number +1 to all those cups facing upwards and -1 to all those cups facing downwards. Now let us consider the sum of numbers appearing on all cups. In the initial situation we had two cups facing upwards and one cup facing downwards. So our initial sum was (+1) + (1) – (1) =+1

If we change the orientation of one cup, we change the sum relatively by two units. It is because if +1 gets converted into -1, the sum decreases by 2 and if -1 gets converted into +1 the sum increases by 2. However owing to the constraint of the problem we are supposed to change the configurations of a couple of cups. Hence any of the three cases may happen during a valid move.

- The two flips in orientation may increase the sum by 2 each. Hence relative change is (+2) + (+2) = +4
- The two flips in orientation may decrease the sum by 2 each. Hence relative change is (-2) + (-2) = -4
- The two flips may cancel out the relative changes which is conversion of a +1 to a -1 and a -1 to a +1. Hence the relative change is (+2) + (-2) = 0

  In the initial situation the sum was +1. It can be understood that the changes occur as a multiple of 4. So we can have +1 – 4 = -3. This implies that there exists a situation in which we can have all three cups facing downwards. It can be here easily understood why we cannot have all three cups facing upwards. It is because the difference between +1 and +3 is 2 which is not a multiple of 4.

  Besides it is also important for us to realize that if we were given one cup facing up and two cups facing downwards, then we could have got three cups facing upwards. This is because (-1) + (+4) =+3. Was not that nice?

  Of course, after all it is physics and simple relative changes.

IV. Chemistry Solution

What the mathematics and physics solutions offered was nothing but simple operations of addition and multiplication which were used to solve the cup problem. People are not really surprised when they see physics and mathematics merged but if I say chemistry, then I am sure they will give a second thought. I am sure you would particularly enjoy the chemistry approach to this question. The fun is, we would be seeing how particularly stereochemistry can help.

To dig deeper into the solution it is important that the reader is clear on all the terminologies in stereochemistry. For example, enantiomers, diastereomers and homomers (identical compounds).

- A compound in which the central atom is attached to four different groups is called a chiral compound. A chiral compound is capable of rotating plane polarized light.
- Depending upon the direction of rotation of plane polarized light, we assign the signs + (plus) or – (minus) to the chiral compound.
- If we take a solution of a chiral compound in which there is an equal amount of + enantiomer and – enantiomer, then the solution is said to be a racemic mixture and the solution is optically inactive.
- Enantiomers- Optically active non superimposable mirror images.
- Diastereomers- Optically active non superimposable non mirror images.
- Homomers- Identical compounds. They may be represented differently through Fischer projections but are actually the same compound.

Let us consider all the cups to be a single system. Let us assign the sign + to any cup facing upwards and the sign – to any cup facing downwards.

In the initial scenario we have two cups facing upwards and one cup facing downwards (+ + -). Now if we flip the orientation of one cup, we may get either all the three cups facing upwards (+ + +) or one cup facing upwards and two cups facing downwards (+ - -). But we are not permitted to do so. We have to flip the configurations of a couple of cups simultaneously.

If we do exactly that then we get only one other case, which is all the three cups facing downwards (- - -).

In a chiral compound, keeping the central atom and two groups fixed, if we interchange the position of the rest of the two groups then the new compound is said to be an enantiomer of the former one. If we now keep the central atom and two groups of the enantiomer fixed, and interchange the rest of the two groups, then we get back the initial compound again. In other words we can say that homomers are enantiomers of enantiomers.

Now here goes the analogy.

- The systems of (+ + -) and (- - -) are homomers. By changing the configuration of two cups we are able to interconvert them.

- The systems of (+ - -) and (+ + +) are homomers. By changing the configuration of two cups we are able to interconvert them.
- The systems of (+ + -) and (+ - -), (+ + -) and (+ + +), (- - -) and (+ - -), (- - -) and (+ + +) are enantiomers. By changing the configuration of only one cup we are able to generate enantiomers.

But as per the question, we are supposed to execute the change of configurations of a couple of cups. Hence it is impossible for us to generate an enantiomer of the initial compound from the initial compound by executing two flips at a time. This is why we cannot we have a situation in which all three cups face upwards.

V. Conclusion

Based on above problem and the solutions, it was concluded that problems can be solved in many ways. The types of solutions depend on individual thinking and their approach. In this paper a case study was taken on real life situation and solved by using three different subjects (Mathematics, Physics and Chemistry). Here our focus is not on how to merge these subjects but to appreciate the fact that abstract thoughts can give delightful results.

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