Volume 1, Issue 2, 2016
ISSN(O): 2455-8613

An Approach to Study Image Denoising using Doubly Sparse Transform Technique

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Abstract – In this Paper the sparse domain of signals in a certain area or dictionary has been widely used in many applications in image, audio, biological and other signal analysis. Analytical Sparse transforms such as discrete cosine transform (DCT) and its counterpart i.e. wavelet transform (WT) have been extensively used in the areas of image compression standards where as synthesis sparsifying dictionaries have become extensively used especially in applications such as image de-noising and medical image reconstruction. In this work, we discuss about the square sparsifying transforms which is the product from a fixed, fast transform so as to consider the DCT and an adaptive constrained matrix to be sparse. Such transforms can be studied and implemented efficiently.

Keywords: Dictionary learning, Sparse representation, Image de-noising, Wavelet transform, Discrete cosine transform

I. Introduction

The major basic challenges in the field of image processing are image de-noising, where the underlying goal is to reconstruct the original image by removing the noise from a contaminated image. With the help of modern acquisition hardware this has become easy to deal with images with high resolution at high shutter speeds. But certain factors also lead that the image capturing devices are very prone to contamination and corrupted by noise. So, in this context the effective image de-noising technique can help imaging device manufacturers to solve this situation. The noisy image may be caused by interactions of different conditions of sensors and environment which are often impossible to avoid in certain practical situations. Therefore, image de-noising plays vital role in a variety range of applications such as image restoration, visual tracking of systems, image registrations, image segmentations, and image classification, where the case of obtaining the original image content is a strong challenge. In this topic many algorithms have been proposed. The purpose of image de-noising, image noise suppression remains an open challenge, especially in situations where the images are captured under poor conditions where the noise amplitude is very high.

Let us consider an ideal image signal x which needs to be measured in the presence of an additive white noise with zero mean and Gaussian noise v distributed uniformly, with σ be the standard deviation. The measured image y is,

thus

$$y = x + v \tag{1}$$

Our aim is to design a de-noising algorithm which can remove the noise from y, getting as close as possible to the original image x.

In this paper, we focus on one specific approach towards the image de-noising problem that we find after simulations, this to be effective, thus the use of sparse and redundant representations over trained dictionaries.

II. Theory

In the study of image processing, the different mathematical models have been studied for image de-noising. Such as Sparse model, (Analysis-synthesis) model, Dictionary learning, K-svd algorithm and Doubly Sparse Transform Model.

A. Sparse Model

There are Two well-known models for sparse representation are the synthesis model and the analysis model [1]. Now days, Sparse models are used for different data processing, these approaches are known in the literature as synthesis dictionary learning [3] and analysis dictionary learning [4]. The synthesis model suggests that a signal may be represented as a linear combination of a small number of columns/atoms from a synthesis dictionary D.

Assumption for synthesis model:

$$y = Dx. \tag{2}$$

Where, $y \in R^n$, $D \in R^{n\times R}$ & $x \in R^R$ being sparse,

i.e., $\|x\|_o << K$.

The 'l_o' quasi norm counts the number of non-zero entries in x. practically, real world signals are expected to deviate from this model. Therefore, the data are generally satisfies

$$y = Dx + e \tag{3}$$

Where 'e' is an error or noise term in the signal domain [5]. When K = n and D is full rank, the dictionary is said to be basis, whereas when K > n, the dictionary is said to be over complete. The signal 'y' and synthesis dictionary 'D' are given, the problem of finding the sparse representation 'x' is the synthesis sparse coding problem [3]. The process of obtaining a sparse representation for a signal or image requires explicit knowledge of the synthesis dictionary D. Many analytical dictionaries have been developed for sparse representation such as Ridgelet, Contourlet and Curvelet dictionaries.

B. Analysis Model

Although the model is not new but another model for sparse representation analysis, which suggests that a signal $y \in R^n$ in sparse domain is in an analysis dictionary $\tau \in R^{m\times n}$ in analysis domain.

Let us assume that $\tau_y \in R^m$ to be sparse i.e., $\|\tau_y\| \ll m$. Where τ is known as analysis dictionary, since it analyses the signal y to produce a sparse result. The zeros of τ_y defines the subspace to which the signal belongs and the total number of zeros is called the co-sparsity [2,5]. When the signal y is noisy, it is better to satisfy a noisy signal analysis model which states that y = q+e with τ_y being sparse and e is the noise added to q.

Given the noisy signal y and analysis dictionary τ, the problem of finding the noiseless vector q is known as analysis sparse coding problem.

Another alternative model for sparse representation is the transform model, which is a generalization of the analysis model [1]. It suggest that a signal y is approximately sparsifiable using a transform $W \in R^{m\times n}$.

The assumption for transform model is that $W_y = x + \delta$, where $x \in R^m$ is sparse, i.e., $\|x\|_o << m$, and δ is a small residual error in the transform domain. The most distinguishing feature of transform model from other two is that in both of which the error term is in signal domain. The transform model is a generalization of the analysis model with τ_y exactly sparse. Moreover, while the analysis model enforces the sparse code (τ_y) to lie in the range space of τ, the sparse representation x in the transform model is not restricted to be in the range of W. This makes the transform model more general than even the noisy signal analysis model [2]. The reason of naming "transform model" is because the assumption Wy ≈ x has been traditionally used in transform coding (with ortho-normal transforms), and the concept of transform coding is older [2] and pre-dates the terms analysis and synthesis [3].

With a known suitable sparsifying transform for the signal, the process of obtaining a sparse code of the given sparsity is known as transform sparse coding problem.

C. Dictionary Learning

For sparse decomposition of image, dictionary structure is a very important aspect. The original dictionary construction method is derived from extending, shifting and modulating window function. But for specific processing of image signal, training dictionary can better satisfy its characteristic. In the existing methods of training dictionary, KSVD algorithm has low computational cost. Therefore, it is more appropriate for processing of image signal.

D. K-svd Algorithm

The problem of sparse representation can be defined by either Eq. 1 or Eq. 2. We assume problem formulation presented in the equations stated earlier and extension of this, to include the entire set of signals observed denoted by the set

$$Y = \{y_i \in [1, K], y_i \in R^n \tag{4}$$

as $$\min_{D,x_i} \|y_i - D_x\|_F, \text{ subject to } \|x_i\|^2 \ll T$$

where X is formed by column stacking all vectors xi and $\|\cdot\|_F$ denotes the Frobenius norm square which is defined as the square of every element in the matrix.

The K-svd algorithm attempts to minimize the cost function iteratively, by first finding a coding for the signals in question using the OMP algorithm using an initial estimate of the dictionary. This coding as such this minimizes the error in representation, and at the same time maintains a sparsity constraint as defined in Eq. 3. Once this sparse coding stage is done, the algorithm proceeds to update the dictionary as if one atom at a given time frame, such that the error term is further reduced.

III. Doubly Sparse Transform Model

Imposing additional structure on the learnt transform leads to computational advantages over the 'unstructured' transforms introduced in [3]. We propose to learn a square (n × n) sparsifying transform W that is a product of two different transforms,

i.e. $$W=B.\omega \quad (5)$$

where

$\Phi \in R^{n\times n}$ is an analytical transform with an efficient implementation, and $B \in R^{n\times n}$ is a transform that is constrained to be sparse (i.e., has few significant non-zero elements).

Such a transform W is said to be 'doubly sparse', since it provides a sparse representation for data and has matrix B that is sparse. The proposed transform structure combines the advantages of trained and analytic transforms: adapting to the data, it provides better representations and denoising than analytical transforms, yet owing to the sparsity of B, it can be stored and applied efficiently. As we show, it can also be learnt more efficiently than unstructured transforms [3]. The structure W=B.ω is expected to be an effective sparsifying transform because ϕ, matrices such as the DCT when applied to natural images (or image patches) produce a result that is already approximately sparse. Therefore, by further modifying the result using only the limited number of degrees of freedom in a sparse transform B, one may be able to produce a highly sparse result.

IV. Problem Formulation

Given the training matrix $Y\in R^{n\times n}$, we propose to minimize the sparsification error given by $\|WY - X\|^2_F$ where $W \in R^{m\times n}$ and X has sparse columns, subject to constraints on W. The notion of sparsification error is justified by the fact that natural signals and image patches are reasonably approximated in a transform domain such as Wavelets using few (say s) significant non-zero transform coefficients.

A. Unstructured Transform Learning

The problem formulation for learning a square unstructured transform W(m=n) is as follows

$$\min_{W,x} \| WY - X \|_F^2 - \lambda \log|\det W| + \mu \| W \|_F^2 \quad (6)$$

Such that $\|X_i\|_0 \le s\forall i$

Here, the log |det W| penalty2 helps enforce full rank on the transform W and eliminates degenerate solutions (such as those with zero, or repeated rows). Many well-known (square) sparsifying transforms such as the DCT, Wavelets, and Hadamard are non-singular. Even the one-dimensional finite difference transform, which typically has more columns than rows, can be appended with a linearly independent row(s), thereby making it non-singular. For the 2-D case, the finite difference transform can be written as a Kronecker product of two non-singular 1-D finite difference matrices [2].The $\|W\|^2_F$ penalty in (P1) helps remove a 'scale ambiguity' [5] in the solution (the scale ambiguity occurs when the data admits an exactly sparse representation), and together with the −log |det W| penalty additionally helps control the condition number of the learnt transform.

B. Doubly Sparse Transform Learning

For doubly sparse transform, W is replaced by B.ω. The problem formulation for doubly sparse transform is as

Follows

$$\min_{B,x} \| B\Phi Y - X \|_F^2 - \lambda \log|\det(B\Phi)| + \mu \| B\Phi \|_F^2 \quad (7)$$

Such that $\|B\|_0 \le r$, $\|X_i\|_0 \le s\forall i$

C. Denoising using K-svd

In this section, we discuss the application of the K-svd algorithm in denoising of images. Here we break up the noisy image into patches and treat the vectorized version of each patch as signals, thereby restricting the dimensionality of each atom in the dictionary. However, the size of the patch has to be chosen such that it encodes enough details of the underlying signal. Dealing with patches as signals, the K-svd algorithm can be effectively scaled to de-noise large images.

For a given image, which can now be thought of as a set of signals Y, the denoising problem can be stated as the ill-posed problem of finding a set of patches Z which are related by Y = Z + η, Where η is assumed to be the additive noise, which corrupts the patches from the image. The noise over the entire image is assumed to be zero mean Gaussian noise. In order to find the denoised image patches Z, we define an optimization problem akin to that defined in Eq. 1 which involves minimization of the cost function.

$$\arg\min(x,z) \| Z - Y \|(2,2) + \gamma\|DX - Z(2,F) + \textstyle\sum_i \mu \quad (8)$$

This can be viewed as solving a set of smaller optimization problems which is defined by

$$\arg\min(x_i,z) \| Z-Y \|(2,2) + \gamma\|DX_i - R_iZ(2,F)+\textstyle\sum_i \mu_i \,| \quad (9)$$

where R_i is defined as the matrix which selects the i-th patch from Z i.e. $z_i = R_iZ$. This cost function allows us to minimize the error between the restored image and the input noisy one, under the assumption that each patch in the input image can be represented as a sparse linear combination of patches in the dictionary D.

V. Numerical Experiments

For experiment, we take a clean image (lena) and add Gaussian noise with standard deviation of 25 and perform the denoising by using a learned dictionary which is used to remove the noise from the input image. It is a denoising process based on dictionary learning from the overlapping patches of images. For doubly sparse transform W=B.ω, we take one fixed transform as DCT and the other transform as K-SVD Dictionary.

For comparing purpose different types of dictionaries are taken. Here DCT Dictionary, Global Dictionary and the dictionary learned from noisy image by using K-SVD are used. The results obtained are as follows:

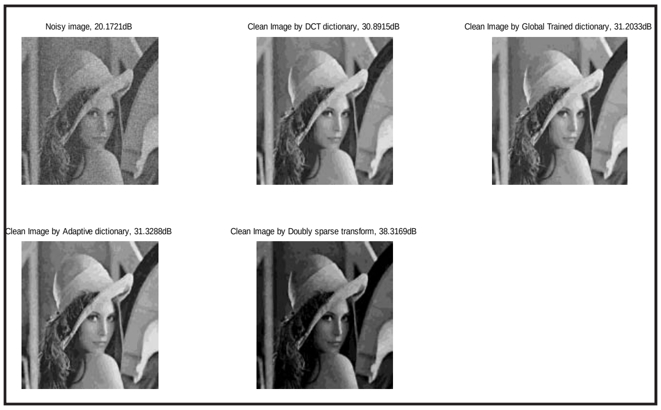

Fig. 1. (a) Noisy image (b) Clean Image by DCT (c) Clean image by global Dictionary (d) Clean image by K-svd (e) Clean image by doubly sparse transform

For the comparison purpose, the same process is applied to different samples at variant conditions.

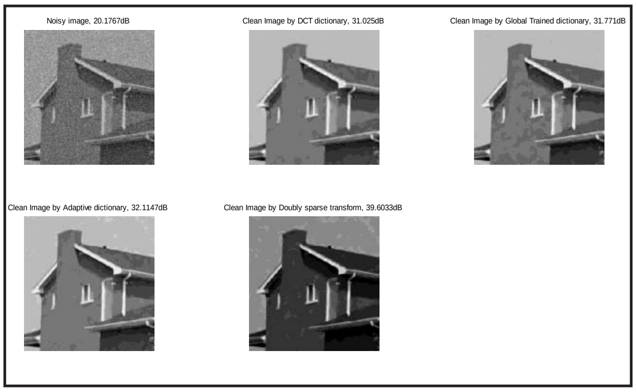

Fig. 2. (a) Noisy Image (b) Clean image by DCT (c) Clean image by Global Dictionary (d) Clean image by K-svd (e) Clean image by Doubly Sparse transform.

Table 1
The PSNR Comparison Table for Lena Image

Different Dictionaries	*PSNR in DB*	
	Single transform	*Doubly transform*
DCT	30.910	38.3178
Global	31.206	38.3213
K-svd	31.298	38.3226

VI. Conclusion & Work Scope

Aim is to get a clean image from noisy image with better accuracy in a very less time. Hence we shall use different transformation techniques for denoising and compare the processing time and further approach to apply the better methods for other applications. Here, we work on the noisy conditions to remove the noise by creating the overlapping patches. We use the sparse method for our work by learning the dictionary from natural images with high resolution and the patches from the noisy image for better accuracy. We use the K-svd algorithm for the operation and try to implement the square sparsifying transform, known as doubly sparse transform to remove the rain streaks from the images. And found that, it can provide better PSNR than the single sparse transform. We compare the doubly sparse transform in three methods by using DCT Dictionary, Global Dictionary and a dictionary learned from noisy image by using K-svd algorithm. From which, it is understood that this provides better result than the others.

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