Supplier Selection Under Incomplete Preference Information

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Abstract

Supplier selection is a Multiple Attribute Decision Making (MADM) problem which is affected by several conflicting factors as the suppliers' information and performances are usually incomplete and uncertain. Several MADM methods have been proposed for solving this problem, one of which is the Analytic Hierarchy Process (AHP). One of the advantages of this approach is the ability to make incomplete comparisons to get a final priority vector (particularly, in combination with Harker's method). Therefore, calculating priorities with incomplete preference information on alternatives is the aim of this paper. Finally, a numerical example for supplier selection is given to illustrate the application of the proposed method. The main findings of this study confirm the effectiveness of the proposed methods.

Keywords: MADM, AHP, Harker's Method, Incomplete Preference Information, Supplier Selection Problem

Introduction

Companies are profit focused. Traditionally, managers have given strong emphasis to sales revenue - the incoming money. Nowadays, due to fierce competition, increasing price has become a difficult strategic choice. Consequently, growing emphasis has been given to the cost – the outgoing money. Naturally, purchasing as the largest expenses to a company has been receiving an increasingly amount of attention and effort. Since, selecting suitable suppliers is the cornerstone of successful purchasing. However, identifying suitable suppliers is not an easy task. One can argue that it is extremely difficult for any single supplier to excel in all criteria. An actual choice of supplier unavoidable involves trade-off among the attribute levels of different suppliers. Therefore, supplier selection is usually a complex multi-criteria problem involving both quantitative and qualitative elements. There is no one proven best method in evaluating and selecting suppliers and companies deploy a variety of different approaches. Choosing the best supplier should meet the goal of receiving the right quantity on the right time with the right cost (Jounio, 2013). The AHP is a Multi Criteria Decision Making (MCDM) method that helps the decision maker facing a complex problem with multiple conflicting and subjective criteria (Ishizaka & Labib, 2009). In addition, AHP is a prominent approach in multi criteria decision-making problems; and in practice, it has found widespread application in supplier evaluation and selection problems, alone or in combination with another tool (Jounio, 2013). According to Saaty (2001), the AHP has a systematic procedure for better judgements.

Generally, AHP is based on three basic principles: decomposition, comparative judgements, and hierarchic composition or synthesis of priorities. The decomposition principle is applied to structure a complex problem into a hierarchy of clusters, sub-clusters, sub-sub clusters and so on. The principle of comparative judgements is applied to construct pairwise comparisons of all combinations of elements in a cluster with respect to the parent of the cluster. These pairwise comparisons are used to derive 'local' priorities of the elements in a cluster with respect to their parent. The principle of hierarchic composition or synthesis is applied to multiply the local priorities of elements in a cluster by the 'global' priority of the parent element, producing global priorities throughout the hierarchy and then adding the global priorities for the lowest level elements [the alternatives] (Forman & Selly, 2001). In standard AHP, an eigenvector (EV) method is used for deriving weights from local matrices; the EV is called the prioritisation method, and the computational procedure is consequently called optimisation. After local weights are calculated at all levels of the hierarchy, a synthesis consists of multiplying the criterion-specific weight of the alternative with the corresponding criterion

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weight and summing up the results to obtain composite weights of the alternative. With respect to the goal, this procedure is unique for all alternatives and all criteria.If all comparisons are performed properly by the DM, then AHP synthesis is straightforward. However, if the DM for various reasons fails to make some judgements, then there are empty cells in the corresponding local matrices. The first case can be treated as decision making with complete information, and the other case with incomplete information (Srdjevic, Srdjevic, & Blagojevic, 2014). The latter is preferred in this paper. Therefore, in this study, the AHP method in combination with Harker's method applied to solve the supplier selection problem under incomplete preference information.

The paper is organised as follow. In the second section, the literature and in the third section, the proposed approach is discussed. Numerical example is provided in the next section. The paper is concluded in the fifth and the last section.

Literature Review

In some real problems, it is impossible or difficult to have comparisons of some pairs of alternatives. Let us call such cases incomplete AHP. It is very important to estimate incomplete comparisons data to have alternatives weights (Gao, Zhang, & Cao, 2010). Several solutions for the above problem are possible, too like Harker's method, Van Uden's method, etc. Here, we will mention some of them. Harker (1987) explored various methods for reducing the complexity of the preference eliciting process from the incomplete pairwise comparisons in the AHP technique. The theory of a method based upon the graph-theoretic structure of the pairwise comparison matrix and the gradient of the right Perron vector is developed, and simulations of a series of random matrices are used to illustrate the properties of this approach. Wedley (1993) developed a multiple regression equations methodology for predicting the consistency index and consistency ratio when pairwise comparisons are incomplete. Wedley *et al.* (1993) studied the effect of different reference items for the first (n-1) pairwise comparisons of the incomplete AHP. The empirical results show that significantly greater initial accuracy is achieved if the items are ranked and the lowest ranked item is used as a common referent for the first (n-1) comparisons.Fan and Ma (1999) proposed a new approach to solve the MADM problem with incomplete preference information on alternatives. The approach is based on an optimisation model which can be used to assess attribute weights and then to select the most desirable alternative. Lu and Yang (2005) focused on the impact of incomplete information that buyers always possessed on their choice behavior in air transportation market. Fedrizzi and Giove (2006) introduced a new method for calculating the missing elements of an incomplete matrix of pairwise comparison values for a decision problem. So, the matrix is completed by minimising a measure of global inconsistency, thus obtaining a matrix, which is optimal from the point of view of consistency with respect to the available judgements. Wedley (2006) enlarged the spanning tree, then put into an expanded matrix with incomplete comparisons and solved using Harker's method. Alonso, Herrera-Viedma, Chiclana, & Herrera (2009) presented two different tools that can be used to solve GDM (Group Decision Making) problems where the experts give their preferences by means of incomplete fuzzy preference relations. Gao et al. (2010) proposed a new method for ranking estimation in incomplete AHP. So that, new least square method is translated into linear system and minimax method, and absolute deviation method are translated into linear programming. It is shown that three proposed methods have fast convergence and smaller computational complexity.Chuang, Kao, Cheng, & Chou (2012) investigated the relationship between the incomplete information and the compromise effect in different choice scenarios. The main findings were that consumers are more likely to choose the middle option when they have incomplete information than when they have complete information. Carmo *et al.* (2013) addressed the aggregation of individual priorities (AIP) in incomplete hierarchies, in AHP. The result shows that only arithmetic mean aggregation of individual priorities is suitable to be used when incomplete hierarchy is considered. Srdjevic *et al.* (2014) proposed a method to fill the gaps in the pairwise comparison matrices generated through elicitation of the DM's semantic judgements.

This paper focuses on the application of the AHP method in combination with Harker's method for solving a supplier selection problem, under incomplete information. In the next section, the proposed method will be considered.

Proposed Approach

A brief discussion of AHP and Harker's method is provided in this section.

Analytic Hierarchy Process (AHP)

In the Analytic Hierarchy Process, first, elements are compared in the form of pair and paired comparison matrix, then formed by use of this matrix to calculate the relative weights of elements to tally a paired comparison matrix shown in the following form in which a_{ij} is the preference of element i to element j. now with the determination of a_{ij}, we want to gain weights W_i of elements

$$A=[a_{ij}], \quad i, j=1,2,\ldots,n. \tag{1}$$

Each paired comparison matrix may be consistent ($a_{ij}=W_i/W_j$) or inconsistent ($a_{ij}\neq W_i/W_j$). In the state, that the matrix is consistent and calculating weight W_i is simple and gained from normalisation of the elements of each column. But in the state that matrixes are inconsistent, four main approaches will be presented for calculation:

1. Least squares method
2. Logarithmic least squares method
3. Eigenvector methods
4. Approximation methods.

Now we explain one of the above methods that we have used for achieving weights in the presented method. In this method, W_i is a determinant in a way that the following relationship is true.

$$\begin{aligned} a_{11}w_1+a_{12}w_2+\ldots+a_{1n}w_n &= \lambda.w_1 \\ a_{21}w_1+a_{22}w_2+\ldots+a_{2n}w_n &= \lambda.w_2 \\ \vdots \\ a_{n1}w_1+a_{n2}w_2+\ldots+a_{nn}w_n &= \lambda.w_n \end{aligned} \tag{2}$$

Here a_{ij} is preference of i-th element to j-th, W_i is weight of i-th element, and is a constant number. This method is one kind of mean that Harker calls it as possible mean in a different way because in this way the weight of i-th element (W_i) according to the above definition is equal to:

$$W_i=1/\lambda.a_{n1}w_1+a_{n2}w_2+\ldots+a_{nn}w_n=\lambda.w_n \tag{3}$$

$i=1,2,...,n$

We can write the above simultaneous equations as follows:

$$A.w = \lambda.w \tag{4}$$

in which A is a paired comparison matrix {mean $A=[a_{ij}]$}, w is a weight vector, and is a scalar (number). According to the definition, this relationship is among one matrix (A), vector (W) and (λ) number, it has been said that w is a special vector and λ is a special amount (Lotfi *et al*., 2012).

Harker's Method

Harker method is based on the following idea. If (i, j) – component is missing, put the artificial value W_i/W_j into the vacant component to construct a complete reciprocal matrix A (W). Then consider the eigen system problem:

$$A(w)\, w = \lambda \quad w \tag{5}$$

Formally, Harker's method is written as follow. Given incomplete matrix $A=(a_{ij})$, define the corresponding derived reciprocal matrix $\tilde{A}=(a_{ij})$ by:

$$\tilde{a}_{ij} = \begin{cases} 1+m_i & \text{if } i=j \\ 0 & \text{if } a_{ij} \text{ is missing} \\ a_{ij} & \text{other wise} \end{cases} \tag{6}$$

where m_i denotes the number of missing components in the i-th row (Gao *et al*., 2010). In other words, enter zero for any missing judgement in that matrix, and add the number of missing judgements in each row to the diagonal element in the row, producing a new matrix A. then, calculate the weight W (Saaty, 2000):

$$\lim_{k\to\infty} A^k{}_e / e^t A^k{}_e = cw \tag{7}$$

where e is the column vector unity, e^t is its transpose, and c is a positive constant.

Numerical Example

To demonstrate the application of the proposed approach in supplier selection context, we use the dataset from Benyoucef, Ding, & Xie (2003) [problem with a known composite answer; Tables 1-13].

Table 1: Pairwise Comparisons of Evaluation Criteria

Objective	Pricing structure	Delivery	Quality	Service	Weights
Pricing structure	1	3	1	3	0.40
Delivery	1/3	1	1/3	1	0.13
Quality	1	3	1	½	0.26
Service	1/3	1	2	1	0.21

Table 2: Pairwise Comparisons of Criterion Dimensions - 1

Delivery	Timeliness	Cost	Weights
Timeliness	1	5	0.83
Cost	1/5	1	0.17

Table 3: Pairwise Comparisons of Criterion Dimensions - 2

Quality	Quality level	Cost	Weights
Quality level	1	1/5	0.17
Cost	5	1	0.83

Table 4: Pairwise Comparisons of Criterion Dimensions - 3

Service	Personnel	Facilities	R&D	Capability	Weights
Personnel	1	1	1	3	0.32
Facilities	1	1	2	½	0.24
R&D	1	½	1	½	0.17
Capability	1/3	2	2	1	0.26

Table 5: Pairwise Comparisons of Suppliers A, B, and C - 1

Price structure	A	B	C	Weights
A	1	1	1/5	0.16
B	1	1	1/3	0.18
C	5	3	1	0.66

Table 6: Pairwise Comparisons of Suppliers A, B, and C - 2

Timeliness	A	B	C	Weights
A	1	5	3	0.66
B	1/5	1	1	0.16
C	1/3	1	1	0.18

Table 7: Pairwise Comparisons of Suppliers A, B, and C - 3

Delivery cost	A	B	C	Weights
A	1	1/3	2	0.23
B	3	1	5	0.65
C	½	1/5	1	0.12

Table 8: Pairwise Comparisons of Suppliers A, B, and C - 4

Quality level	A	B	C	Weights
A	1	½	1/3	0.16
B	2	1	½	0.30
C	3	2	1	0.54

Table 9: Pairwise Comparisons of Suppliers A, B, and C - 5

Quality cost	A	B	C	Weights
A	1	1	½	0.25
B	1	1	½	0.25
C	2	2	1	0.50

Table 10: Pairwise Comparisons of Suppliers A, B, and C - 6

Personnel	A	B	C	Weights
A	1	1/5	2	0.20
B	5	1	3	0.65
C	½	1/3	1	0.15

Table 11: Pairwise Comparisons of Suppliers A, B, and C - 7

Facilities	A	B	C	Weights
A	1	1	1	0.33
B	1	1	1	0.33
C	1	1	1	0.33

Table 12: Pairwise Comparisons of Suppliers A, B, and C - 8

R&D	A	B	C	Weights
A	1	1/3	1/3	0.14
B	3	1	1	0.43
C	3	1	1	0.43

Table 13: Pairwise Comparisons of Suppliers A, B, and C - 9

Capability	*A*	*B*	*C*	*Weights*
A	1	3	1	0.42
B	1/3	1	¼	0.12
C	1	4	1	0.46

The result is as follow:

$$\begin{vmatrix}0.157\\0.185\\0.685\end{vmatrix}.0.40+\begin{vmatrix}0.586\\0.240\\0.174\end{vmatrix}.0.13+\begin{vmatrix}0.230\\0.259\\0.507\end{vmatrix}.0.26+\begin{vmatrix}0.280\\0.400\\0.320\end{vmatrix}.0.21=\begin{vmatrix}0.260\\0.250\\0.490\end{vmatrix}$$

From the above results, it can be concluded that, the ranking is as follow:

C>A>B

In this section assume that, DM for each reasons fails to make some judgements (for instance, for Table 1, 4, 5, 7, 8, 10, 12, and 13, respectively).Therefore, there are empty cells in the corresponding local matrices (Table 14-21) i.e. for Table 1 (based on formula No. 6 and 7), we have (Table 14):

Table 14: Incomplete Pairwise Comparisons of Evaluation Criteria (Based on Table 1)

Objective	*Pricing structure*	*Delivery*	*Quality*	*Service*	*Weights*
Pricing structure	1	?	1	3	0.386
Delivery	?	1	?	1	.174
Quality	1	?	1	½	.217
Service	1/3	1	2	1	.223

$$A=\begin{vmatrix}2 & 0 & 1 & 3\\0 & 3 & 0 & 1\\1 & 0 & 2 & 1/2\\1/3 & 1 & 2 & 1\end{vmatrix}=A^1;\ e=\begin{vmatrix}1\\1\\1\\1\end{vmatrix}$$

$$A^1.\,e=\begin{vmatrix}6\\4\\3.50\\4.33\end{vmatrix};\ e^T.A^1.e=\begin{vmatrix}1 & 1 & 1 & 1\end{vmatrix}.\begin{vmatrix}6\\4\\3.50\\4.33\end{vmatrix}=17.833$$

Then in first iteration:

$W^1=A^1.e/e^T.A^1.e$=(6/17.833=0.336,4/17.833=0.224,3.50/17.833=0.196, and 4.33/17.833=0.243).

Similarly (details are omitted):

W^2=(0.369 0.211 0.196 0.224)

W^3=(0.375 0.200 0.204 0.222)

W^4=(0.377 0.192 0.208 0.223)

W^5=(0.380 0.186 0.211 0.223)

W^6=(0.382 0.182 0.213 0.223)

W^7=(0.384 0.179 0.214 0.223)

W^8=(0.385 0.177 0.215 0.223)

W^9=(0.385 0.176 0.216 0.223)

W^{10}=(0.386 0.175 0.216 0.223)

W^{11}=(0.386 0.174 0.217 0.223)

Finally after 12 iterations:

W^{12}=(0.386 0.174 0.217 0.223)

As can be seen, the process has convergence in twelfth iteration and the calculation was stabled. In other words, W^{12} is the final solution (the last column of Table 14).

Similarly;

Table 15: Incomplete Pairwise Comparisons of Evaluation Criteria (based on table 4)

Service	*Pricing structure*	*Delivery*	*Quality*	*Service*	*Weights*
Pricing structure	1	**1**	1	3	0.334
Delivery	**1**	1	**2**	½	.240
Quality	1	**1/2**	2	0	.185
Service	1/3	2	0	2	.241

Table 16: Pairwise Comparisons of Suppliers – 1 (based on Table 5)

Price structure	*A*	*B*	*C*	*Weights*
A	2	0	1/5	0.131
B	0	2	1/3	0.217
C	5	3	1	0.652

Table 17: Pairwise Comparisons of Suppliers – 2 (based on Table 7)

Delivery cost	*A*	*B*	*C*	*Weights*
A	2	1/3	0	0.217
B	3	1	5	0.652
C	0	1/5	2	0.131

Table 18: Pairwise Comparisons of Suppliers - 3 (based on Table 8)

Quality level	*A*	*B*	*C*	*Weights*
A	1	1/2	1/3	0.167
B	2	2	0	0.334
C	3	0	2	0.449

Table 19: Pairwise Comparisons of Suppliers - 4 (based on Table 10)

Personnel	*A*	*B*	*C*	*Weights*
A	2	0	2	0.334
B	0	2	3	0.500
C	1/2	1/3	1	0.167

Table 20: Pairwise Comparisons of Suppliers - 5 (based on Table 12)

R&D	*A*	*B*	*C*	*Weights*
A	3	0	0	0.333
B	0	2	1	0.333
C	0	1	2	0.333

Table 21: Pairwise Comparisons of Suppliers - 6 (based on Table 13)

Capability	*A*	*B*	*C*	*Weights*
A	2	3	0	0.376
B	1/3	1	1/4	0.125
C	0	4	2	0.499

The new result is as follow:

$$\begin{vmatrix}0.131\\0.217\\0.652\end{vmatrix}0.386+\begin{vmatrix}0.584\\0.240\\0.176\end{vmatrix}0.174+\begin{vmatrix}0.236\\0.264\\0.500\end{vmatrix}0.217+\begin{vmatrix}0.344\\0.339\\0.317\end{vmatrix}0.223=\begin{vmatrix}0.280\\0.258\\0.461\end{vmatrix}$$

From the above results, it can be concluded that, the ranking is as follow:

C>A>B

A comparison of the test results is given in Table 22.

Table 22: Comparison of Results

Method	*Priorities*
AHP (under complete preference information)	C > A > B .490 .260 .250
AHP and Harker's method (under incomplete preference information)	C > A > B .461 .280 .258

Comparative results shown in Table 22 indicate that results obtained by AHP (under complete preference information) were not different from those obtained using the AHP and Harker's method (under incomplete preference information). Moreover, these results implicitly indicate the effectiveness of the proposed models.

Concluding Remarks

In this paper, we proposed a model for supplier evaluation using AHP and Harker's method to evaluating and ranking supplier under incomplete preference information (the results were tested by the example with a known composite answers). Then a comparative analysis is performed (Table 22). The results indicate that the ranks obtained by the AHP and AHP-Harker's method are not different. Therefore, the findings in this paper confirm the effectiveness of proposed method (because these results implicitly indicate the accuracy of the applied methods). In addition, further research can apply this proposed approach to other managerial issues or compared with another method to estimate incomplete comparison data.

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