A New Aggregation Rule for Ranking Suppliers in Group Decision Making under Multiple Criteria

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Abstract

Aggregation rule has always played a major role in group decision-making process. Therefore, how to aggregate the individual pre-orders in a single collective pre-order is very crucial to Multiple Attribute Group Decision Making (MAGDM) models. According to literature, three of the most commonly used aggregation rules in group decision-making (especially, MAGDM) are 1. Ranks mean (average), 2. Borda, and 3. Copeland method. Theoretically, there is no reason to be restricted to these methods. Therefore, in this paper, a new MAGDM approach (based on the improved Analytic Hierarchy Process [AHP] models) is proposed, and can more assure the results by applying a systematic model. In addition, the proposed method is compared with an existed model (TOPSIS and Borda's Function approach). Compared results in supplier selection context show that the proposed method is effective and feasible.

Keywords: MAGDM, AHP, Improved AHP Model, Aggregation Rules, Means Rank, Borda, Copeland

Introduction

Decision problems are usually complex (Dhingra & Singh, 2015). Therefore, in most organisations, decision-making is made by two or more people, regardless of whether the organisation is public or private and the decision problem is local or global. Although all members do not need to be located in the same physical location, they are aware of one another and act as a member of the group which makes a decision. The apparent purpose of the group is to reach a decision, that is, to choose an alternative which seems to be acceptable and agreeable for all members. However, it is sometimes difficult to reach a consensus among group members (Entani, 2009). Nevertheless, behavioural scientists, economists, and decision theorists have proposed a variety of models/methods describing how a DM (Decision Maker) might arrive at a preference judgment when choosing among multiple attribute alternatives such that, different MADM (Multiple Attribute Decision Making) techniques yield different results for the same problem. Therefore, it is not clear which approach is better (Azadfallah, 2015a). On the other hand, supply chain management plays a vital role in the growth of the industry, survival in the market, the production rate, and the dynamic interaction among the suppliers and the customers (Kumar, 2014). In addition, According to Izadikhah (2012), success of a supply chain is highly dependent on selection of good suppliers. However, selecting the wrong supplier could be enough to deteriorate the whole supply chains financial and operational position. Since, supplier selection, the process of finding the right suppliers who are able to provide the buyer with the right quality products and/or services at the right price, at the right time and in the right quantities, is one of the most critical activities for establishing an effective supply chain. Moreover, it is a hard problem since supplier selection is typically a multiple criteria (also often called attributes) group decision-making problem. The definition of MAGDM is described specially as follow, multi DMs make judgments or evaluations by virtue of respective knowledge, experience and preference for a decision space (i.e., a finite set of alternatives) under multi attributes to rank all the alternatives or give evaluation information of each alternative, and then decision results from each DM are aggregated to form an overall ranking result for all the alternatives (Azadfallah, 2016).

Generally, the practice of MAGDM is to invite internal experts, external experts, or their combination of related fields to evaluate each attribute of every alternative individually. In the MAGDM, the experts usually have diverging opinions, because they often come from different specialty fields and each expert has his/her unique characteristics with regard to knowledge, skills, experience and personality. Thus, how to obtain the maximum degree of consensus or agreement from these experts for the given alternatives is an interesting and important research topic (Azadfallah, 2015b). Since, how to aggregate individual preferences or a set of ordinal rankings into a group preference or consensus ranking is a typical group decision-making problem (Wang, Yang, & Xu, 2005).

In the current literature, several solutions for the above problem are possible. Some of the popular methods are being explained as follows.

1. **Ranks Mean (average) Technique**: In this technique, the alternatives are prioritised based on the achieved arithmetic mean of ranks from different MADM methods.
2. **Borda Technique:** This technique is based on majority rule, and rank of each pair in different ranking ways is compared with each other.
3. **Copeland Technique:** This technique can be called correction of previous techniques, since, in addition to the number of wins, the numbers of lost are also considered in prioritisation (Tajvidi Asr *et al.*, 2015). In this paper, different aggregation techniques (based on improved AHP models) concerning supplier selection are discussed and the advantage of the proposed method are illustrated and compared. Although, most of studies are limited to combined results of different MADM methods (i.e., Hojjati & Anvari, 2014; Hashemi & Zamani, 2015, etc.), Azadfallah (2016) used the aggregation rules (especially, Borda's method) for to unify the results of each DM (in-group decision-making process), where same MADM method is used. However, in this paper, both the aforementioned aggregation rules concerning supplier selection are discussed. So, two numerical examples are examined to illustrate the validity and practability of proposed method where, three aggregation rules (Ranks mean, Borda, and Copeland techniques) will be applied to aggregate the 1. four sets of preference orderings (SAW, ELECTRE, TOPSIS, and WPM methods), and 2. individual preferences into a group preference (TOPSIS and Borda's function approach). In addition, comparisons with proposed method will also be made.

The paper is organised as follows. In the second section, the literature and in the third section, the proposed approach is discussed. Numerical example is provided in the next section. The paper is concluded in the fifth and the last section.

Literature Review

The problem of aggregating the individual pre-orders of decision makers in a single collective pre-order has been the target of several studies in the literature on group decision making. Historically, the first papers, which tackled this problem, were by Borda and by Condorcet (Alencar, Almeida, & Morais, 2010). On the other hand, according to Hojjati and Anvari (2014), decision makers are not limited just to one method of MADM in fatal issues, because it is possible that various MADM methods gain different results. To get over this problem, aggregate method has been introduced. Voogd (1982) demonstrated that, in some qualitative and quantitative models (9 and 23 models, respectively), it can be said that there is a minimum of 40% chance that a technique results in another ranking than any other technique. Zanakis, Solomon, Wishart, and Dublish (1998) described the inconsistency in such results occurs because 1. the techniques use weights differently in their calculations, 2. algorithms differ in their approach to selecting the best solution, 3. many algorithms attempt to scale the objectives, which affects the weights already chosen, and 4. some algorithms introduce additional parameters that affect which solution will be chosen. Therefore, in order to get over this problem, aggregate method (Ranks mean, Borda, Copeland, and some of extension methods) has been introduced. During recent years, various studies have been under taken (However, most of them are limited to Borda's method). Here, we will mention some of them. Wang *et al.* (2005) developed a preference aggregation method for ranking alternative courses of actions by combining preference rankings of alternatives given on individual criteria or by individual decision makers. In the method, preference rankings are viewed as constraints on alternative utilities, which are normalised, and linear programming models are constructed to estimate utility intervals, which are weighted and averaged to generate an aggregated utility interval. Luo and Jennings (2007) provided a framework for identifying various types of compromise operators that decision-making agents might adopt when aggregating opinions. Garc´ıa-Lapresta, Mart´ınez-Panero, and Meneses (2008) used linguistic labels as inputs in the Borda count. Fu (2008) developed three extended TOPSIS models. The pre-model, post-model, and inter-model, associated with three approaches to aggregating group preferences (the pre-operation, post-operation, and inter-operation), which depends on Dempster's rule or its modifications, some social choice functions (i.e., Borda's function and some mean approaches). Shahbandarzadeh and Haghighat (2010) used LINMAP for target market selection so that LINMAP technique was used separately for each level (strengths, weaknesses, opportunities, and threats), and at last unified the results of each level by using Borda method. Charakhlou, Faraji Sabokbar, and Givehchi (2010) used TOPSIS, Fuzzy, and SAW techniques for making changes in the route network to facilitate rescue operations in urban disasters. Next, the results were combined by means of Borda method. Kim and Ye (n.d.) proposed a general fuzzy grey decision making method, which takes into consideration of the grey degree of the weight and the attribute value at the same time, for the MADM where attributes have the generalised super mixed-type values given by real number, interval value, linguistic value and uncertain linguistic value. Then, their obtained four ranks by four MADM methods of plan evaluation such as the evaluation by the relative approaches degree of grey TOPSIS. Finally, the weighted Borda method is used to obtain final rank by using the results of four methods. Kim, Jong, Liu, and Shong (n.d.) suggested an interval weight determining method and three methods of grey interval relation decision making: the evaluation of plans by the relative approach degree of grey TOPSIS method, the evaluation by the relative approach degree of grey incidence, and the evaluation by the relative approach degree of grey incidence using maximum entropy estimation. The final rank of plans has been obtained by weighted Borda method considering the above three ranking results. Alcantud, Andrés, and Cascon (2013) proposed a unifying model that generates a consistent decision in terms of the individual preferences and then measures the consensus that arises from it. They focused their inspection on two relevant and specific cases: the Borda and the Copeland rules under a Kemeny-type measure. Liu, Li., Cai, Xu, and Xia (2014) developed a fuzzy Multiple Attribute Decision Analysis approach (FMADA) for supporting the evaluation of water resources security in nine provinces within the yellow river basin. In continuation, four conventional MADA methods were implemented in the evaluation model for impact evaluation, including Simple Weighted Addition (SWA), Weighted Product (WP), Cooperative Game Theory (CGT), and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). Next, several aggregating methods (including Average ranking, Borda, and Copeland methods) were used to integrate the ranking results. Tajvidi Asr *et al.* (2015) used two MADM methods (including TOPSIS and Linear Assignment) for selection of a proper support system for Beheshtabad water transporting tunnel from among the six proposed support systems whereas, aggregating the results of ranking by the Ranks Mean, Borda, and Copeland techniques led to the suggestion of a support system. Finally, Azadfallah (2015_b, 2016) focused on the application of MAGDM models, particularly TOPSIS and Borda's function approach for solving the supplier selection problem. In the presented model, first TOPSIS is used to find the individual preference ordering (with same criteria set for each DMs (Azadfallah, 2015_b), and while each expert have own criteria set (Azadfallah, 2016), respectively. Then, Borda's function is used to find the collective preference orderings.

In general, in the present investigation, the different aggregation techniques (improved AHP models) concerning supplier selection are discussed. In addition, comparisons with other existing techniques (Ranks Mean, Borda, and Copeland method) will also be made. In the next section, the proposed method will be considered.

Proposed Approach

In this section, brief description presented as follows.

Existing Aggregation Methods

Different approaches of MADM aggregation were adopted in this paper, including:

The Rank Mean (or average ranking procedure)

The average ranking procedure is the simplest technique among the three aggregation methods. This technique is based on the concept of statistical calculation and ranks the alternatives according to the average rankings from the MADM methods.

Borda Method

It is based on the concept of voting and it compares each pair of alternatives separately and forms an N*N matrix. For each pair of alternatives Aj and Aj', the number of votes is defined as the number of "supporting" methods in which Aj is more preferable than Aj'. Then an N*N matrix is generated such that Xjj'=1, if Aj receives more votes than Aj', Xjj'=0, otherwise. Sj indicates the number of "wins" that Aj has received against other alternatives and it is calculated by summing the Xjj' in each row of the matrix. Hence, the alternative with the highest Sj is considered the most preferable.

Copeland Method

This is an extension of the Borda method, which is also based on the voting concept. It is believed that the aggregation utility of Aj does not only depend on the number of "wins", but the number of "losses" also needs to be taken into account. The number of "losses", denoted as Sj', is used to compensate the utility value of Sj. Sj' is calculated by summing the values of each column of the matrix and the aggregation utility is simply defined as the difference of Sj from Sj'. As with the Borda method, the Copeland method ranks the alternatives in descending order of their aggregation utilities from largest to smallest. Although using these aggregation methods may still result in inconsistencies among the rankings, some useful patterns can easily be observed by the decision maker according to the analyzed information (Liu *et al*., 2014).

TOPSIS and Borda's Function Approach

In this approach, TOPSIS can be used to find the individual preference ordering. Then Borda's function can be used to find the collective (social) preference orderings. With m candidates in A, scores of m-1, m-2, …, 1, 0 can be assigned to the first ranked, second ranked, …, last ranked candidate by each committee member. The Borda scores (the sum of the committee members scores) can be determined for each candidate. Finally, the candidates are ranked according to their Borda scores (Hwang & Lin, 1987).

Proposed Aggregation Method

In this paper, the new aggregation approach based on pair-wise comparison (in direction of the improved AHP models) is proposed. According to Asgharpour (2003, pp. 64-70), Azadfallah and Azizi (2015_c), and Azadfallah (2015_d), assume that there are m alternatives (or Attributes); with k members from an expert team. The decision makers' priorities obtain from pair wise comparison.

If we consider k_{ij} as the number of voter, where, A_i is preferred to A_j (and inversely, for k_{ji}; so that, $k_{ij} + k_{ji} = k$). Then, group decision-making matrix ($D_{m.m}$); are as follows.

$$D_{m.m} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_{i=j} \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccccc} A_1 & A_2 & \dots & A_j & \dots & A_m \end{array} \\ \left| \begin{array}{cccc} 1 & K_{12}/K_{21} & K_{1j}/K_{j1} & K_{1m}/K_{m1} \\ K_{21}/K_{12} & 1 & & \\ & \cdot & & \\ K_{i1}/K_{1i} & & 1 & K_{im}/K_{mi} \\ & & \cdot & \\ K_{m1}/K_{1m} & K_{m2}/K_{2m} & & 1 \end{array} \right| \end{array} \quad (1)$$

(Preference Ratio Matrix)

The element of $a_{ij} = k_{ij} / k_{ji}$ denoted that; the ratio of voters that support A_i over A_j, to the voters that support A_j over A_i (in other words, this ratios so called relative preference intensities).

D is a positive reciprocal matrix. Therefore, it has the reciprocal property and can be decomposed to eigen value and eigen vectors. On the other hand, in this matrix, it is not needed to do the consistency test. However, $a_{ij} = 1 / a_{ji}$, but in this model we have:

$$a_{il} . a_{lj} \neq a_{ij};_{\ j \neq 1} ; \text{Because of } (k_{il} / k_{li} . k_{lj} / k_{jl} \neq k_{ij} / k_{ji} \text{ OR } a_{ij} \neq a_{il} / a_{jl}) \quad (2)$$

PERRON-FROBENIUS says that a positive squared matrix has a real eigen value and dominant **(λ max),** so that the corresponded eigen vector is weight vector priorities and unique.

The eigen value **"λ max"** and its corresponding eigen vector can be obtained from:

I. Decomposing of D squared matrix[1] to eigen value and eigen vector:

$$D.W = \lambda . I . W \quad (3)$$

$$(D - \lambda . I) W = 0 \quad (4)$$

$$\det \left| D - \lambda . I \right| = 0 \quad (5)$$

And or **II**. Can be approximated by using the geometric mean:

$$g_i = (\prod^{m}_{j=1} a_{ij})^{1/m} ; i = 1, \dots , m. \quad (6)$$

The current method characteristics are neutrality, anonymity, monotonicity, homogeneity, and parato optimality.

Numerical Example:

In order to illustrate the application of the proposed method, two examples are given as follows.

Example 1: In this example, we try to find a set of preference orders of alternatives (suppliers) by employing four MADM methods (i.e. SAW, TOPSIS, ELECTRE, and WPM) simultaneously. Whereas, it is not the purpose of this paper to explain in detail the SAW, TOPSIS, ELECTRE, and WPM methodologies, we shall only point to the results (interested readers may refer to Hwang and Yoon, 1981 and Triantaphyllou, 2000, for

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[1] The foundation of decomposition decision matrix ($D_{m.m}$) to eigen value and eigen vector is the Dogson matrix (Asgharpour, 2003).

more discussions on the existing approaches). Assume that, a comparison of different method results (for ranking 5 suppliers) is as follows (Table 1).

Table 1: Preference Orders (ranks) by Four MADM Methods

Method / Alternative (or Suppliers)	SAW	TOPSIS	ELECTRE	WPM
S1	2	1	3	1
S2	3	2	2	3
S3	1	3	1	5
S4	5	4	4	4
S5	4	5	5	2

As seen in Table 1, different preference orders of alternatives (suppliers) was obtained using these 4 methods. Since, in order to have a consensus about different preference orders, aggregate method which includes Ranks Mean, Borda and Copeland methods have been employed.

Rank Mean Procedure

The average preference orders (over four MADM methods) are shown in the last column of Table 2 (labeled "average").

Table 2: Preference Orders by the Rank Mean Procedures

Method / Alternative (or Suppliers)	SAW	TOPSIS	ELECTRE	WPM	Average
S1	2	1	3	1	1.75
S2	3	2	2	3	2.5
S3	1	3	1	5	2.5
S4	5	4	4	4	4.25
S5	4	5	5	2	4

As seen in Table 2, based on the average rank, the aggregate rank order is $S_1>S_2\approx S_3>S_5>S_4$ where the symbol "≈" means be indifferent to.

According to Hwang and Yoon (1981, p. 216), if two alternatives have the same average rank (i.e. S_2 and S_3), the one with the smallest standard deviation can be ranked ahead. We find that the two alternatives (S_2 and S_3) generate the different standard deviation; S_2=0.50 and S_3=1.66 (based on formula: $\sigma^2_x = \sum (X_i-\mu_x)^2 / N$, $\sigma_x = \sqrt{\sigma^2_x}$), for each alternative. Therefore, the new rank order is $S_1>S_2>S_3>S_5>S_4$.

Borda Method

It is based on a majority rule relation (Hwang & Yoon, 1981). Therefore, the number of methods preferring A_j to $A_{j'}$ is greater than the number of methods preferring Aj' to Aj; hence A_j has a majority vote over $A_{j'}$, denoted $X_{jj'}$=1 [$X_{jj'}$=0, otherwise] (Table 3).

Table 3: Majority Vote between Alternatives (or Wins)

-	S_1	S_2	S_3	S_4	S_5	$\sum S_j$
S_1	-	1	1	1	1	4
S_2	0	-	1	1	1	3
S_3	1	1	-	1	1	4
S_4	0	0	0	-	1	1
S_5	0	0	0	1	-	1

As seen in Table 3, the highest Borda rank has been assigned to the alternatives S_1 and S_3 and the second highest to the alternative S_2, the group ranking is:

$S_1\approx S_3>S_2>S_4\approx S_5$

Copeland Method

This procedure starts where the Borda method stops. The Copeland methods consider not only how many "wins" an alternative has but also explicitly includes the "losses". The Copeland score is determined by subtracting the "losses" for an alternative from its "wins" (Hwang & Yoon, 1981). The result is shown in Table 4.

Table 4: Majority Vote between Alternatives (or Wins and Losses)

-	S_1	S_2	S_3	S_4	S_5	$\sum S_j$
S_1	-	1	1	1	1	4
S_2	0	-	1	1	1	3
S_3	1	1	-	1	1	4
S_4	0	0	0	-	1	1
S_5	0	0	0	1	-	1

$\sum S_{j'}$	1	2	2	4	4	-

The last row of Table 4 represents the number of "losses" for an alternative. For instance, the score of S_1 is (4-1) =3. The scores for S_1 through S5 are 3, 1, 2,-3, and -3, respectively.

From the above results, it can be easily derived that, the implied ranking is as follow.

$$S_1 > S_3 > S_2 > S_4 \approx S_5$$

Now, to aggregate the preference ordering into a consensus ordering, the new proposed model is used.

Proposed Model

In this method (the improved AHP models), the group decision-making model based on pair wise comparisons is proposed. Whereas, this is done by transforming the values from $a_{ij}=w_i/w_j$ to $a_{ij} = k_{ij} / k_{ji}$ (Azadfallah & Azizi, 2015c).

As noted earlier, among the most common MADM methods we can point out to SAW, TOPSIS, ELECTRE, and WPM. First, the preference (rank) of alternatives (suppliers) is determined for each of existing methods [we shall only point to the results] (Table 5).

Table 5: Preference Rankings by Four MADM Methods (based on table 1 information's)

MADM methods	preferences
SAW	$S_3>S_1>S_2>S_5>S_4$
TOPSIS	$S_1>S_2>S_3>S_4>S_5$
ELECTRE	$S_3>S_2>S_1>S_4>S_5$
WPM	$S_1>S_5>S_2>S_4>S_3$

Next, the following preference ratio matrix is created.

$$D_{m.m} = \begin{array}{c|ccccc} & S_1 & S_2 & S_3 & S_4 & S_5 \\ \hline S1 & 1 & 3/1 & 2/2 & 4/0 & 4/0 \\ S2 & 1/1 & 1 & 2/2 & 4/0 & 3/1 \\ S3 & 2/2 & 2/2 & 1 & 3/1 & 3/1 \\ S4 & 0/4 & 0/4 & 1/3 & 1 & 2/2 \\ S5 & 0/4 & 1/3 & 1/3 & 2/2 & 1 \end{array} \tag{7}$$

(Ratio preference matrix)

One issue needs to be mentioned here is that, in the illustrative example, we obtained a matrix with some zero ratio elements (for instance, a_{14}). However, this may not happen always. On the basis of the results above (table 5), the number of methods preferring S_1 to S_4 is greater than the number of methods preferring S_4 to S_1. In other words, S_4 is always less preferable to S_1, since S_{14}= 4/0.

Here, two points are noteworthy. First, according to Forman and Selly (2001, p. 62), pair wise comparisons are basic to the AHP methodology. On the other side, zero may not be used for pair wise comparison (Saaty & Vargas, 1991, p. 22; Wedley, 2007, p. 6). This is because division by zero intensity is undefined. Something (an intensity), can be divided by something else (another intensity), but something cannot be divided by nothing [zero] (Wedley & Choo, 2008). Therefore, AHP and its variants (especially, our proposed model in this paper), is not capable of considering the zero intensity. Hence, the critical question in this condition is, what do we do? Second, according to Asgharpour (2003, p. 70), the group at large with many individuals (experts), to be used has a significant effect to get a good approximation to the w_i/w_j from $a_{ij} = k_{ij} / k_{ji}$. Since, consistency and accuracy of proposed method can increase with increasing the number of experts. In other words, if the diversity is high, the outcome is considered more reliable. As noted earlier, in the illustrative example, we obtained a matrix with some zero ratio elements. This could cause some bias in the final result (considering all above-mentioned points it should also be considered). Nevertheless, for the purpose of the experiment in this paper, a simple approach (way) is proposed to resolve this limitation.

From the above debates, it can be easily derived that, the comparisons would always be equal to or greater than one. Since, we incorporated the additional step to the conventional approach in the conditions that the pair wise comparisons matrix with some zero ratio elements are present. So, the given matrix D can be converted into matrix D* (or revised matrix D), by the following transformation.

Two matrices [A] and [B] can be added only if they are the same size. The addition is then shown as (Kaw, 2002):

$$[C] = [A] + [B] \tag{8}$$

where $\quad c_{ij} = aij + b_{ij.}$

Have

$$[D^*] = [D] + [B] \tag{9}$$

where $\quad d^*_{m.m} = d_{m.m} + b_{m.m.}$

So, $d^*_{m.m}$=revised matrix, and $b_{m.m}$ include same positive elements (or constant, and b≥1), except the diagonals element (because of the diagonal of the matrix only contains 1's). Since (D, Eq. 1):

$$
B_{m.m} = \begin{array}{c|ccccc}
 & S_1 & S_2 & S_3 & S_4 & S_5 \\
S1 & 0 & 1/1 & 1/1 & 1/1 & 1/1 \\
S2 & 1/1 & 0 & 1/1 & 1/1 & 1/1 \\
S3 & 1/1 & 1/1 & 0 & 1/1 & 1/1 \\
S4 & 1/1 & 1/1 & 1/1 & 0 & 1/1 \\
S5 & 1/1 & 1/1 & 1/1 & 1/1 & 0
\end{array}
$$

(Positive elements [constant] matrix)

If b=1/1, when the Eq. (9) is applied, then the following values are derived. For instance, for a_{15} (4/0):

A15= (K15/K51) +b= (K15+1/K51+1) = (4/0+1/1)/(0/4+1/1) =5/1

The transformed matrix D (D*) is as follows.

$$
D^*_{m.m} = \begin{array}{c|ccccc}
 & S_1 & S_2 & S_3 & S_4 & S_5 \\
S1 & 1 & 4/2 & 3/3 & 5/1 & 5/1 \\
S2 & 2/4 & 1 & 3/3 & 5/1 & 4/2 \\
S3 & 3/3 & 3/3 & 1 & 4/2 & 4/2 \\
S4 & 1/5 & 1/5 & 2/4 & 1 & 3/3 \\
S5 & 1/5 & 2/4 & 2/4 & 3/3 & 1
\end{array}
$$

(The revised Ratio preference matrix)

For determining the weight of the alternatives (suppliers), from the group judgments (here, four MADM methods), decomposition of matrix method (discussed earlier), is considered.

Decomposition of Matrix Method

$\det|D - \lambda.I| = 0$

$$
D^*_{m.m} = \left|\begin{array}{c|ccccc}
 & S_1 & S_2 & S_3 & S_4 & S_5 \\
S1 & 1-\lambda & 5/3 & 4/4 & 6/2 & 6/2 \\
S2 & 3/5 & 1-\lambda & 3/3 & 5/1 & 5/3 \\
S3 & 4/4 & 4/4 & 1-\lambda & 5/2 & 5/3 \\
S4 & 2/6 & 2/6 & 3/5 & 1-\lambda & 4/4 \\
S5 & 2/6 & 3/5 & 3/5 & 4/4 & 1-\lambda
\end{array}\right| = 0 \quad (10)
$$

Results are the following:

$\lambda\ 1 = \lambda max\ 5.1778$

$\lambda\ 2 = -\ 0.0065+0.8323\ i$

$\lambda\ 3 = -\ 0.0065-0.8323\ i$

$\lambda\ 3 = -\ 0.0824+0.4680\ i$

$\lambda\ 3 = -\ 0.0824-0.4680\ i$

Next, consider the following homogeneous linear equations:

$$
D_{m.m} = \begin{array}{c|ccccc|}
 & S_1 & S_2 & S_3 & S_4 & S_5 \\
S_1 & -4.1778 & 5/3 & 4/4 & 6/2 & 6/2 \\
S_2 & 3/5 & -4.1778 & 4/4 & 6/2 & 5/3 \\
S_3 & 4/4 & 4/4 & -4.1778 & 5/3 & 5/3 \\
S_4 & 2/6 & 2/6 & 3/5 & -4.1778 & 4/4 \\
S_5 & 2/6 & 3/5 & 3/5 & 4/4 & -4.1778
\end{array} . \begin{vmatrix} W1 \\ W2 \\ W3 \\ W4 \\ W5 \end{vmatrix} = 0 \quad (11)
$$

Results as follows (after normalisation):

$$
\longrightarrow \begin{vmatrix} W1 \\ W2 \\ W3 \\ W4 \\ W5 \end{vmatrix} = \begin{vmatrix} 0.369 \\ 0.235 \\ 0.226 \\ 0.078 \\ 0.092 \end{vmatrix} \quad (12)
$$

$\mathbf{W^t} = \{0.369, 0.235, 0.226, 0.078, 0.092\}$

The results indicate that the weight of importance is in the order of :

$S_1=0.369>S_2=0.235>S_3=0.226>S_5=0.092>S_4=0.078$

As seen from the above results, the first supplier is the most preferred one.

According to Wang (2007), one evaluating procedure is to examine the stability of a Multi Criteria Decision Making (MCDM) methods mathematical process by checking the validity of its proposed rankings. This is the problem we wish to address here. Therefore, this paper investigates the stability of the proposed method results by **1.** Checking the preference ranks via different MADM methods. In continuation, similarities and differences in the behaviour of these methods are investigated. A comparison of test results is given in Table 6.

Table 6: Comparison Results

Methods	Preferences
Ranks Mean	$S_1>S_2>S_3>S_5>S_4$
Borda	$S_1\approx S_3>S_2>S_4\approx S_5$
Copeland	$S_1>S_3>S_2>S_4\approx S_5$
Proposed Method	$S_1>S_2>S_3>S_5>S_4$ (0.369 0.235 0.226 0.093 0.078)

As can be seen from Table 6, S_1 ranking is best for the all methods. In addition, the proposed method is the same results as compared with Ranks Mean method. However, these results indicate that the proposed method and the other method (i.e. Borda and Copeland methods) are not the same whereas there is disagreement among another alternative orderings (except S_1). In addition, according to Azadfallah (2015a), different MADM techniques may yield different results for the same problem. Nevertheless, the proposed method seems to be more satisfactory than the other methods for solving decision problems because of, a) the results in both ordinal and cardinal form are presented, and b) the results can more assurance by applying a systematic model (AHP based model). Another question is what happens if we converted the revised ratio preference matrix (D*, b≥1) to D** (b≥2), and redo our assessment of alternatives.

The transformed matrix D** is as follows (clearly, this is an assumption that is made here in order to study different conditions):

$$D^{**}_{m.m} = \begin{array}{c|ccccc} & S_1 & S_2 & S_3 & S_4 & S_5 \\ \hline S_1 & 1 & 5/3 & 4/4 & 6/2 & 6/2 \\ S2 & 3/5 & 1 & 4/4 & 6/2 & 5/3 \\ S_3 & 4/4 & 4/4 & 1 & 5/3 & 5/3 \\ S_4 & 2/6 & 2/6 & 3/5 & 1 & 4/4 \\ S_5 & 2/6 & 3/5 & 3/5 & 4/4 & 1 \end{array}$$

(Re- revised Ratio preference matrix)

The computation are available for D** and are not repeated here. In brief, the result is as follows.

(0.316, 0.230, 0.227, 0.107, and 0.119)

A comparison of test results is given in Table 7.

Table 7: Comparative Results

Transformed Matrix	Priorities
D*	$S_1>S_2>S_3>S_5>S_4$ (0.369 0.235 0.226 0.093 0.078)
D**	$S_1>S_2>S_3>S_5>S_4$ (0.316 0.230 0.227 0.119 0.107)

In practice, it was shown that the priorities obtained by the two different transformed matrix (D* and D**), are quite close to each other (D**≈D*). Since, it is evident that priorities obtained from transformed matrix D** is compatible with the result of transformed matrix D*. So, if priorities for D* and D** is D**≈D*, then may be say D*≈ D. nevertheless, this could cause some bias in the final result. So, more studied are needed (This is beyond the scope of this study).

Example 2: In this example, TOPSIS and Borda's function approach are discussed. So, in the present model, first TOPSIS is used to find the individual preference ordering. Then, Borda's function is used to find the collective preference orderings. As noted earlier, it is not the purpose of this paper to explain in detail the TOPSIS and Borda's function methodologies; we shall only point to the starting and final results (interested readers may refer to Hwang & Lin, 1987, for more discussion on the existing approaches).

Assume that there are six committee members, who are experts (Experts; E_1, E_2… E_6), five alternatives (Suppliers; S_1, S_2… S_5), and three criteria (C_1= shorter lead times, C_2= reduced cost, and C_3= higher quality). As you see, the performance values for each expert are shown in Tables 8-13. Moreover, several researchers have argued that the equal weight rule is often a highly accurate simplification of the decision making process (Birnbaum, 1998). Thus, W_j= (0.333, 0.333, and 0.333).

In the process of group decision making, each committee member (expert), provides the performance values, the alternative with respect to each attribute. For expert 1:

Table 8: Performance Value for E_1

Criteria / Alternative	C_1*	C_2*	C_3*
S_1	5	1	9
S_2	7	5	9
S_3	1	9	7
S_4	1	5	5
S_5	7	1	9

*.note, that all criteria's are the benefit type (in all tables).

For another expert is as follows.

Table 9: Performance Value for E_2

Criteria / Alternative	C_1	C_2	C_3
S_1	7	3	7
S_2	7	9	1
S_3	3	9	9
S_4	5	3	1
S_5	7	1	3

Table 10: Performance Value for E_3

Criteria / Alternative	C_1	C_2	C_3
S_1	**7**	**1**	**7**
S_2	**5**	**9**	**1**
S_3	**3**	**5**	**7**
S_4	**3**	**1**	**5**
S_5	**5**	**3**	**9**

Table 11: Performance Value for E_4

Criteria / Alternative	C_1	C_2	C_3
S_1	1	9	9
S_2	1	7	7
S_3	5	1	5
S_4	9	1	7
S_5	7	3	1

Table 12: Performance Value for E_5

Criteria / Alternative	C_1	C_2	C_3
S_1	9	9	9
S_2	1	3	3
S_3	7	7	1
S_4	1	9	7
S_5	9	5	1

Table 13: Performance Value for E_6

Criteria / Alternative	C_1	C_2	C_3
S_1	7	3	9
S_2	3	9	9
S_3	3	1	9
S_4	9	7	5
S_5	7	5	3

A comparison of the test results (individual prefer ordering by TOPSIS for 6 experts), is given in Table 14.

Table 14: The Comparative Results

Rank by TOPSIS / Expert	priorities
E_1	$S_2>S_3>S_5>S_1>S_4$ (0.277 0.235 0.191 0.155 0.142)
E_2	$S_3>S_1>S_2>S_5>S_4$ (0.324 0.241 0.215 0.132 0.088)
E_3	$S_2>S_5>S_3>S_1>S_4$ (0.250 0.228 0.225 0.189 0.107)
E_4	$S_1>S_4>S_2>S_5>S_3$ (0.247 0.225 0.211 0.170 0.149)
E_5	$S_1>S_4>S_5>S_3>S_2$ (0.394 0.207 0.173 0.161 0.066)
E_6	$S_4>S_2>S_1>S_5>S_3$ (0.266 0.241 0.192 0.177 0.124)

In this section, Borda's function is used to find the collective preference orderings. So, 4, 3, 2, 1, and 0 scores are assigned to the first rank, second rank … and last rank. i.e., for S_1:

$$S_1 = 1+3+1+4+4+2=15$$

Similarly, $S_2=15$

$S_3=10$

$S_4=10$

$S_5=10$

From the above results, it can be easily derived that, the implied ranking is as follows.

$$S1 \approx S_2 > S_3 \approx S_4 \approx S_5$$

Now, in order to compare proposed method in this paper with the TOPSIS and Borda's function approach, we will use the same numerical example (Table 13). We will only comment on the starting (ratio preference matrix) and final results by the methodology on the previous example.

$$D_{m.m} = \begin{array}{c|ccccc} & S_1 & S_2 & S_3 & S_4 & S_5 \\ \hline S1 & 1 & 3/3 & 3/3 & 5/1 & 4/1 \\ S2 & 3/3 & 1 & 4/2 & 3/3 & 5/1 \\ S3 & 3/3 & 2/4 & 1 & 3/3 & 2/4 \\ S4 & 1/5 & 3/3 & 3/3 & 1 & 3/3 \\ S5 & 2/4 & 1/5 & 4/2 & 3/3 & 1 \end{array} \quad (14)$$

$\mathbf{W^t}$ **= {**0.29630**,** 0.29615**,** 0.13901, 0.13506, 0.13347**}**

The results ($S_1>S_2>S_3>S_4>S_5$) show that, the first supplier is the most preferred one. A comparison of the results is provided in Table 15.

Table 15: The Comparative Results

Method	Priorities
TOPSIS and Borda's function	$S1\approx S_2>S_3\approx S_4\approx S_5$
Proposed method	$S_1>S_2>S_3>S_4>S_5$ (0.29630**,** 0.29615**,** 0.13901, 0.13506, 0.13347)

A comparative analysis is performed (Table 15), and the proposed method seems to be more satisfactory than the other method (TOPSIS and Borda's function approach), for solving decision problems because of1. the results (priorities) are present in both ordinal and cardinal form, and 2. the results can give more assurance by applying a systematic model (AHP based model). These advantages make proposed method a major MADM method as compared with other related techniques such as Mean Rank, Borda, and Copeland methods.

Conclusion

In this paper, we propose a new MADM approach to aggregating the individual pre-orders to single collective pre-order whereas, the improved AHP model of Asgharpour (2003) is adopted in supplier selection context for aggregating individual decision to reach consensus. In addition, the advantage and disadvantage of proposed method are compared with an existed aggregation models (Means Rank, Borda, Copeland, and TOPSIS and Borda's function). In order to illustrate the application of the proposed method, two examples are given. In Example 1, comparative results (Table 6) indicate that preference orders by proposed method are the same as compared with Ranks Mean method. In addition, Supplier 1 (S_1) ranking is best for the all methods (Ranks Mean, Borda, and Copeland method), and there is disagreement among another alternative orderings. In Example 2, there is not even any resemblance in the rankings derived by using both approaches (TOPSIS and Borda's function, and proposed method), except that both approaches rank S_1 as the best alternative. Moreover, according to the results of Table 15, we can find the priority is $S1\approx S_2>S_3\approx S_4\approx S_5$. This priority is different from the proposed method ($S_1>S_2>S_3>S_4=>S_5$). This means that the proposed method is more sensitive to small differentiation. Also, in Compare with Ranks Mean, Borda, Copeland, and TOPSIS and Borda's function approach, the proposed method in this paper seems has some attractive advantages. First, the results (priorities) are present in both ordinal and cardinal form. Next, the results can more assurance by applying a systematic model. Further research can apply this proposed approach to other managerial issues or compare it with another MADM models.

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