Design of Gear Tooth Profile Using Free Form Curve for Optimum Stresses

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Abstract

Gear is a key component which finds many applications in numerous engineering sectors and its design plays an important role in a power transmission system. This paper presents the new design concept of a gear tooth profile using free form curve in order to reduce bending stress. The designed tooth profile is tested with a series of simulation results which show the effect in reduction of stress under the variation of face width and speed. The results obtained from simulation test are satisfactory and effective from previous one.

Keywords: Tooth profile, Free form curve, Stress Analysis

INTRODUCTION

The primary function of gear is to transmit from one axis to another. The gear finds application in industries, for the power transmission point of view. So, its demand is higher in load- carrying capacity and increased fatigue life. The design of gear is a complex process and it needs large set of iteration. The bending fatigue is the most critical failure in gear which leads to the breakage of gear. In order to take off from these types of failure many methods were applied.

Fernandes “[1]” discussed the characteristics of gear failure mode and a number of definite case studies are presented which show the occurrence of bending fatigue failure mode in practice. The bending stress in the root fillets of gear teeth having either convex as well as circular-arc profiles has been calculated by Chen “[2]” using a new analytic model for variable surface pressure distribution. Pedersen “[3]” presents a technique of shape optimization of the tip of the standard cutting tool which leads to the minimization of bending stress of involute teeth. S. Sankar “[4]” uses a profile modification technique in which they replace a Trochoidal root filet with Circular root filet. The results show minimization in the deflection.

Celniker “[5]” gives the idea about the development of an improved computer based freeform design methodology capable of interactively defining fair shapes with a minimal amount of user input. The uses of Free-form parametric curves are extensively employed in various fields such as: computer graphics, computer vision, robotics, and geometric modeling “[6]”. Campbell “[7]” gives detailed data describing the shape of an object offers the computer vision practitioner new ways to recognize and localize free-form objects. Dorai propose a new and general surface representation scheme for recognizing objects with free-form (sculpted) surfaces.

Hebert “[8]” approach begins by representing an object by a discrete mesh of points built from range data or from a geometric model of the object. They also shows the similarity between reference model and observed data can be easily computed using this representation. Kunjin “[9]” gives the concept about the Region of Interest (ROI). They create a user- defined freeform feature.

As freeform curve is used in complex products. In general Non Uniform Rational B-Spline (NURBS) is prevalent mathematical representation for both freeform and standard analytical surface in Computer Aided Design (CAD). Thomas “[10]” used NURBS as a deviation function to design a new tooth profile which is capable

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of carrying higher loads than gears of equivalent sizes. Yahaya “[11]” uses the transition curves with curvature continuity as the degree of smoothness to redesign a spur gear.

FREE FORM CURVE

The curve plays an important role to design a gear tooth profile and for this a Bezier curve as a free form curve is considered. The following procedure in order to design a profile is considered.

1. Gear parameters selection
2. Construction of tooth profile using Bezier Curve
3. Evaluation of Gear

Gear Parameters Selection

As from the case studies the regular failures in gear occurs due to the bending stress. So, based on the objective to minimize the bending stress with face width and speed the following gear specification is considered:

Table 1: Gear Parameters

Gear tooth Type	*Standard Involute full Depth*
Number of teeth (z)	24
Module (m)	4 mm
Pressure angle (α)	20°
Helix angle (β)	Spur gear
Pitch circle diameter (D)	96 mm
Addendum circle diameter	104 mm

Design of tooth profile using Bezier Curve

The Bezier curve with four degree and having five control points is used for the generation of tooth profile. The equation of the curve as follows:

$$P_u = \sum_{i=0}^{n} P_i B_{i,n}(u) \quad 0 \le u \le 1 \qquad 1$$

Where, Pi is a control point

n is degree of cure

i is number of control points

Bi,n is a bending function and it is given by

$$B_{i,n} = {}^{n}C_i u^i (1-u)^{n-i}$$

$$\frac{n!}{i!(n-i)!} u^i (1-u)^{n-i} \qquad 2$$

With expanding the equation 1 in terms of x and y coordinates to obtain the points. With the help of MATLAB R2009a the plotting of the curve points is done.

EVALUATION OF GEAR

The last and final step considered the evaluation of newly designed gear. From the case studies of gear failure in industrial background in testing point of view it reveals the two most high stress regions that normally a source of failure. These are the region of root fillet and the surface in contact of the teeth. So based on this regions the most critical failure due to bending stress of the new gear is analyzed. Also the deflection is examined.

Tooth Bending Stress

At the first the bending stress of the newly designed gear will be evaluated. The gear tooth will be considered as a cantilever beam having a rectangular cross section used for the evaluation of bending stress. It follows some assumptions which are tooth is straight, isotropic and consist of a cross section which is undergoing pure bending, and then we can apply a flexure formula. In, those case the highest stress occurrence at the top and bottom of this cross-section.

$$\sigma = \frac{My}{I} \qquad 3$$

Where, the moment of inertia, $I = \frac{bt^3}{12}$ 4

b is face width and t is thickness,

M is the maximum bending moment at filled support,

$$M = W_t h \qquad 5$$

h is height of application of load

W_T and W_R are tangential and radial load acting on tooth as shown in fig.1.

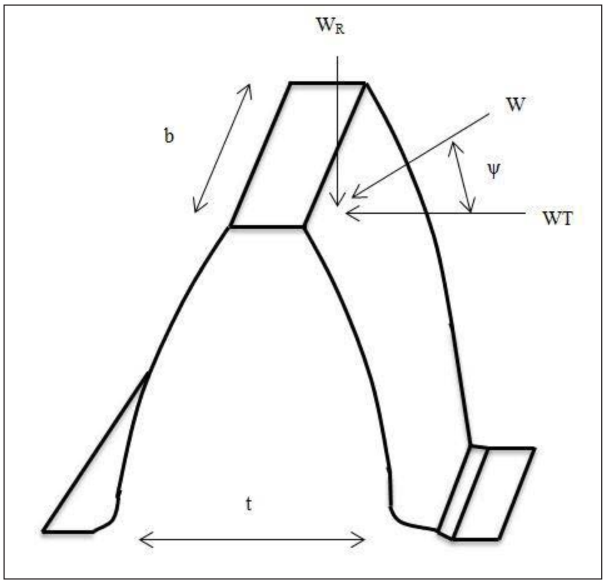


Fig. 1. **Tooth Loading Condition**

GEOMETRIC MODELING

Based on the Bezier curve control points the tooth profile is generated in MATLAB R2009a and the same points are imported in the Autodesk Inventor 2014 design and modeling software in order to design a new gear tooth profile. Parametric modeling gives the dependency of one parameter over the other if any parameters get changed then relatively all parameters changes. Fig. 2 shows the 3D view of the Spur gear which is designed in Autodesk.

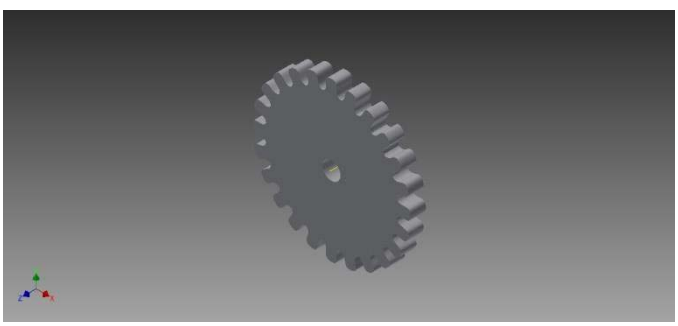


Fig. 2. **3D Modeling of Spur Gear in Autodesk Inventor 2014**

Finite Element Analysis

For the FEA procedure only the tooth part of gear is considered. The analysis work is done over anAnsys Workbench 14.5. By considering the gear conditions as power transmitted is 4 KW at speed of 318 rpm. The material of gear having modulus of elasticity E= 210 GPa and poisons ratio = 0.3.

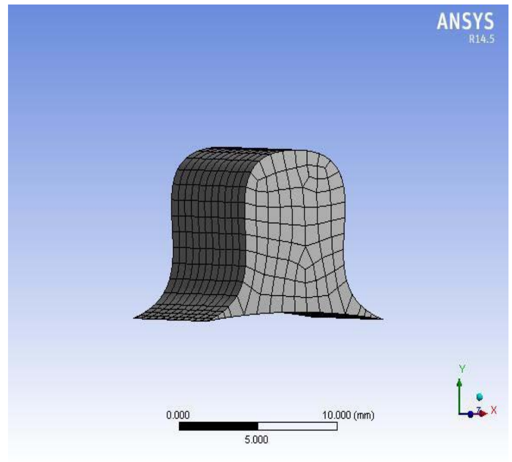
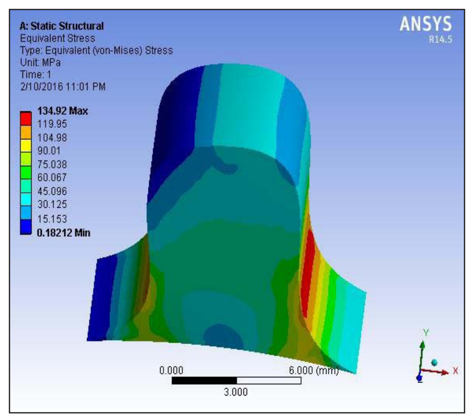
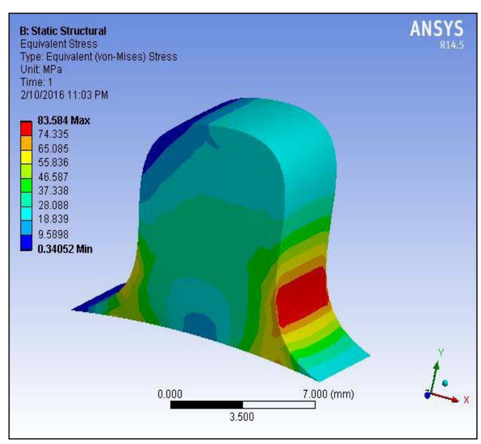


Fig. 3. **Gear tooth with mesh**

Fig 3 represents a tooth, with having mesh, which is designed with the help of Bezier curve. And accordingly the variation in face width with different stress analysis has been shown in the fig 4.



(a)



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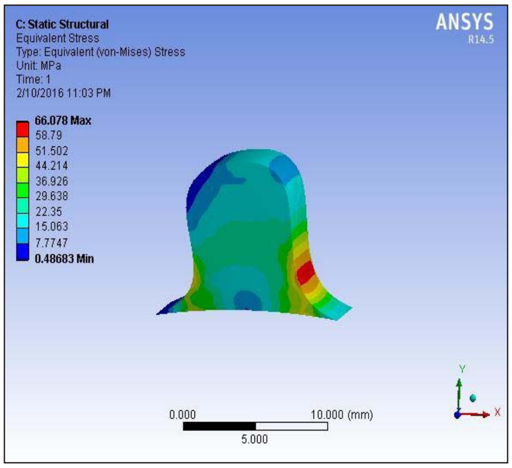

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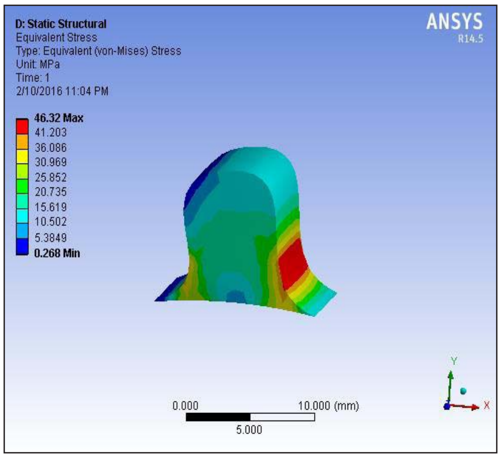

(d)

Fig. 4. Ansys Result for different face width (a) 10mm, (b) 15mm, (c) 20mm and (d) 25mm.

RESULTS AND DISCUSSION

The table 1 gives the information and comparison in between the existing model and the new designed model.

For theoretical bending stress calculation taking h=6.4 mm.

Table 2: Bending stress at different face width

Face width mm	*Existing Model MPa*	*Bezier Curve MPa*	*Theoretical MPa*
10	152.51	134.95	146.53
15	101.15	83.58	97.69
20	73.43	66.08	73.26
25	57.45	46.32	58.61

The variation of stress along with the change in face width is shown in fig 5. And it shows the decrease in the bending stress in the newly designed gear.

Another graph shows the change in bending stress for the different number of cycles or speed. This graph as shown in fig 6, states the effect of width and speed on the bending stress.

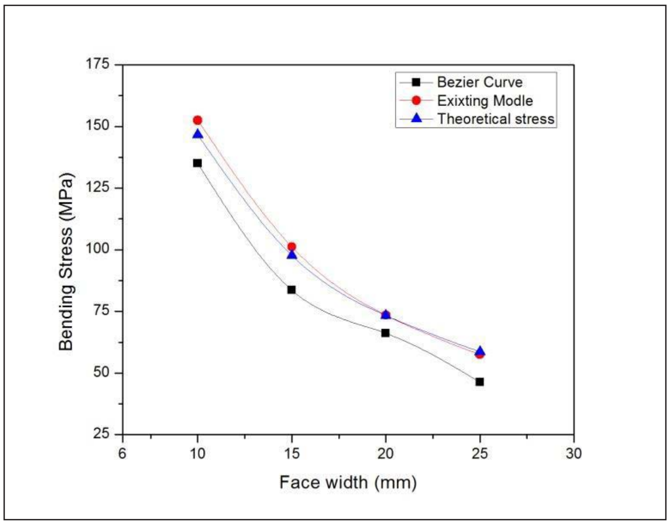

Fig. 5. Graph of Bending stress Vs Face width

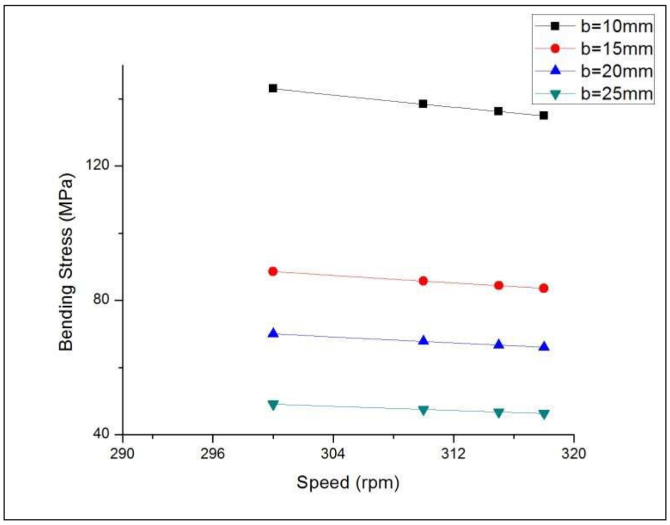

Fig. 6. Graph of Bending stress Vs Speed with different face width.

CONCLUSION

The results obtained from the theoretical and simulation analysis it is found that the develop methods to design the gear tooth profile using proposed quartic Bezier curve gives optimum bending stress.

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