P-Hub Model with Mode of Transportation and Fuzzy Flow and Fixed Cost

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ABSTRACT

Hubs are special facilities which work as connecting and replacement points. Instead of providing source-destination pair service, hub facilities concentrate the flow to utilise its consequential economical savings. In this article, after presenting the prominent P-Hub model, the cost of hub location will be added to the price of the objective function of this model, the limitation of the number of hubs will be eliminated from this model and then, different forms of transportation will be discussed and analysed. Furthermore the way it is added to the model will be explained as well. Afterwards, this model will be formulated with the current in fuzzy form and in order to clarify the achieved improvement, a case study according to the data from Iranian airports is presented. Finally the two heuristic algorithms in accordance with the proposed model, is presented and their efficiency is analysed and discussed.

Keywords: P-Hub Problem, Multi Transportation Modes, Uncertain Parameters, Heuristic Algorithms

INTRODUCTION

An average of fifty percent of total logistics cost is due to transportation. Efficient transportation system provides a competitive edge to the retailer. Distribution centre and transportation network are remarkably expensive if not controlled effectively (Parkhi, Jagadeesh & Kumar, 2014). The hub location problem is concerned with locating hub facilities and allocating demand nodes to hubs in order to route the traffic between origin-destination pairs (Ernst, Jiang, Krishnamoorthy & Baatar, 2011). In some cases, direct communication between different nodes is very expensive. In order to use the scale savings, it's better to move goods via other nodes, called hubs. So, instead of using a direct connection between source and destination, indirect connection is used (connections to hubs). In such cases, the objective of network setup is allowing the flow of goods through non-hub nodes, by hub nodes (Yaman & Elloumi, 2012). In practice, economy of scale means that larger and more efficient aircrafts in lines that connecting hubs are considered and incase of communication networks, use of optical fibers with a higher capacity to communicate between the coupling hubs is considered. Because of many applications of modern transportation and communication systems, the hub location problems have obtained great importance in recent decades. Hub collection is responsible for transmission and distribution flows in distributed systems. Hub location problem is using in delivery and communication systems, postal networks, airline systems and transportation networks (Zanjirani Farahani, Hekmatfar, Boloori Arabani & Nikbakhsh, 2012).

The hub location problem has a short history. The first published article in this case was issued by Hakimi (1964). But it took a long time to be known as hub location problem. Then, the hub model was developed by O'Kelly (1987). He played an important role in early development, especially in modeling. Campbell continued to play a key role in completing the various hub models. His and his colleagues' article in 1771, is one of the most important papers in variety of hub models. In addition, Klincewicz and Aykin also had a significant role. The articles trend over time has been upward. Since1776, papers were significantly faced with growth that seems it reaching maturity hub problem.

Many applications can be found in this area. One of these classic examples is the Hungary railway in which Budapest is a hub. Two major USA airlines with headquarters in Atlanta and Chicago have hubs. The Dubai International Airport also is a hub of many international flights in the Middle East. Hubs are special facilities that are used as connecting and switching points offlow from source to destination. Unification of the source path to destination and between the hub and the hub takes place. When there is a need to locate the hub facilities, there is no need to specify the number of hubs and the model should determine the most economical number of hubs based on the cost. Cost per unit of goods which are transporting between nodes should be determined based on the vehicle that transfers goods (Zarandi, Davari & Sisakht, 2012). Accordingly, in this paper after proposing the famous P-Hub model, the cost of the hub is added to the objective function and limitation of p-hubs from the model is removed. In the next section, the model is formulated. Finally, to clarify the development of the paper we provide a detailed example. Given that the solution time of the problem significantly increases as the number of nodes increases, in some cases, the GAMS software is not able to solve the problem. In the end, two heuristic algorithms are proposed that reduced solution time significantly as well as their performance is analysed.

P-HUB MODEL

This problem first was provided tby the O'Kelly. The objective function of this problem is to minimise the total cost. Each node must only be connected to a hub and for traveling between two nodes, at least one and at most two hubs are met (Campbell & O'Kelly, 2012). A hub network is a full and unlimited capacity. In this model, Cij is the cost (time or distance) between two nodes, i, j and α is cost factor of this two hubs (0≤α ≤1).

hij: flow between *i* and *j*

Yij: equal to 1 if node*i* is allocated to the hub located at j and otherwise equal to zero.

Yjj: equal to 1 if node *j* is selected as the hub, and otherwise equal to zero.

The mathematical model is as follows:

$$\min \sum_{i}\sum_{k} C_{ik}\ Y_{ik}\left(\sum_{j} h_{ij}\right) + \sum_{k}\sum_{i} C_{ik}\ Y_{ik}\left(\sum_{j} h_{ji}\right) + \propto \sum_{i}\sum_{j}\sum_{k}\sum_{m} h_{ij} C_{km} Y_{ik} Y_{jm}$$

$$\sum_{j} Y_{ij} = 1\ \forall i \qquad 1$$

$$\sum_{j} Y_{ij} = P \qquad 2$$

$$Y_{ij} - Y_{ij} \le P \qquad 3$$

$$Y_{ij} \in Y_{jj} \le 0\ \forall i,\ j \qquad 4$$

Objective function minimises the total cost of the network. Constraint (1) states that every non-hub node must be assigned to only one hub. Constraint (2) specifies the number of hubs and constraint (3) ensures that if j is a hub, node i can be allocated to it.

MODEL EXTENSION

In order to make the model closer to reality, the number of hubs constraint from the p-hub is removed and instead of it, the fixed cost of the hub construction () is added to the model. It makes your model obtain the optimal hub numbers by itself and in this paper, it defines the mode of transport as a type of vehicle should be assigned to the links (Alumur & Kara, 2008). Considering this provision that the vehicles have their own capacity include strucks, trailers, etc. in road transport and a variety of digital transmitters with different capacity (O'Kelly, 1992).

In fact considering the various modes of transport, α coefficient is modified, which is unprecedented in the literature.

Modeling and definition of the parameters is as follows:

A_l: capacity of lth vehicle

Y_{kml}: zero and one variable, if one takes the total amount of current that passes between nodes k and m, l do not exceed the capacity of the vehicle.

αl: discount factor cost by L th.

Cij^T: Adjusted cost (distance or time) between nodes i, j with respect to the mode of transport

$$\min \sum_{i}\sum_{k} C^T_{ik}\ Y_{ik}\left(\sum_{j} h_{ij}\right) + \sum_{k}\sum_{i} C^T_{kj}\ Y_{ik}\left(\sum_{j} h_{ji}\right) + \propto \sum_{i}\sum_{j}\sum_{k}\sum_{m} h_{ij} C^T_{km} Y_{ik} Y_{jm} + \sum_{j} F_j Y_{jj}$$

s.t.

$$C^T_{ik} = \left(\sum_{i} \alpha_l\ Y_{ikl}\right) C_{ik} \qquad 5$$

$$C_{km}^{T}=\left(\sum_{l}\alpha_{l}\ Y_{kml}\right)C_{km} \quad 6$$

$$C_{mj}^{T}=\left(\sum_{l}\alpha_{l}\ Y_{mjl}\right)C_{mj} \quad 7$$

$$\left(\sum_{l=2}^{L}A_{l}\ Y_{ikl}\right)\leq\sum_{j}h_{ij}Y_{ik}\ \forall\, i,k \quad 8$$

$$\left(\sum_{l=2}^{L}A_{l}\ Y_{kml}\right)\leq\sum_{i}\sum_{j}h_{ij}Y_{ik}\ Y_{mj}\ \forall\, k,m \quad 9$$

$$\left(\sum_{l=2}^{L}Al\ Y_{mjl}\right)\leq\sum_{i}h_{ij}Ymj\ \forall\, m,j \quad 10$$

$$\sum_{i}Y_{ikl}=1\ \forall\ i,k \quad 11$$

$$\sum_{i}Y_{mjl}=1\ \forall\ m,j \quad 12$$

$$\sum_{i}Y_{kml}=1\ \forall\ m,k \quad 13$$

$$\sum_{i}Y_{ij}=1\ \forall\ i \quad 14$$

$$Y_{ij}-Y_{jj}\leq 0\ \forall i,j \quad 15$$

$$Y_{ij},Y_{jijl}\in(0,1) \quad 16$$

The first part of the objective function shows the cost of non-hub node *i* to hub node *k*. The second part of the objective function shows the cost between a hub node *k* to other hubs node *j*. The third part of the objective function represents the cost between hub nodes and the fourth shows the fixed costs of hubs. This fixed cost causes that the model determines economical number of hubs by itself. Constraints (8-10) give values to $Y_{i,k,l}$, $Y_{k,m,l}$, $Y_{m,jl}$ zero-one variables according to the capacity of the vehicle and the amount of flows that can be transmitted between nodes and accordingly, the Constraints (5-7) constraint obtain values and Constraints (11-13) do not allow more than one obtaining value for each of the $Y_{i,k,l}$, $Y_{k,m,l}$, $Y_{m,jl,}$variables. In the above model, given that the objective function is a kind of minimizing and$\alpha_{1\leq\ldots\leq}$ α_L, therefore, according to the Constraints (5-7), the model is trying to establish more flow between two nodes in order to use a vehicle with more capacity and use more additional discount. The smaller discount rate used in the objective function reduces their costs. Other restriction is such as the P-hub model.

FUZZY FLOW BETWEEN NODES

In today's uncertain and turbulent markets, supply chain vulnerability has become an issue of significance for many companies. The challenge to business today is to manage and mitigate that risk through creating more resilient supply chains (Parveen, Arif & Nishat, 2014). Considering the importance of this issue, in this paper the flow between nodes is given by Lai and Hwang approach, as triangular Fuzzy numbers and we solve the problem considering the low coefficients w_1 for lower bound, w_2 for middle bound and w_3 for upper bound for triangular numbers that changing the coefficients w, can change the results ($w_1+w_2+w_3=1$).

The flow between nodes are shown with h symbol, that is considered with a triangular fuzzy number (h^L ,h^m , h^u).

CASE STUDY

Many people have done studies on solution approaches, such as García *et al*. (2012). To clarify the problem, in this section a detailed example is given. In this example (adopted from Karimi & Bashiri, 2011), 10 cities of Iran are considered as nodes and information about the distance between cities and the flow between them is the number of passengers.

First, the non-linear model is changed to linear model. Because the objective function was set up of multiplying two binary variables and solving the problem of non-linear model, it had a time solution as well as it didn't obtain optimum solution or even it was not near the optimal answer. The model was developed using GAMS software and it was implemented by CPLEX solver. Obtained solution after linearisation is optimal with no gap.

The problem was solved in two modes:

1. Solving the proposed solution in third section of the paper, regardless of the mode of transport (in p-hub model only difference is that the limited number of hubs is removed and the cost is added to the objective function).
2. The proposed solution is presented in third section of this article, as well as the flows have been considered using Fuzzy transport (final version).

In the first case (proposed model in third section of this article, regardless of the mode of transport), as can be seen in Fig.1, Isfahan node is selected as hub among 10 nodes and all of the nodes are connected to this node. In this case, transportation capacity is uncertain.

In the second case (model proposed in third section of this article, as well as the flows are considered Fuzzy), as can

Fig. 1: Optimum Transportation of Model 1

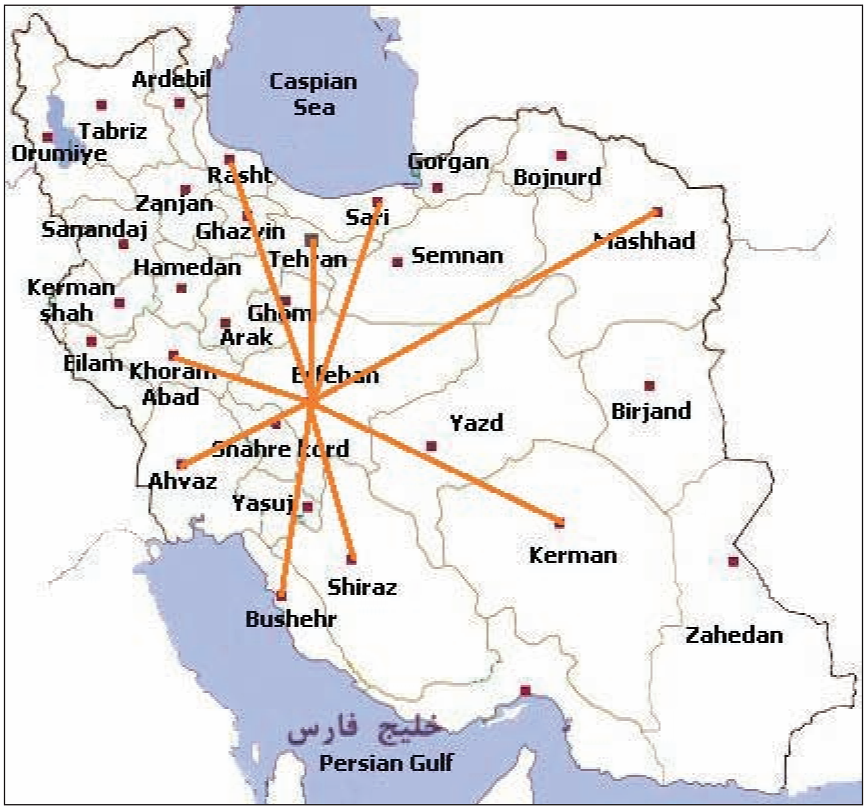

Table 1: Case Study Information

	Number of vehicle	Vehicle capacity	Fuzzy flow	No. of obtained nodes	Hob nodes	gap
First case	3	infinite	-	1	4	0%
Second case	3	50 and 120 and infinite	*	2	8 and 4	0%

be seen in Table 1 and Fig. 2, out of the 10 nodes, Tehran and Isfahan nodes are considered as the hub nodes.

Now it should be noted that we solve the above problem without considering the Fuzzy flow in which we get results exactly the same as in the second case. But if we consider flows as Fuzzy, the results won't change. As we once solve this problem with 8 cities, and regard the Fuzzy flow that here hubs and hub and non-hub connections between the nodes of the two things were different.

Considering flow state as fuzzy and transportation, according to the above examples, they can answer the problem as well as change connections between nodes. Of course, it depends the issue of data and parameters (coefficients discounts and capacity of vehicles, etc.).

HEURISTIC ALGORITHMS

Since solving the proposed model by GAMS software needs a lot of time and if the number of nodes increase, the GAMS software cannot reach to an optimized answer, here we present two algorithms for solving the proposed model. Finally, notice that the results are very good and take very little time to solve.

The steps of the two algorithms are as follows:

First Heuristic Algorithm

Step 1: Get problem data (n, C, h)

Step 2: Sort the nodes in descending order according to their input flows and put them into a set as w. Specify the node with the most current and set

Fig. 2: Optimum Transportation of Final Model with Fuzzy Transport

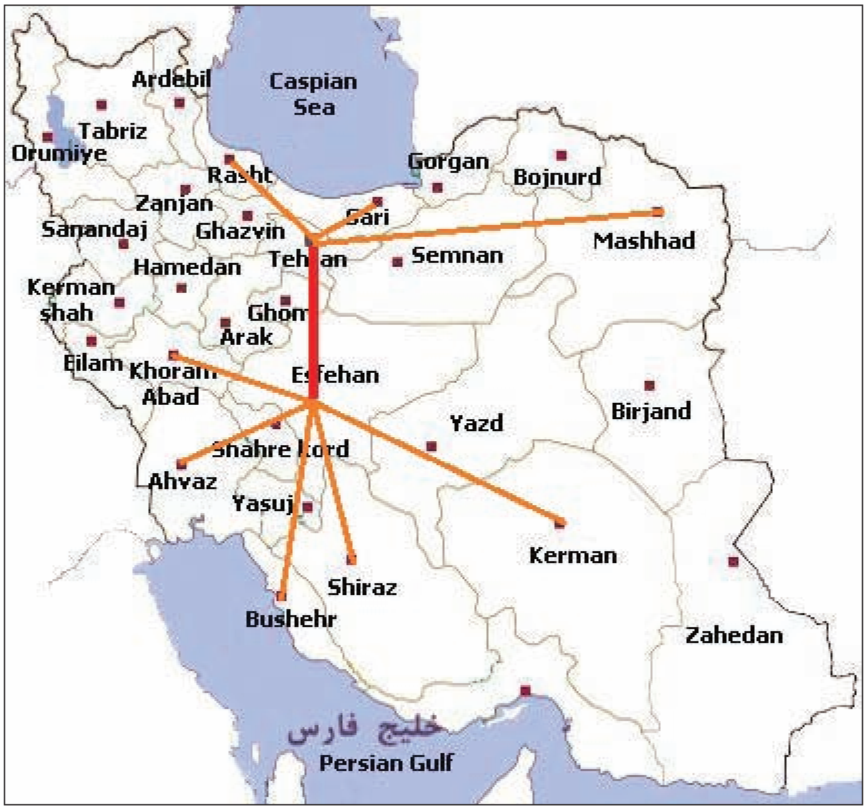

Table 2: Solve by GAMS

	No. of hubs	Solution time	Objective function value	Gap (%)
Using 5 nodes	1	1.8 seconds	3400	0%
Using 10 nodes	2	11 minutes and 35 seconds	18172	0%
Using 37 nodes	-	-	-	-

it as a hub and calculate connectivity matrix and calculate the objective function.

Step 3: Update the minimum objective function and matrix connections.

Step 4: For k=2to n, iterate step 5 to 10

Step 5: For all modes (selecting K-1 nodes of K+2 nodes from w, except the node with most stream) iterate step 6 to 10

Step 6: Put K-1 nodes and the node with the most current at T.

Step 7: Connect each of the nodes to nearest node in the T and set up the connection matrix.

Step 8: Calculate the cost of this connection with K hubs (calculate objective function).

Step 9: If the cost is less than minimum value of the objective function you must go the next step.

Step 10: Update minimum objective function and joints by now (joints matrix).

Step 11: Show minimum objective function and joints by now (joints matrix).

Second Heuristic Algorithm

Step 1: Get problem data (n, C, h)

Step 2: Specify a node with the highest inflow and put it as hub and calculate matrix of connections and objective function.

Step 3: Update the minimum objective function and matrix connections.

Table 3: Solve Using Second Heuristic Algorithm

	No. of hubs	Solution time	Objective function value	Gap (%)
Using 5 nodes	1	0.17 seconds	3400	0%
Using 10 nodes	2	1.66 seconds	18172	0%
Using 37 nodes	5	8 minutes and 36 seconds	123890	-

Table 4: Solve Using First Heuristic Algorithm

	No. of hubs	Solution time	Objective function value	Gap (%)
Using 5 nodes	1	0.02 seconds	3400	0%
Using 10 nodes	2	0.078 seconds	18172	0%
Using 37 nodes	4	61 seconds	121550	-

Step 4: For k=2 to n, iterate steps 5 to 10

Step 5: For a given number (like 50 times) iterate steps 6 to 10

Step 6: Select K-1 of n nodes randomly. Put K-1 nodes and the node with the most flow at T.

Step 7: Connect each of the nodes to nearest node and set up the connection matrix T.

Step 8: Calculate the cost of this connection with K hubs (calculate objective function).

Step 9: If the cost is less than minimum value of the objective function you must to go the next step.

Step 10: Update minimum objective function and joints by now (joints matrix).

Step 11: Show minimum objective function and joints by now (joints matrix).

In this article, we found that the proposed objective function is a convex function and we used it as a stop condition for heuristic algorithms and in the following section, using numbers we will show that objective function is convex. In order to heuristic algorithms steps to be specific, we will describe the first heuristic algorithm. In the first heuristic algorithm, first we determine the input stream to each node, and sort them in descending order. We choose the node which has maximum stream (we considered that the cost of hubs are equal; if considered them different, we use flow to cost ratio here) and connect all nodes to this node and according to the objective function of the proposed model, we calculate the value of the objective function. Now for calculating objective function value in two hubs mode, we select one of the top three nodes flowand the node which is chosen as the single hub connect choice and connect non-hub nodes to the nearest hub node and we calculate objective function value and we do this for each top three nodes. Each node that has the lowest cost, is selected as the two hubs. If the cost is more than one hub, the algorithm stops (because the objective function is convex), otherwise continue. To calculate the cost in the three hubs mode, except for the most flow of nodes, we choose two nodes out of four nodes with the most flow and calculate its cost. This process is done for all modes and we choose lowest cost node as the three hubs mode and we continue the algorithm as before.

In order to evaluate performance of our two heuristic algorithms, we have proposed heuristic algorithms by MATLAB and we have solved them for 5, 10 and 37 nodes. We compare their results with each other and analyse them. The results of this analysis are summarized in Tables 2, 3 and 4.

We first solved the problem with 5 cities, i.e. Abadan, Ahvaz, Bushehr, Isfahan, and Yazd. The GAMS software used Isfahan city as a hub. The objective function value was 3400 and the solution time was 1.8 seconds. Both the algorithms chose Isfahan as hub and obtained a cost equal to 3, 40, but the first algorithm obtained this result in 0.02 seconds while the second algorithm obtained this result in 0.17 seconds. Here, the objective function of the both algorithms with one hub is 3400, with two hubs is 4993, with three hubs is 7105, with 4 hubs is 5, 8996 and with 5 hubs is 11,067.It shows that the objective function is convex. Next, the problem with Abadan, Ahvaz, Bushehr, Isfahan, Mashhad, Kerman, Khorram Abad, Tehran, Sari, and Rasht was solved by GAMS software that obtained Tehran and Isfahan as hubs with 18,172 objective function value. It obtained solution time of 11 minutes 35 seconds. Both heuristic algorithms chose Isfahan and Tehran as the hubs and the obtained cost of 18,172 that was same as GAMS software. But the solution time using first heuristic algorithm and second heuristic algorithm is 0.078 seconds and 1.66 seconds respectively. If we want to show convex objective function here, we must first review the obtained objective function value by the algorithm for different number of hubsi.e. 22004 for one hub, 18172 for two hubs, 19106 for three hubs, 20,720 for four hubs, 22,156 for five hubs that shows the convexity of the objective function.

Notice that in the GAMS software, if the number of nodes is 10 or more, the solution time is not optimal. Even with the linear model all the answers have gap and very high solution time. For example, this software may answer the problem for 37 cities in 24 hours, and it is certainly a very large gap. Now we want to examine these two algorithms with 37 cities.

When we solve these two algorithms using 37 cities, the first algorithm solved the problem in 62 seconds and considered Tehran, Isfahan, Shiraz, and Mashhad cities as hubs with 121, 550 objective function value. The second algorithm solved the problem in 8 minutes and 36 seconds and considered Tehran, Shiraz, Mashhad, Shahr Kord, and Sirjan cities as hub with 123,890objective function value.

Finally, we find that both algorithms are optimal in most cases they obtain this answer in shorter time than the GAMS software. But if you want to do a comparison between these two algorithms, we know that the first heuristic algorithm is better than the second heuristic algorithmin terms of the objective function and the solution time.

CONCLUSION

It is obvious that direct communication between different nodes is very expensive. In order to use the economy of scale, it's better to move goods via other nodes, called hubs. So, instead of using a direct connection between nodes, indirect connection is used (connections to hubs). In such cases, the objective of network setup is allowing the flow of commodities through non-hub nodes, by hub nodes. In many models, the objective is minimizing total costs. In P-hub model, the number of hubs is fixed and equal to P, and each node must only be connected to a hub and for traveling between two nodes, at least one and at most two hubs are linked.

In order to close the p-hub to reality, first we have removed the hub number's limit, then the cost of establishing hubs is added to the objective function of the model, so that model gains the economic hub numbers by itself (solved example in the first case). Were moved the restrictions on the number of hubs and the objective function determines the economic number of hubs. The model was closer to reality and we observed that the number of hubs and connections between nodes have changed.

Now many companies use variety of vehicles. According to this, we added the mode of transport to the model as vehicle capacity determines the coefficient costs in the objective function. Since the real world flows are often not definitive, we considered flows as triangular fuzzy numbers. We solved a few examples based on the above models and finally, we found that the removal of restrictions on the number of hubs in most cases will change the answer and adding the mode of transportation to the model closes the resulting solutions to reality. Finally, changing flows from definitive state to fuzzy, can change answer and give a better result.

Then, we have proposed two heuristic algorithms for the model, and we show that both algorithms perform better than GAMS software in terms of solution time and in most cases, they reach the optimal solution. According to the comparison that was made between these two algorithms, we conclude that the first algorithm is better than the second algorithm in term of objective function value and solution time.

SUGGESTIONS FOR THE FUTURE

As a suggestion for the future, it can be proposed that you can use other proposed fuzzy methods for the flows among nodes which may be practical in some other issues and even considering the distance as fuzzy closes the model to reality. Considering reliability of hub location problems can be another research area. Since so far, the issues except minimising the transportation costs has not been raised, modeling a stable hub location and considering social and environmental issues are interesting and practical issues. Other proposed topics for future research are given as below:

- Considering the impact of traffic on the located hubs
- Hub location dynamic modeling
- Combining hub location models with other location problems
- Modeling hubs in random and uncertainty situation
- Development of solving methods of problems in a larger scale

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