Non-Linearity and Chaotic Behaviour in Cyprus Stock Market

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Abstract

The intention of this research is to investigate the aspect of non-linearity and chaotic behaviour of the Cyprus stock market. For this purpose, we use non-linearity and chaos theory. We perform BDS, Hinich-Bispectral tests and compute Lyapunov exponent of the Cyprus General index. The results show that existence of non-linear dependence and chaotic features as the maximum Lyapunov exponent was found to be positive. This study is important because chaos and efficient market hypothesis are mutually exclusive aspects. The efficient-market hypothesis which requires returns to be independent and identically distributed (i.i.d.) cannot be accepted.

Keywords: Efficient Market Hypothesis, Nonlinear Dependence, Chaos in Financial Time- Series

JEL classification C01, C12, D53, G14, F65

Introduction

In the early decades, the approach of efficient-market hypothesis (EMH) was used to explain phenomena in the financial branch. Historically, (EMH) was firstly expressed by Bachelier and followed the papers of Fama. This notion makes the following fundamental assumptions:

a. All information is reflected by prices.
b. Investors are risk-averse - they know what information is relevant for and know how the information should be interpreted.
c. Markets do not have any memory, i.e. yesterday's events do not influence today's events,
d. Returns are independent, normally distributed. In other words, capital markets are efficient when there is no arbitrage opportunity and security price variations are unpredictable.

For many years, these assumptions were tested by application of autocorrelation test, variance ratio and run tests, which could not conclude about the existence of non-linearities. If the investigated return series rejected linear dependence, then Share price movements were considered unpredictable under linear modelling, although they could be forecastable with non-linear dynamics and chaos theory. According to Hsieh 1989 and 1993, the problem was that, rejection of linear dependence is not sufficient for accepting independence because of non-normality of series. Presence of non-linearity provides opportunities to market participants to make excess profit, and moreover, it contradicts EMH assumptions.

In general, nonlinearity and chaos theory make the following assumptions:

a. Each individual may interpret information in different ways and at different times.
b. There is evidence indicating that investors are not rational, they may not know how to interpret all the known information, and investors tend to be risk-seeking when losses are involved.
c. Investors are influenced by experiences and circumstances, their expectations about the future are shaped by their recent experiences.
d. A number of empirical studies on probability distribution of price changes indicate that price changes are not normally distributed (prices fall faster than they rise).

This paper searches the existence of nonlinearity and chaos in the Cyprus stock market.

Review of Literature

In the 90s period, an increased number of studies Scheinkman and Lebaron 1989, Hsieh papers 1989 and 1993, Serletis and Gogas 1999, focused on finding out if there was any existence of nonlinear dependence

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and chaos in financial time series. Low dimension deterministic non-linear processes mimic random walk behaviour and allow for significant and unpredictable fluctuations, such as those seen in stock market crises. According to Schatzberg, this is due to that Share prices do not always adjust instantaneously to new information.

Confirmation of nonlinearity comes from other researchers, such as Bonilla who argue that financial market series exhibit non-linearity. Sewell finds evidence of dependency in the market index series in Japan, Hong Kong, South Korea, Singapore and Taiwan. Malaysian financial market, according to the papers of Cheong (et al.) 2006 also rejects the efficient market hypothesis. Most of these financial structures share the characteristic of the emerging capital market.

Brief Profile of the Cyprus Capital Market

The Cyprus Stock Exchange is the primary stock market in Cyprus and is considered to be a small emerging capital market with a very short history. On March 29 1996, transactions started taking place. The main index for this market is the General Price Index. Alternatively, FTSE/CySE 20 is another important index designed to provide a real-time measure of the Cyprus Stock Market on which index-linked derivatives can be traded. It was constructed with the cooperation of the Cyprus Stock Exchange, the Financial Times and the London Stock Exchange in November 2000.

One way to validate market efficiency qualitatively is the study of long memory in stock markets. A Hurst exponent value above 0.5 implies persistent behaviour for the time series. According to the research of Bougioukou 2012, Hurst exponent was found to be 0.65, for the return series of the Cyprus General Index, showing trend reinforcing time series and rejection of market efficiency, also there is no evidence of Gaussian probability distribution. These facts motivate the present research. We support previous conclusions by applying nonlinearity tests and measuring the rate of chaos with the maximum Lyapunov exponent.

Database and Research Methodology

We consider the General Price index and use data from 3/1/2005 to 30/12/2012, the sample size is N = 1963 values. Data is available for download from the Cyprus Stock exchange website, after excluding the weekend and holiday periods. All computations were made with the MATLAB software.

In econometrics field a common practice to compute continuously compounded return series r_t for the period t is:

$$r_t = \text{In}(X_t) - \text{In}(X_{t-1})$$

The first differences may ensure the stationarity of time series.

The specific study requires the following steps to be done: Non-stationarity study, Nonlinearity study and Chaos test.

I. Non-stationarity Study

Linear regression analysis is a common technique to build models capable of describing time-series and predict. However, interpretation of the results of regression models which is estimated using time series data must be done very carefully in order to avoid the problem known to econometricians as "spurious regression". One aspect of this problem is the low value of the autocorrelation statistic in the Durbin-Watson test when we check the autocorrelation of the estimated residuals; or the estimation of parameters of the Ordinary Least Square Regression model produces statistically significant results between time series that contain a trend and are otherwise random. In order to encounter the problem Granger and Newbold suggested that for econometric applications time series must be stationary. In this study, the Augmented Dickey-Fuller (ADF) test was employed to test the stationarity.

II. Nonlinearity Study

At first step, we check the Normality of the log returns series. A rejection of Normal distribution is an indication of nonlinearity. One way to study normality is the Jarque-Bera test and the QQ-plot. Another way to study normality is the Hinich Bispectral test.

III. Jarque-Bera Test

The Jarque-Bera test is used when the data distribution is unknown and its parameters should be estimated. It is a two-sided goodness-of-fit test and the corresponding test statistic is

$$JB = \frac{n}{6}\left(\sigma^2 + \frac{k-3}{4}\right),$$

Where:

n: sample size,

σ: sample skewness

k: sample kurtosis.

For large sample sizes n, the test statistic is chi-square distributed with two degrees of freedom.

IV. QQ-plot

QQ-plot is a quantile-quantile plot of the sample data quantiles against the theoretical Gaussian quantiles. A normal distributed sample is almost approximated by a straight line.

Secondly, we examine the linear dependence of the return series. We use the modified Q-statistic of the Ljung-Box papers. This test is a "portmanteau" test that accesses the null hypothesis a series of residuals exhibits no autocorrelation for a fixed number of lags L against the alternative that some autocorrelation function coefficient $\rho(\kappa)$ is nonzero for $\kappa = 1,\ldots,L$. The test statistic is

$$MQ(\kappa) = n(n+2)\sum_{i=1}^{\kappa}\frac{\rho(\kappa)^2}{n-\kappa},\ \kappa = 1,\ldots,L$$

Where:

n: sample size.

Under the null hypothesis, the asymptotic distribution of MQ is chi-square distribution with L degrees of freedom.

We should note that linear independence is neither a necessary nor a sufficient condition to accept or reject the random walk hypothesis.

V. BDS Test

The BDS test can only be used to produce indirect evidence about non-linearity because the sampling distribution of the test statistic is not known. This statistic is useful to test for patterns that occur more (or less) frequently than would be expected in independent data. The null hypothesis may be formulated as:

H_0: Pure whiteness, independent data, data generated by an i.i.d. stochastic process, market efficiency,

H_1: Non-linear dependence, absence of market efficiency.

The BDS test was proposed by Brock et al.1996 and is a non-parametric test. The BDS statistic is based on the correlation integral developed by Grassberger and Procaccia 1983.

Let $(X_t,\ t = 1,\ldots,N)$ be a sample of a time series of size N.

Consider vectors of m-histories.

The correlation integral is:

$$C_m(\varepsilon,T) = \frac{2}{N_m(N_m-1)}\sum_{t<s} I_\varepsilon(X_t^m, X_s^m).$$

The parameter m is called the embedding dimension and $N_m = N - m + 1$ is the maximum number of overlapping vectors that can be formed by a sample size N.

The indicator function I_ε is:

$$I_\varepsilon = \begin{cases} 1, & \left\|X_t^m - X_s^m\right\| < \varepsilon \\ 0, & \text{otherwise} \end{cases}.$$

A pair of vectors X_t^m, X_s^m is said to be ε apart, if the maximum norm $\|\bullet\|$ is greater or equal to ε. If the null hypothesis is valid, then

$$C_m(\varepsilon) = C_1(\varepsilon)^m.$$

Under this relation the BDS statistic is

$$BDS(m,\varepsilon) = \frac{C_m(\varepsilon,N) - C_1(\varepsilon)^m}{\sigma_m(\varepsilon,N)/\sqrt{N}},$$

where $\sigma_m(\varepsilon,N)/\sqrt{N}$ is the standard deviation of the difference between the two correlation measures $C_m(\varepsilon,N)$ and $C_1(\varepsilon)^m$.

For large samples, under the null hypothesis, the BDS statistic has a standard normal limiting distribution. If the null hypothesis is not true, then $C_m(\varepsilon) > C_1(\varepsilon)^m$.

According to Brock et al parameter ε is chosen to be equal 0.5 1.0, 1.5 and 2.0 times standard deviations of the data. Embedding dimension m can be varied from 2 to 10 according to Hsieh.

A rejection of the null hypothesis accepts dependence in the returns that could result from a linear stochastic process, non-stationarity, a non-linear stochastic process. For exclusion of linear dependency BDS test should be used to a filtered return series. Serial dependence in the return series can be removed using a $AR(p)$ model and the resulting residual series is tested to reveal possible non-linear hidden structures.

VI. Hinich Test

Hinich Bispectral test works in frequency domain and tests for Gaussianity and linearity. For a stationary time series Hinich estimates the third order Cumulant, then for a Gaussian process its bi-spectrum is zero and for a Gaussian and linear its bi-coherence is flat (constant).

The test of Gaussianity makes the following assumptions:

H_0: The bi-spectrum of the process generated the data is zero, Gaussian process.

H_1: The bi-spectrum of the process generated the data is not zero, non-Gaussianity.

Let $P_{xx(\omega)}$ denote the power spectrum and $B_{xxx}(\omega_1, \omega_2)$ is the bi-spectrum. The normalized bi-spectrum (or bi-coherence) is defined as

$$|bic_{xxx}(\omega_1, \omega_2)|^2 = \frac{|B_{xxx}(\omega_1, \omega_2)|^2}{(P_{xx}(\omega_1+\omega_2)P_{xx}(\omega_1)P_{xx}(\omega_2))}.$$

The test of Gaussianity is based on the mean bi-coherence power

$$S = \sum |bic_{xxx}(\omega_1, \omega_2)|^2 . \quad (1)$$

The statistic in (1) is a central chi-squared random variable with 2P degrees of freedom, P are the points where the squared bi-coherence is summed over in the non-redundant region. The statistical test determines whether the observed statistic *S* is consistent with a central chi-squared distribution by using a probability-of-false *Pfa*.

Thus for a non-Gaussian process (non-zero bispectrum) we formulate the linearity hypothesis test as:

H_0: The bi-coherence of the process generated the data is a constant, absence of nonlinear dependence

H_1: The bi-coherence of the process generated the data is not constant, existence of nonlinear dependence.

For a non-Gaussian process linearity hypothesis assumes, the estimated bi-coherence will be flat (constant). For all ω_1, ω_2, we obtain an estimate of the constant value by computing the mean value of the bi-coherence over the points in the non-redundant region. Let λ denote this mean value, we expect

$$|bic_{xxx}(\omega_1, \omega_2)|^2 \sim \chi^2(\lambda, 2),$$

Where λ is non central parameter.

The statistical test compares interquartile ranges, the estimated range which is computed as diference between medians of values above/below median and the theoretical interquartile range of $\chi^2(\lambda, 2)$ distribution. The linearity is ensured if they are approximately equal.

VII. Chaos Test

Lyapunov exponents measure the mean exponential divergence of initially close orbits (trajectories) with time in phase space. The more rapid this divergence is within a certain period of time, the more chaotic the system is. This is a clear indication of sensitive dependence on initial conditions. Exponential divergence of the initially nearby orbits, measured by at least one positive Lyapunov exponent, and according to Wolf et al., this estimates the level of chaos in the dynamical system. For a given *n*-dimensional system there are *n* Lyapunov exponents, but we need to estimate only the largest one, λ_1.

Consider initial distance $\delta_0 = x_i - x_{i'}$ of two trajectories at time t_0. After time *t* the distance is $\delta_t = x_{i+t} - x_{i'+t}$. If there is a relation $\delta_t = \delta_0 e^{\lambda 1 t}$, then λ_1 (with probability one) is the largest Lyapunov exponent and defined as:

$$\lambda_1 = \lim_{t\to\infty} \frac{1}{t-t_0}\left(\log\frac{\delta_t}{\delta_0}\right).$$

There are three possible values for the Lyapunov exponent:

$\lambda_1 < 0$, a negative exponent characterizes dynamical system as convergent,

$\lambda_1 = 0$, a zero exponent indicates dynamical system which is in some extent in steady state mode,

$\lambda_1 > 0$, a positive exponent characterizes the dynamical system as chaotic.

Implementation of TISEAN software gives the estimations of the exponent for the daily returns of the General

Index.

Results and Analysis of the Cyprus Stock Index

I. Non-Stationarity Test

For the stationary property the ADF test was employed and represented in Table 1. The values of ADF test statistic in any case are much larger than critical values at 1%, 5% and 10% significant levels, therefore return time series was found to be stationary for all significance levels.

Table I: ADF Test of Returns

	No Trend *T – statistic* *– 39.84589*	*Trend* *T – statistic* *– 40.07717*
a % sig.lev	Crit. values	Crit. values
1	– 3.437041	– 3.96813
5	– 2.864394	– 3.41426
10	– 2.568387	– 3.12833

II. Non-linearity Study

We begin the analysis by calculating the first four statistics from the sample: mean standard deviation, skewness and kurtosis (represented in Table 2). The skewness and kurtosis can give a hint about how the empirical distribution and consequently the distribution of the generated process differ from the Gaussian one.

Table 2: Descriptive Statistics and Jarque-Bera test of General index

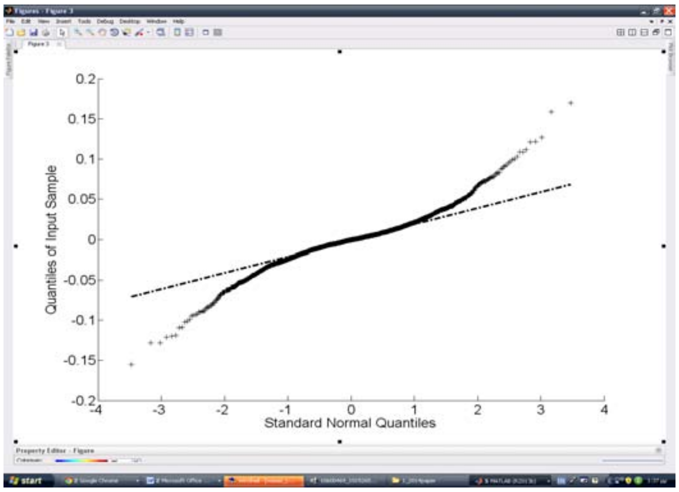
Mean	St. Dev.	Skewness	Kurtosis	J.-B. test
-0.0011	0.02929	0.06723	6.69252	1116.68466*

Note: *Significant at the 5% level.

Sample return series looks like to be positively skewed, kurtosis is larger than 3, hence fat tail distribution appears. Distributions with fat tails of returns decay slower than those being Gaussian. This implies that return generated process is not a random walk. Lastly the Jarque-Bera test with null hypothesis, the return series follows the Gaussian distribution with unknown mean and variance against the alternative: the return series does not follow a Gaussian distribution, suggests no normality for the return series.

The QQ plot, Fig. 1, shows a deviation from the straight line for the returns of General index. The graph reveals that both positive and negative shocks are responsible for the non-normality of the series.

Fig. 1: QQ Plot of the Return Series General Index Return Series Against the Normal.



Next step is examination of serial correlation. If price changes turn out to be serially correlated, it would not necessarily imply inefficiency. Spurious autocorrelation may exist due to institutional factors such as non synchronous trading which may induce price adjustment delays into the trading process see Hasbrouck and Ho. For the Cyprus stock index we calculate the results of Table 3. All autocorrelations coefficients up to lag 30 are statistically significant. We accept serial correlation and we proceed to study the nonlinear independence assumption.

Table 3: Test for Serial Correlation of the Return Series

Lags L	*Modified statistic MQ*
5	26.6333*
10	28.736*
15	44.164*
20	48.916*
25	67.200*
30	68.447*

Note: *significant at the 5% level.

III. BDS Test

Before applying BDS test we seek the best *AR*(*p*) model to fit the data. This is important because it removes linear dependence from the produced residuals series. An advantage of using the residuals of *AR*(*p*) model is that it reduces the effect of infrequent trading, which is more pronounced in price indices of thinly traded stock markets, see Stoll and Whaley.

Implementation of *AR*(*p*) is made in Matlab according to Neumaier et al 2001. This software determines the optimum order of *AR*(*p*) model with use of Akakai 1971 logarithm criteria and in the case of General index return series is modelled as *AR*(3).

To testify how much adequate are *AR*(*p*) models we compute the time series of residuals. Significance level of the modified Li-McLeod portmanteau (LMP) statistic is computed and a model passes this test if significance level > 0.05. In our case significance level of the modified Li-McLeod portmanteau (LMP) statistic is 0.19757 and *AR*(3) is adequate.

The computed residual time series is tested for non-linearity and the BDS statistic produces the results of *Table IV.* We use the following embedded dimension m = 2, 3, 4, 5, 6 and distance values $\varepsilon = 0.5\sigma, 1\sigma, 1.5\sigma, 2\sigma$. All test statistics are significant at 5% level, and reject the null Hypothesis. We cannot accept data generated by an i.i.d. stochastic process therefore, efficiency for the Cyprus financial market.

Table 4: BDS Statistics of the Residuals from *AR*(3) Model of the Return Time Series

	e			
(m)	*0.5 σ*	*1 σ*	*1.5 σ*	*2σ*
2	11.61852	10.32383	9.63750	9.22787
3	17.77967	14.16587	12.78246	11.46529
4	25.87570	17.59612	14.97690	13.17808
5	38.28707	21.44506	17.25329	14.80362
6	57.45134	25.82196	19.55426	16.25579
7	91.55172	31.41101	21.75549	17.43845
8	150.8920	38.48837	24.11390	18.55483

Note: (*m*) is the embedded dimension, ε is the bound, significant at the 5% level. The critical values for BDS is 1.96

IV. Hinich Test

Hinich bi-spectral test checks the nonlinear dynamics on third Cumulant and is invariant under linear filters, hence, we apply Hinich bi-spectral test on return series. Hinich bi-spectral test was computed using MATLAB and HOSA toolbox (Higher order spectral analysis) with GLSTAT.M.

As reported in Table 5, the test rejects the null hypothesis of Gaussianity, giving a probability of false $pfa = 0$ for the General index. The null hypothesis of linearity is also rejected as there is a difference between the empirical Re and theoretical interquartile Rt ranges.

Table V: Hinich Bi-Spectral Test of General Index

Gaussianity test	
Test statistic 1199.9103, *df* = 170	*pfa* = 0
Linearity test	
Re = 16.7933 Rt = 9.69356	

Note: *Re* empirical and *Rt* theoretical interquartile ranges.

V. Chaos Test

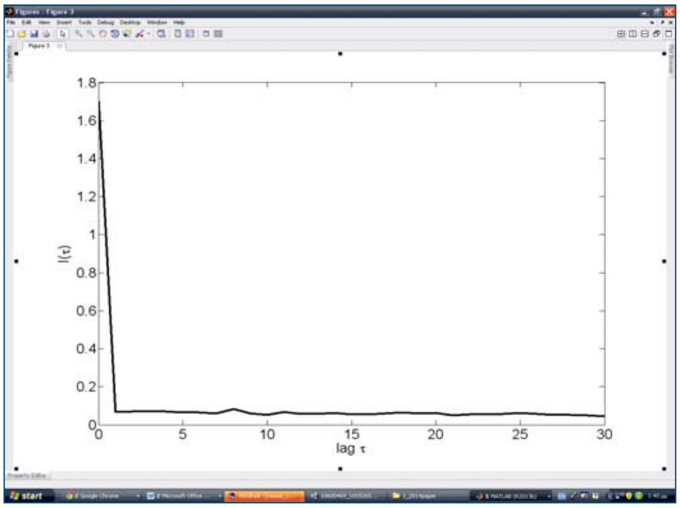
We investigate the nature of nonlinearity by searching for chaotic features in the Cyprus financial market. We calculate the largest Lyapunov exponent using the Tisean software and specifically the maxlyapunov.m routine which requires calculation of the time delay τ. The optimum value of time delay t was selected by applying the method of mutual Information. This method takes into account linear and non-linear correlations and searches for the first minimum of the function of mutual Information I(τ) see Hegger et al. For the General index time delay was calculated $\tau = 2$, see Fig. 2.

Exponents were estimated with embedding dimensions from two up to four according to Kantz and Rosenstein. The results are all positive hence, they allow us to accept chaos for the financial time series, see Table 6.

Table 6: Lyapunov Exponents of General Index Returns Series

Emb. dim (m)	*max Lyap.* λ_1
2	0.00694
3	0.00898
4	0.0181

Fig. 2: Calculation of Time Delay of General Index Returns



Conclusion

The Cyprus Stock Exchange is a small emerging capital market with a very short history. A previous study conducted by Bougioukou for a smaller sample of the General index (N=1730) had rejected Gaussian distribution and found the value 0.65 for the Hurst exponent, giving implications of nonlinear aspects and chaotic behavior. We search the Cyprus General index for non-linear dynamic aspects and chaotic behavior. The QQ Plot of return series and Jerga-Bera test indicate no-Gaussian distribution. This result is also confirmed by the Gaussianity Hinich Bispectral test. Both tests, BDS and Hinich accept non-linearity and absence of market efficiency. Evidence of nonlinearity leads to a research of chaotic features. The positivity of the largest Lyapunov exponent confirms chaos and contradicts the hypothesis that share price changes are independent and identically distributed.

Cyprus as an emerging capital market in Europe is considered in general, not efficient in semi-strong form or strong form according to Birau. Emerging capital markets given their functional, structural and institutional malfunctions and limitations are less efficient than the developed capital markets. In the most optimistic case, empirical research studies have identified a weak form of efficiency.

We also consider the fact that existence of possible noise in the data may give wrong results for Lyapunov exponents. According to Dingwell when Lyapunov exponents are estimated from recorded time series data, there is some level of stochastic noise (which itself might include measurement noise, as well as system noise, or both, etc.). Noise, by itself, will create "sensitivity to initial conditions" that can trigger findings of positive Lyapunov exponents even for systems that are not remotely "chaotic".

Finally, the length of the time series is also important. Harrison et al. support that the accuracy of tests results improves with the increase of the length of the time series. Perhaps, a larger number of data will produce contradicted results. Therefore, we suggest further study with larger sample size. Also, utilization of differed methods from those reported in this paper could be helpful to clarify the financial situation in Cyprus.

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