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Bulk Queues: An Overview

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Abstract

In this article we present an overview on the contributions of the researchers in the area of bulk queues with their applications in several realistic queueing situations such as in communication networks, transportation systems, computer systems, manufacturing and production systems and so forth. The concept of bulk queues has gained a tremendous significance in recent time; mainly due to congestion problem faced in communication / computer networks, industrial/ manufacturing system and even in handling of air, land and water transportation. A number of models have been developed in the area of queueing theory incorporating bulk queueing models to resolve the congestion problems. Through this survey, an attempt has been made to review the work done on bulk queues under different context such as unreliable server, control policy, vacation, etc. The methodological aspects have been described. We provide a brief survey and related bibliography of some notable contributions done in the area of bulk queueing systems.

Keywords: Bulk Arrival, Bulk Service, Markovian and Non-Markovian Queues.

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1. Introduction

Queue is an unavoidable aspect of modern life that we encounter at every step in our daily activities whether it happens at the checkout counter in the super market or in accessing the internet. The queueing phenomenon arises wherever a shared facility needs to be accessed for service by a large number of jobs or customers. Queueing Theory has emerged as one of the foremost areas of research because of its utility in simulating many real life systems. It is applicable in a wide variety of situations that may be encountered in business, commerce, industry, healthcare, and public service and engineering. It constitutes a powerful tool in modelling and performance analysis of many complex systems, such as computer networks, telecommunication systems, call centers, flexible manufacturing systems and service systems etc.

In theory of queues, a bulk queue (sometimes called batch queue) is a generalized queueing model where jobs arrive in and/or are served in groups of random size. There has been a great interest to analyze batch arrival queues during the last three and a half decades, both from theoretical as well as practical points of view. These queues are frequently encountered in many real time applications; details of such queues are discussed by Chaudhary and Templeton (1983). In ordinary queueing problems it is assumed that customers arrive singly at a service facility. However, this assumption is violated in many real-world queueing situations. Letters arriving at a post office, ships arriving at a port in convoy, people going to a theater, restaurant, are few examples of queueing situations in which customers do not arrive singly, but in bulk or groups. Also, the size of an arriving group may be a random variable or a fixed number.

We now turn to queueing situations in which arrivals occur singly, but service is performed in bulk. Bulk service queues have potential applications in many areas e.g. in loading and unloading of cargoes at a seaport, in traffic signal systems, in computer networks where jobs are processed in batches, manufacturing/ production systems, in restaurants, cinema and an elevator. Bulk-arrival and bulk-service systems are abundant in the real world. In the air-cargo delivery system, in amusement parks, and in the manufacturing setting, are few more real life situations of bulk queues.

In the present article, we provide a brief survey with methodological aspects and applications on bulk queues. The classification of existing literature on bulk queueing models has been done to study in different frameworks. The rest of the article is organized as follows. Some classical models with relevant references are discussed in section 2 to highlight the significance of the topic. Historical development of bulk queueing systems has been presented in this section. section 3consists of various techniques/methodologies in order to provide better solution of queueing problems have appeared. Specific models that have appeared in the literature are presented together in section 4. Finally section 5 highlights the status of the current research in Bulk queues.

2. Bulk queues: Some Preliminary Concepts with Historical Review

Bulk queueing systems are practically very important because of the typical trade-off between various costs of providing service and the costs associated with waiting for the service or leaving the system without being served. In general, high quality fast service is much expensive, then the waiting costs of the customers in the queue. On the other hand, in some situations, the long queues may cost a lot because customers do not work while waiting in the queue or customers leave because of long queues. Much of the literature on this topic focuses on the development of the theory for waiting time and number in such queues. A general class of bulk queues with poisson input was studied by Nuets [1].

The problem on bulk queues is studied widely by many researchers [2-11]. The first treatment of a group queue occurred in the work of Bailey [2], and Down ton [12], who in their model allowed customers still arrived singly but were serviced in groups of 's'. The study of bulk-arrival queues several years after Bailey work on bulk service. Miller [3] described the general model for a bulk queue as "group of entities" arrive at a service line and are serviced in groups; the service groups do not necessarily coincide with arrival groups.

Mejia-Tellez and Worthington, 1994 modeled the queue length in bulk-arrival, bulk-service queues. Medhi [13] considered the bulk-service models. Chang et al.[14] developed performance measures for finite-buffer bulk-arrival, bulk-service queues using transform methods. Armero and Conesa [15] modeled a bulk-arrival queueing system for the analysis to-stock production system. Chen [16] discussed a fuzzy bulk queue and provides some interesting examples of cable cars and elevators. Bar-Lev et al. [17] studied a bulk-service queue with variable batch size for a group-testing center that has applications in a medical testing center.

3. Methodological Aspects

There are various techniques/methodologies to provide better solution of bulk queueing problems in different frameworks. Most powerful queue theoretic approaches for evaluating system performance of congestion problems are based on the stochastic process and theory of Markov chain. We now briefly discuss some techniques frequently used in the queueing characterization of bulk-arrival and bulk service queues. The worth mentioning techniques to be reviewed in this regard are supplementary variable technique, embedded Markov chain, probability generating function, maximum entropy analysis, and many others. Now we describe some basic features of these techniques as follows:

3.1 Probability Generating Function (PGF)

There are some transforms particularly L-transform and z-transform, which are widely applied in queueing theory in order to simplify complex calculations. The probability generating function is one of the powerful methods providing the solution of difference equations invented by great mathematician Euler. Let X be a non-negative discrete random variable with $P(X=n) = p_n$, n = 0, 1, 2,…then. The probability generating function G(Z) of X is defined as follows:

$$G(Z) = \sum_{n=0}^{\infty} p_n z^n$$

Note that $|G(z)| \leq 1$ for all $|z|$. Also $G'(1)$,

$$E[X^2] = G''(1) - G'(1)$$

3.2 Supplementary Variable Technique (SVT)

Over the last few decades, different methodologies have been developed to predict the performance of different variants of queueing systems. Supplementary variable technique is frequently used in order to change non-Markovian model into Markovian model in continuous time by the implementation of one or more supplementary variables. Cox [18] was the first who introduced the SVT in order to study the non-markovian queueing models such as M/G/1 queueing model, M/G/c queueing model, etc. There are numerous works in the field of bulk queueing models using SVT.

3.3 Embedded Markov Chain (EMC)

Embedded Markov chain is an important classical method which has been widely used in queueing theory for non-Morkovian model. The fundamental idea behind this method is that we wish to simplify the description of state from the two dimensional description [N(t), X(t)] into a one-dimensional description N(t). Here N(t) is the number of customers, including those waiting and those in service, in the system at time t and X(t) is the number of customers in the queue at time 't'. EMC is a semi-Markov process with discrete state space in which the state transition takes place at customers' departure instant or just prior to arrival instant. The EMC at these instants is defined to be the number of customers present in the system immediately following the departure.

3.4 Maximum Entropy Principle (MEP)

Maximum entropy approach is efficient enough for practical purpose and is a feasible method for approximating the solution of complex queueing systems. The goal of the maximum entropy principle is to provide a uniquely correct method of inference for estimating an unknown probability distribution. This approach is based on the principle of measure of uncertainty suggested by Shannon [19]. Maximum entropy approach consists of maximizing an entropy function subject to given constraints, where the constraints are formed by some well known exact results. The probability distribution is thus uniquely determined. Table 1.1 explicitly illustrates the technique wise relevant research work done on bulk queueing models.

Table 1.1. Categorization based on technique wise for bulk queues:

Probability \ Generating Function	Madan [20], Jayaraman, etal[21], Gupta and Sikdar [22] Arumuganathan and Jeyakumar [23], Li, et al[24]
Embedded Markov Chain Technique	Abolnikov & Dshalalow [25], Agarwal & Dshalalow [26], Tadj & Ke[27], Choudhury et al[28], Kahraman, & Gosavi [29]
Supplementary Variable Technique	Reddy et al[30], Choudhury and Paul [31]
Maximum Entropy Principle	Wang, et al[32], Ke and Lin [33], Pandey and Tripathi[34]

4. Some Specific Models

4.1 Markovian Models

Markovian models satisfy memoryless property. The exponential distribution is the only continuous distribution which exhibits the memory less property. In such models, it is assumed that inter arrival time as well as service time of the customers are exponentially distributed. Markovian models with bulk arrival and bulk service can be formulated to examine a variety of genuine congestion situations of many real life problems. Some Markovian models with bulk arrivals are depicted in table 1.2.

Table 1.2. Markovian models with batch arrivals:

Models	Features	Authors
$M^x/M/1$	Generalized vacation	Borthakur and Choudhury [35]
	Setup time, Vacation period	Choudhury[36]
	N-policy, Unreliable server, Single vacation	Ke [37]

4.2 Non-Markovian Models

Non-Markovian queueing model does not follow memory less property which is feature of Markovian models. Some notable works on Non-Markovian models with bulk arrivals are reported in table 1.3.

Table 1.3. Work done on bulk queues with Non-markovian models.

Models	Features	Authors
$M^x/G/1$	Single vacation	Baba [11]
	Single/Multiple vacation, N-policy	Lee et al.[38-39]
	Retrial, Multiple vacation, Starting failure	Krishnakumar and Madheswari [40]
	Two-stage arrival, N-policy, Bernoulli vacation schedule	Choudhury and Madan [41]
	Two-phase service, Bernoulli vacation schedule, Multiple vacation policy	Choudhury[28]
	Two-phase service, Bernoulli vacation schedule	Choudhury and Tadj[42]

4.3 Vacation Models

Bulk models with server vacations have been well investigated by many researchers and extensively used in various practical systems such as polling protocols, frequently used in high speed telecommunication networks, manufacturing systems with machine breakdown and processor schedules in computer and switching systems, etc. Server vacations are useful for the system in which the server wants to utilize his idle time for different purposes. For comprehensive and excellent surveys on queueing systems with server vacations are made by Doshi [43] and Takagi [44].

Lee and Srinivasan [45]were the first to study N policy $M^X/G/1$queueingsystems with vacations. Choudhury [46] examined an $M^x/G/1$queue with a startup period and a vacation period. Other related works on bulk queues with vacation are shown in the table 1.4.

Table 1.4. Bulk queueing models with vacation

Models	Authors	Features
$M^x/M/1$		N-policy, Vacation, Startup
	Ke[47]	Vacation, Startup/ Closedown times
	Ke and Lin[48]	Delaying vacation
	Kumar & Madheswari[40]	Retrial queue, Multiple vacations
	Ke[37]	N-Policy, Single vacation
	Omey and Gulck[49]	Multiple vacation

4.4 Retrial Models

Bulk queueing model with retrial jobs can be used for telephone switching systems, digital cellular mobile networks and computer networks, etc. In many real time systems the customers arriving to a busy or broken down system are compelled to leave the service area and go to some virtual place referred as an orbit. These customers try their luck again for service from the orbit after a random amount of time. During the last three decades considerable attention has been paid in studying retrial queue under different context. Some worth noting works on retrial models with batch arrivals are summarized in table 1.5.

Table 1.5. Bulk queueing models with retrial

Models	Authors	Features
$M^x/G/1$	Kumar & Madheswari[40]	Retrial queue, Multiple vacations
	Jain and Pandey [50]C houdhury[46]	Retrial queue, Setup Retrial queue, two phase service

5. Recent development in Bulk queue models

5.1 Phase Type Service Models

There are various areas in which service is performed in different phases, for example in a manufacturing or assembly-line process in which units proceed through a series of work station, each station performing a given task. In a registration process of the system, the registrant must visit a series of desks. A clinical physical examination procedure where the patient goes through a series of stages is also an example of phase type service.

Kim, and Kim[51]studied the processor-sharingqueuewithbulkarrivals and phase-type services. A bulk arrival queue with two phase service and Bernoulli vacation schedule under multiple vacation policy was analyzed by Choudhury [28]. Recently, Choudhury et al.[46] studied batch arrival retrial queues with two phases of service and service interruption.

5.2 *Optional Service Interruption Models*

During last few years, some papers have appeared in queueing literature for the analysis of unreliable queueing system with second optional service. According to this feature all arriving jobs request for first essential service while some of them request for second optional service which is provided by the same server.

Bulk queues with optional service have been proposed by Ke (42]. Madan and Al-Rawwas [52] studied bulk queue with feedback, optional service and single vacation. Recently, Maurya [53] studied Maximum entropy Analysis of $M^X/(G_1, G_2)/1$ retrial queueing model with second phase optional service and Bernoulli vacation schedule.

6. Fuzzy parameters

In the queue literature, customer inter-arrival times and customer service times are required to follow certain probability distributions with fixed parameters. However, in many real-world applications, it is more adequate to describe the arrival and service patterns by linguistic terms, such as "frequently arrivals" or "fast, slow or moderate services", rather than by probability distributions. That is to say that both inter arrival times and service times are more possibilistic than probabilistic in many practical situations. Consequently, control and design problems with fuzzy queues are more realistic and applicable than with the crisp queues.

Thus, fuzzy queues are potentially much more useful and realistic than the commonly used crisp queues. Li and Lee 1989 investigated the analytical results for two typical fuzzy queues (denoted $M/F/1/\infty$ and $FM/FM/1/\infty$, where F represents fuzzy time and FM represents fuzzified exponential distributions) using a general approach based on Zadeh's extension principle. Negi and Lee, 1992 proposed a procedure using α-cuts and two-variable simulation to analyze fuzzy queues. Chen, 2005developed $FM/FM^{[K]}/1/\infty$ fuzzy systems using the same principle. Ke and Lin[48] and Lin et al.[54] who respectively analyzed fuzzy queues with unreliable-server and retrial systems using the same approach (Zadeh's extension principle). Through α -cuts and Zadeh's extension principle, we transform the fuzzy queues to a family of crisp queues. As α varies, the family of crisp queues is described and solved using parametric nonlinear programming (NLP).

7. Conclusions

The present survey article provides an extensive overview on the literature related to the queue with bulk arrivals. Several models have been studied by many researchers due to versatility of batch arrivals in every field of life and make clear in diverse part of this authentic world. We have surveyed the literature, which are categorized model wise (Markovian and non-Markovian models, vacation models, retrial models, etc.) as well as techniques wise (supplementary variable, matrix geometric method, maximum entropy principleetc.).

This survey which includes most of the concepts related to real time situations with the concept of bulk queues provides a valuable insight to those who are interested in the performance modeling of real time systems working in machining environments wherein bulk queues are an unavoidable phenomenon.

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