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Fractal Dimensional Analysis in Financial Time Series

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Abstract

A predictability index for time series of a financial market vector consisting of chosen market parameters is suggested providing a measure of long range predictability of the market. It is based on fractional Brownian motion that includes Brownian motion as a particular case followed by the time series of financial market parameters. By analysing respective time series, these indices are computed for parameters like volatility, FII investments in the local market, IIP numbers, CPI numbers, Dow Jones Index, different stock market indices, currency rates, and gold prices.

Keywords: Fractional Brownian motion, Hurst Exponent, Fractal Dimension, Predictability Index

1. Introduction

A financial market is manifested by financial parameters like stock prices, volatility, various stock market indices, currency rates, interest rates, oil prices, gold prices, etc. During the evolution of the market, a financial parameter generates a time series exhibiting values of the parameter over a specified period of time. In the philosophy of Black-Schole-Merton theory for the option pricing formula a financial time series is assumed to be completely random and follows a Brownian motion (BM). This completely rules out predictability in the behavior of time series, the increments in the Brownian motion being independent random variables.

On the other hand, identifying and understanding trends (either long range or short range), if any, in a financial time series is of considerable interest for policy makers, investors, practioners as well as academicians interested in engineering financial markets. This naturally suggests (as was first proposed by Mandelbrot and Van Ness (1968)) to use fractional Brownian motion (fBM), instead of Brownian motion, to model a financial time series, as fBM incorporates quantitatively long range predictability in its formalism.

The fBM intrinsically brings in Hurst exponent H associated with fBM; equivalently, its fractal dimension D = 2 – H. When D = 1.5, the fBM becomes BM, and hence is hardly predictable. Thus variation of D from 1.5 provides a measure for long range predictability of the series called predictability index (PI) of the parameter. This helps classify time series as persistent, anti-persistent, or Brownian.

All these raise the basic issue: Does the market follow fBM? In the present note, we contribute to this issue with data analysis carried out on available data base. We consider a financial market vector $X = (X_1, X_2, \ldots, X_n)$ consisting of appropriately chosen market parameters $X_1, X_2, \ldots, X_n$. The vector X holistically represent state of market in totality. Its predictability index vector $PI(X) = (PI(X_1), PI(X_2), \ldots PI(X_n))$ is an element of a vector space $\mathbb{R}^n$ which is measured in terms of various norms on $\mathbb{R}^n$. We apply this formalism to volatility vector, $V = (V_O, V_H, V_L,$

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V_C) (on a particular day, V_O = volatility at the opening, V_H = highest volatility, V_L = lowest volatility, V_C = volatility at the closing) and stock market vector, M = (*FII*, *CPI*, *IIP*, *DJI*) (Foreign Institutional Investors, Consumer Price Inflation, Index of Industrial Production, Dow Jones Index) and also to each of stock market indices like S&P, Dow Jones, BSE, and Hangseng, as well as to currency rates of different countries, and to gold prices. For all these, we use the available database from different sources (including bloomberg.com, globalfinancialdata.com, gold.org, mospi.nic.in, nseindia.com, sebi.gov.in) over appropriate time periods and compute respective predictability indices. We use rescaled range $\frac{R}{S}$ analysis to compute Hurst exponents H with MATLAB and KaotiXL.

2. Review of Literature

Fractional Brownian motions were implicitly considered by Kolmogorov (1940), Hunt (1951), Lamperti (1962), and Yaglom (1958, 1965). In 1965, Fama suggested that real market distributions need to be Gaussian. It was Mandelbrot (1968), who recognised the relevance of this random process and suggested its use for the financial market in his seminal paper with Van Ness in 1968. The Levy process allows incorporating a wide range of distributions into financial models. Also, Lo & MacKinley (1988) infer that the processes of observable market values seem to exhibit serial correlation, there by suggesting the use of fBM. Starting with Rogers (1997), there is an ongoing debate on proper usage of fBM in option pricing theory especially due to its inability to incorporate no-arbitrage pricing. However, as shown by Rostek (2009), arbitrage can be made to disappear by assuming that market participants cannot react instantaneously.

Taking BM as the source of randomness, the celebrated Black-Schole-Mortion (BSM) theory (1973) develops option pricing formula involving Stochastic Calculus which is based on integration with respect to the Brownian motion, the underlying assumption being that the market follows a Brownian path. The Black-Scholes equation (Black & Scholes, 1973; Merton, 1973) is a partial differential equation, which describes the price of the option over time. The equation is

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0$$

where $V(s, t)$ is the price of a derivative as a function of time and stock price, r is the annualised risk-free interest rate continuously compounded, σ is the volatility of the stock's returns which is the square root of the quadratic variation of the stock's log price process, and t is time in years. We generally use present time, $t = 0$, and expiry time, $t = T$. The key financial insight behind the equation is that one can hedge the option by buying and selling the underlying asset in just the right way and consequently "eliminate risk". This hedge, in turn, implies that there is only one right price for the option, as given by the BSM formula.

Rangarajan and Sant (1997) have introduced climate predictability index PI based on fBM representing a weather parameter time series. They have shown that it is useful in studying various climatic components which are of random nature. They have suggested that this concept would be useful in other areas also. On the basis of this, we have introduced predictability index PI in the financial market (Bhatt *et al*., 2013).

3. Database and Research Methodology

The data used in this paper for study are collected from different sources (including bloomberg.com, globalfinancialdata.com, gold.org, mospi.nic.in, nseindia.com, sebi.gov.in). In order to proceed for the methodology, we have to introduce a few terminologies. We begin with the definition of a discrete Brownian motion and discrete fractional Brownian motion.

Definition: A discrete Brownian motion(BM) is a real valued stochastic process $t_i \to B(t_i)$ over discrete time values t_i, $i = 0, 1, 2, ...$ such that

(1) $B(t_0) = B(0) = 0$;

(2) The increments $\Delta B(t_j, t_i)$ and $\Delta B(t_l, t_k)$ $(0 \le i < j \le k < l)$ are independent random variables, where $\Delta B(t_n, t_m) = B(t_n) - B(t_m)$;

(3) The increments $\Delta B(t_j, t_i)$ are normal random variables with the mean $E(\Delta B(t_j, t_i)) = 0$ and variance $\mathrm{Var}(\Delta B(t_j, t_i)) = t_j - t_i (j > i)$.

Definition: A discrete fractional Brownian motion (fBM) is a real valued stochastic process $t_i \to B_H(t_i)$ over discrete time values t_i, $i = 0, 1, 2, ...$ such that

(1) $B_H(t_0) = B_H(0) = 0$;

(2) The increments $\Delta B_H(t_j, t_i)$ are normal random variable with the mean $E(\Delta B_H(t_j, t_i)) = 0$ and the variance $Var(\Delta B_H(t_j, t_i)) = (t_j - t_i)^{2H}$ $(j > i)$, where H is the Hurst exponent satisfying $0 < H < 1$

when $H = 0.5$, it becomes a BM. Thus fBM includes BM.

The fBM process $t_i \to B_H(t_i)$ satisfies the following [12]. For all $0 \le i < j$,

(1) $E(B_H(t_i)) = 0$;

(2) $E(\Delta B_H(t_j, t_i)) = 0$;

(3) $E(\Delta B_H(t_j, t_i)^2) = (t_j - t_i)^{2H}$;

(4) $E(B_H(t_i)\, B_H(t_j)) = \frac{1}{2}\left[t_i^{2H} + t_j^{2H} - (t_j - t_i)^{2H}\right]$;

(5) $E(\Delta B_H(t_j, t_i)\, \Delta B_H(t_i, t_0)) = \frac{1}{2}\left[t_i^{2H} + t_j^{2H} - (t_j - t_i)^{2H}\right]$.

So except $H = 0.5$, the increments of fBM are not independent.

An important feature of fBM is fractality of resulting time series. Fractal curves and fractal surfaces are discovered by Mandelbrot (1977). Technically, he defined a fractal to be a set in a metric space for which the Hausdorff-Basicovitch dimension is greater than topological dimension. A fractal exhibits self-similarity. According to box counting, the fractal dimension of a fractal set in a two dimensional Euclidian space is

$$D = \lim_{r \to 0} \frac{\log m(r)}{\log\left(\frac{1}{r}\right)}$$

where $m(r)$ is the number of boxes in a square grid of size r required to cover the object. This comes from the scaling law $m(r) = C\left(\frac{1}{r}\right)^D$. A fractal dimension need not have to be an integer (Falconer, 2003; Sagan, 1994). AnfBM is known to exhibit self-similarity, the resulting curve is a fractal whose fractal dimension is given by D = 2 – H.

Using the Hurst exponent we can classify time series into types and gain some insight into their dynamics (Mulligen, 2004; Qian & Rasheed, 2004). Here are some types of time series and the fractal dimension associated with each of them.

3.1. Brownian Time Series

In a Brownian time series (also known as a random walk or a drunkard's walk) there is no correlation between the observations and a future observation. Series of this kind are hard to predict. A fractal dimension close to 1.5 (i.e. $H = 0.5$) is indicative of a Brownian time series. In this case, the market follows the efficient market hypothesis (EMH).

3.2. Persistent Time Series

In a persistent time series, an increase in values will most likely be followed by an increase in the short term; and a decrease in values will most likely be followed by another decrease in the short term. A fractal dimension D between 1 and 1.5 (i.e. $0.5 < H < 1$) indicates persistent behaviour; the smaller the D value, the stronger the trend. However when $D \approx 1$, there is high risk of abrupt changes, because in this case, the time series is Cauchy distributed. The series is auto correlated for $D < 1$; and for $D = 0$, it is Levy distributed.

3.3. Anti-Persistent Time Series

In an anti-persistent time series (also known as a mean-reverting series) an increase will most likely be followed by a decrease or vice-versa (i.e., values will tend to revert to a mean). This means that future values have a tendency to return to a long-term mean. A fractal dimension D between 1.5 and 2 (i.e. $0 < H < 0.5$) is indicative of anti-persistent behaviour. If the value of D is closer to 2, the tendency is stronger for the time series to revert to its long-term mean value. In this case, market exhibits more volatile behaviour than accommodated by EMH which promotes competition and innovation. When $D \approx 2$, the market exhibits ergodicity strongly disconfirming EMH.

Motivated by climate predictability index in Atmospheric Science introduced by Rangrajan & Sant (1997, 2004), we suggest predictability index for stock market parameters (Bhatt *et al.*, 2013). Let X be any market parameter. Predictability index of X is defined as

$$PI(X) = 2|D(X) - 1.5|.$$

Here $D(X)$ is the fractal dimension of time series for X; viz. $D(X) = 2 - H(X)$; and Hurst exponent $H(X)$ for X can be computed using rescaled range $\frac{R}{S}$ analysis. The number $PI(X)$ gives a measure of long range predictability in behaviour of X We note that $\frac{R}{S} = cT^H$, where c is constant, T is time span, and H is the Hurst exponent. The quantities R and S are defined as follows (Qian & Rasheed, 2004).

$$R(T) = \max_{1 \le t \le T} X(t, T) - \max_{1 \le t \le T} X(t, T)$$

$$S(T) = \left[\frac{1}{T} \sum_{t=1}^{T} \{\xi(t) - E(\xi)_T\}^2 \right]^{\frac{1}{2}}$$

where $E(\xi)_T = \frac{1}{T} \sum_{t=1}^{T} \xi(t)$ and $X(t, T) = \sum_{u=1}^{t} [\xi(u) - E(\xi)_T]$

More generally, let the market be represented by a vector $X = (X_1, X_2, ..., X_n)$, where $X_1, X_2, ..., X_n$ are market parameters are chosen conveniently like volatility, GDP, CPI, FII investment, interest rate etc.Then its predictability index $PI(X)$ is a vector in $\mathbb{R}^n$ given by $PI(X) = (PI(X_1), PI(X_2), ..., PI(X_n))$.

We consider the following norms for $PI(X)$ like

$$\|PI(X)\|_S = |PI(X_1)| + |PI(X_2)| + ... + |PI(X_n)|$$

$$\|PI(X)\|_M = \max\{PI(X_1), PI(X_2), ..., PI(X_n)\}$$

$$\|PI(X)\|_g = [|PI(X_1)|^2 \, |PI(X_2)|^2 + ... + |PI(X_n)|^2]^{1/2}$$

They give various measures for long range predictability of the market as a whole collectively taking into account the parameters chosen. At an instant, the values of the parameters X_i are real numbers and we consider norms of market vector like $\|X\|\ S$, $\|X\|M$, $\|X\|E$.

4. Results and Analysis

4.1. Volatility

The volatility index (VIX) is considered to be a premier parameter (Suppannavar, 2008). It (calculated as annualized volatility, denoted in percentage) is a measure of the amount by which an underlying index is expected to fluctuate, in the near term, based on the order book of the underlying index options. We have analysed data for India VIX (Bhatt *et al.*, 2013) whose underlying is Nifty option prices. The volatility of the market is the vector $V = (V_O, V_H, V_L, V_C)$ where V_O is the volatility at the opening of the day, V_H is the highest volatility during the day, V_L is the lowest volatility during the day, and V_C is the volatility at the closing of the day. Assuming each of these parameters V_O, V_H, V_L, V_C following fBM, the predictability index vector of volatility is the quadruple

$$PI_V = (PI_O, PI_H, PI_L, PI_C)$$

For India VIX we have analysed total 1000 data point from June 2009 to May 2013 (nseindia.com). The predictability index vector PI_V has turned out to be (0.7756, 0.7828, 0.7746, 0.7818) (Bhatt *et al.*, 2013). The norms of PI_V are $\|PI_V\|_S = 3.11$, $\|PI_V\|_M = 0.7828$, $\|PI_V\|_E = 1.5574$. India VIX exhibits highly persistent behaviour. We have reported this analysis in our earlier paper (Bhatt *et al.*, 2013). Here they are briefly included for comparison with other parameters considered in the present paper.

4.2. A Stock Market Vector

As a second example we consider the parameters which may affect the dynamics of a stock market. We consider the following factors.

(1) FII (Foreign Institutional Investors) for investment in the local equity market.

(2) IIP (Index of Industrial Production) to measure the level of industrial activity.

(3) CPI (Consumer Price Inflation) as an indicator of the rate of inflation.

(4) DJI (Dow Jones Index) for effect of global market movement.

Table 1: PI of Different Market Parameters for 10 years

Parameter	*H*	*D*	*PI*
FII	0.638676	1.361324	0.277352
CPI	0.979533	1.024670	0.950660
IIP	0.561325	1.438675	0.122650
DJI	0.636309	1.363691	0.272618
BSE	0.652381	1.347619	0.304762

$PI_M(10) = (0.277352, 0.950660, 0.122650, 0.272618)$ and PIBSE(10) = 0.304762.

We have collected the data of last 10 years and analysed them for 10 years, 5 years and 3 years of time frame (mospi.nic.in, sebi.gov.in). The market predictability index (PI_M) is defined as the vector in $\mathbb{R}^4$ as $PI_M = (PI_{FII},$

PI_{CPI}, PI_{IIP}, PI_{DJI}) Here we consider 4-dimensional vector M = (FII, CPI, IIP, DJI). We can also study the inter-relationships between the four components, affecting the market behaviour, from the viewpoint of fractal dimensions. We have used the software KaotiXL from XLpert to find Hurst exponent.

We observe that the CPI exhibits high predictability with either a high volatility or high stability, IIP is not predictable; whereas FII and DJI are less predictable. Also, we observe that FII and DJI behave similarly in the long run. Moreover, BSE Index depends on FII and DJI in the long run.

Table 2: PI of Different Market Parameters for 5 Years

Parameter	H	D	PI
FII	0.644172	1.355828	0.288344
CPI	0.916871	1.083129	0.833742
IIP	0.572620	1.427380	0.145240
DJI	0.473372	1.526628	0.053256
BSE	0.586764	1.413236	0.173528

$PI_M(5)$ = (0.288344, 0.833742, 0.145240, 0.053256) $PI_M(5)$ = (0.288344, 0.833742, 0.145240, 0.053256) and $PI_{BSE}(5)$ = 0.173528.

We observe that the CPI exhibits high predictability, IIP and DJI are not predictable, while FII is less predictable. Also, we see that BSE Index depend on IIP in the medium run.

Table 3: PI of Different Market Parameters for 3 Years

Parameter	H	D	PI
FII	0.540911	1.459089	0.081822
CPI	0.859096	1.140904	0.718192
IIP	0.586501	1.413499	0.173002
DJI	0.641843	1.358157	0.283686
BSE	0.490752	1.509248	0.018496

$PI_M(3)$ = (0.081822, 0.718192, 0.173002, 0.283686)

and $PI_{BSE}(3)$ = 0.018496.

We again observe that the CPI exhibits high predictability. Trends of IIP and FII are not confirming having predictability while DJI is less predictable. Also, we see that BSE Index depend on FII in the short run.

4.3. Stock Market Indices

We have analysed different stock market indices for 200 days (bloomberg.com) and have come out with the following result.

Table 4: PI of Different Indices for 200 Days

Index	H	D	PI
Dow	0.655696	1.344304	0.311392
S & P	0.517800	1.482200	0.035600
BSE	0.482797	1.517203	0.034406
Nifty	0.520778	1.479222	0.041556
Hangseng	0.594871	1.405129	0.189742
Nikkei	0.555871	1.444129	0.055871
Shanghai Composite	0.786000	1.214000	0.572000
St. Times	0.660021	1.339979	0.320042
Kospi	0.580600	1.419400	0.161200

A noticeable amount of predictability is found in Shanghai Composite. Little less predictability is found in Dow Jones and Straight Time Index. The S&P, BSE, Nifty, Nikkei approximate the Brownian motion and are therefore unpredictable.

4.4. Different Currencies

We consider currencies of different Asian countries against US $ for last one year movement (globalfinancialdata.com). The result is shown in Table 5.

Table 5: PI of Different Currencies for 1 Year

Country /US $	H	D	PI
India	0.7530	1.2470	0.5060
China	0.4089	1.5911	0.1822
Hong Kong	0.6654	1.3346	0.3308
Indonesia	0.6062	1.3938	0.2124
Japan	0.6219	1.3781	0.2438
Malaysia	0.4705	1.5295	0.0590
Philippines	0.5547	1.4453	0.1094
Saudi Arabia	0.1160	1.8840	0.7680
Singapore	0.5633	1.4367	0.1266
Thailand	0.6368	1.3632	0.2736

The currencies of Saudi Arabiaand China exhibit anti-persistent behaviour indicating either more stability or more volatility, whereas those of Malaysia, Philippines, and Singapore more or less follow random walk. The rest shows persistent behaviour against US $.

4.5. Gold Prices

We have analysed the data of gold prices in different currencies for 9100 days (30 years) price movement and 200 days price movement (gold.org). The rationale behind the study in different currencies is that the demand for gold is likely to rise as the world heads towards a multi-currency reserve system under the impact of uncertainty about the stability of the dollar and the euro, the main official assets held by central banks and sovereign funds. The role of gold in the international monetary system is likely to be further enhanced in the coming 10 years. For central banks, concerned with preserving value and naturally politically cautious, gold will prove a haven from currency storms.

Table 6: PI of Gold Prices for 30 years

Currency	*H*	*D*	*PI*
US $	0.5775	1.4225	0.1550
India`	0.4869	1.5131	0.0262
Saudi Riyal	0.5645	1.4355	0.1290
South Africa Rand	0.4903	1.5097	0.0194
China Yuan	0.5345	1.4655	0.0690

The time series over 30 years of daily closing price of gold in different currencies follow more or less a Brownian path.

Table 7: PI of Gold Prices for 200 Days

Currency	*H*	*D*	*PI*
US $	0.5386	1.4614	0.0772
India `	0.6441	1.3559	0.2882
Saudi Riyal	0.5388	1.4612	0.0776
South Africa Rand	0.5934	1.4046	0.1868
China Yuan	0.5619	1.4381	0.1238

We observe that 200 days daily closing price of gold exhibits some predictability for India. Others follow a Brownian path.

5. Conclusion

We have considered fBM to model a financial time series associated with a market parameter. This formalism is more general than Brownian motion. Indeed it includes Brownian motion as a particular case. It also incorporates long range predictability in the time series which is quantified as a predictability index measuring the variation of fractal dimension D of the series from the BM value of D which is 1.5. More generally, we represent state vector of a financial market as $X = (X_1, X_2, ..., X_n)$ an n-tuple of financial market parameters X_i taken together. Then the predictability index of X is quantified as value of suitable norm of the corresponding predictability index vector. This formalism is applied to existing data base to check whether volatility vector, stock market vector and parameters like currency rates and gold prices follow fBM or not. The data analysis exhibits that CPI does indeed follow fBM exhibiting appreciable predictability, whereas IIP is not much predictable. In stock market indices, Shanghai Composite follows fBM exhibiting noticeable predictability; whereas S & P, BSE, NIFTY and Nikkei are very much close to BM indicating absence of predictability. On the other hand, the time series over last 30 years of daily closing price of Gold in different currencies follow more or less a Brownian path ruling out long range predictability. Such market behavior needs to be understood by analyzing the market forces responsible for these. The fBM behavior of some of the market parameters is in conformity with the findings of Lo and MacKinlay (1988) confirming that the observable market values exhibit serial correlation suggesting that they follow fBM. This suggests the promising nature of fBM and PI to understand market; and further data analysis with available data base is warranted. On the other hand, on the theoretical level, there is a non-conformity in the use of fBM in option pricing theory due to its inability to incorporate non-arbitrage in the formalism (Rostek, 2009). This has resulted in efforts to exclude arbitrage from theoretical formulation involving fBM through appropriate modifications like restrictions on trading strategies and regularization of local path behavior in fBM. Thus understanding financial market as fBM is an important active area of theoretical studies. The present study contributes to numerical experimental support for employing fBM to understand financial behavior.

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