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General Rough Properties of Soft Topological Space

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Abstract

In this paper, the soft general topology is defined.The general (lower) rough approximation is defined on the soft topological space over the soft general rough formal context which associates with the rough set, formal context and soft sets together using the arbitrary relation between the objects set and attributes set.The general rough properties of the soft topological space is discussed over the soft general rough formal context.

Keywords: Soft General Topological Space, General Lower (Upper) Approximation, Soft Set, Soft General Rough Formal Context

1. Introduction

Formal concept context (Ganter & Wille, 1999), rough set (Pawlak, 1982), soft set (Molodtsov, 1999) are the topics that are extensively applied in uncertainty reasoning, uncertainty of data modelling, and problems in engineering, physics, computer sciences, economics, social sciences, medical science and many other diverse fields. These may be due to the uncertainties of natural environmental phenomena, such as vagueness or uncertainty in the boundary between states or between urban and rural areas and so on. Many researchers have studiedthese uncertainties and the related research results are flourishing.

Li & Zhen (2009) defined the rough formal context, and discussed its properties. Shabir and Naz (2011) initiated the study of soft topological spaces. Hussain& Ahmad (n.d.) continued investigating the properties of soft topological spaces. Li (2013a) defined the rough soft formal context, and discussed the rough properties of rough formal context in soft set. Li (2013b) defined the soft general rough formal context and discussed their properties. Li (2014) discussed the topological structure of the rough soft formal context.

In this paper, we discuss the general rough properties of topological structure based on the general rough soft formal context . The rest of this paper is organized as following.

In the second section, we review some basic concepts and properties of rough concept formal context, soft sets and soft topology space. In the third section, we discuss soft topological space over the soft general rough formal context, define the general rough approximation operations over the soft general topological space which associates with the rough set, formal context and soft sets together using the arbitrary relation between the objects set and attributes set, and discuss general rough properties of the rough lower (upper) approximations based on the soft topology over the soft general rough formal context. Conclusions are given in the fourth section.

2. Basic Concepts

Definition 2.1 (Molodtsov, 1999): Let U be an initial universe set and E be a set of parameters . Let P(U) denotes the power set of U and $A \subset E$. Then a pair (F, A) is called a soft set over U, where $F : A \rightarrow P(U)$ is a mapping.

That is, the soft set is a parameterized family of subsets of the set U. Every set F(e), $\forall e \in E$, from this family may be considered as the set of e-elements of the soft set (F,

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E), or considered as the set of e-approximate elements of the soft set. According to this, we can view a soft set (F, E) as a collection of approximations: (F, E) = {(F(e), e) | e ∈ M}.

Definition 2.2 (Li & Zhen, 2009): Let (G, M, R) be a rough formal context; G is objects set, also called the universe; M is attributes set. A pair (F, B) is a soft set over G, where B ⊆ M, and

F : B → P(G) is a set-value mapping over G, furthermore, the lower and upper rough approximations of pair (F, B) are denoted by R(F, B) = (F, B), R(F, B) = (F, B), which are soft sets over G with the set-valued mappings given by F(x) = B(F(x)) and F(x) = B(F(x)), where x ∈ B.

The operators R, R are called the lower and upper rough approximation operators on soft set (F, B).

If R = R, we say that the soft set (F, B) is definable, otherwise, (F, B) is rough.

we call such quadruple tuple (G, M, R, F) as rough soft formal context, and, such soft set (F, B) on the rough soft formal context (G, M, R, F) which is called rough soft formal set.

Obviously, ∀x ∈ B ⊆ M, F(x) ⊆ G is a parameterised family of subsets of G, and F(x) is the set of x- approximate elements in (G, M, R, F).

We replace the equivalent relation by any relation R ⊆ U ×M, where∀x ∈ U, ∀y ∈ M, if (x, y) ∈ R, then we say object x has the property y. Moreover, we call rough formal context (U, M, R) with any relation R as the general rough formal context.

Definition 2.3 (Li, 2013a): Let (G, M, R, F) be the rough soft formal Context; F1, F2 : B → P(G) are two set-value mapping over G on (G, M, R, F). (F1, B1) and (F2, B2) are two rough soft formal sets over (G, M, R, F).

(i) The union of (F1, B1) and (F2, B2) is the rough soft formal set (H, C), where C = B1 ∪ B2, and ∀e ∈ C, denoted as (F1, B1) ⊔ (F2, B2) = (H, C) = (H, B1 ∪ B2),where H(e) = F1(e), if e ∈ B1 − B2

F2(e), if e ∈ B2 − B1

F1(e) ∪ F2(e), if e ∈ B1 ∩ B2

(ii) The intersection of (F1, B1) and (F2, B2) is the soft rough formal set (H, C) is denoted as (F1, B1) ∩ (F2, B2) and is defined as (F1, B1) ⊓ (F2, B2) = (H, C), where

C = B1 ∩ B2, and ∀e ∈ C, H(e) = F1(e) ∩ F2(e).

Definition 2.4 (Li, 2013b): Let G = (U, M, R) be a general rough formal context, U is objects set, also is the universe, M is attributes set, R ⊆ U × M is an arbitrary relation.

Rs(x) = {y ∈ M | xRy, x ∈ U} means the set of attributives common to the objects; and

Rp(y) = {x ∈ U | xRy, y ∈ M} means the set of all objects which have the property x.

A pair S = (F, B) is a soft set over U which is called the soft general rough set, where B ⊆ M, and F : B → P(U) is a set-value mapping over U. We call pair P = (G, S) as a soft general rough formal context which is also a soft approximation space based on P , ∀B, B ⊆ M, ∀X, X ⊆ U;the lower and upper rough approximations is defined as apr P X = {x ∈ U | ∃y ∈ B s.t. x ∈ F(y) ⊆ X}; aprPX = {x ∈ U | ∃y ∈ B s.t. F(y) ∩ X □= ∅}.

The operatorsapr P X, apr PX are called the lower and upper general rough approximation operators on soft set (F, B), simply called lower and upper soft general rough approximation operators, respectively.

Definition 2.5 (Li, 2013b): Let G = (U, M, R) be a general rough formal context, S = (F, B) be a soft set over U, and

P = (G, S) be a soft general rough formal context, let

B1, B2 ⊆ M, then (F, B),(F1, B1) and (F2, B2) are soft sets over U. F, Fi : Bi → P(U), i = 1, 2 are two set-value mappings over U on P.

(1) if B1 ⊆ B2, and F1(x) ⊆ F2(x), ∀x ∈ B1 ⊆ B2, then the soft sets (F1, B1) is a soft subset of the soft set (F2, B2), denoted as (F1, B1) ⊑ (F2, B2).

(2) Two soft sets (F1, B1) and (F2, B2) on the soft general rough formal context P are said soft equal , if (F1, B1) ⊑ (F2, B2), and (F2, B2) ⊑ (F1, B1). We simply denote by (F1, B1) = (F2, B2).

(3) The relative complement of (F, B) is denoted by (F, B) c and is defined by (F, B) c = (F c, ¬B), where F c : ¬B → P(U) and F c (x) = U − F(x), ∀x ∈ B.

(4) (F, B) is said to be a relative null soft general rough formal set denoted by N , if ∀x ∈ B, F(x) = ∅, if B = U, then is called absolute null rough soft formal set.

(5) (F, B) is said to be a relative whole soft general rough formal set denoted by W, if ∀x ∈ B, F(x) = U, B ⊆ M.

Definition 2.6 (Li, 2014): Let T = (G, M, R, F) be a rough soft formal context over the object set G and attributes set M, Bi ⊆ M, τ = {(Fi, Bi) | (Fi, Bi) is a soft set over G} which is the collection of soft sets on the rough soft formal context (G, M, R, F), if

(1) ∅, Ge belong to τ .

(2) The union of any number of soft sets in τ belongs to τ, that is, τ is closed for the any union of soft sets over T .

(3) The intersection of any two soft sets in τ belongs to τ , that is, τ is closed for the finite intersection of soft sets over T.

Then the collection τ is calleda soft topology over the rough soft formal context T (simply called soft rough topology).

The triplet (G, τ, M) is called a soft topological space over the rough soft formal context T (simply called soft rough topological space), and the members of τ are soft open sets in T , the relative complement (F, B) c = (F c, B) is said to be a soft closed set in T if (F, B) c∈ τ .

3. The General Rough Properties of the Soft Topology

Definition 3.1: Let G = (U, M, R) be a general rough formal context, S = (F, B) be a soft set over U, and P = (G, S) be a soft general rough formal context, τ = {(Fi, Bi) | (Fi, Bi) is a soft set over U, Bi ⊆ M} which is the collection of soft sets on the general rough soft formal context P = (G, S), if

(1) ∅,Ue belong to τ .

(2) The union of any number of soft sets in τ belongs to τ, that is, τ is closed for the any union of soft sets over P.

(3) The intersection of any two soft sets in τ belongs to τ, that is, τ is closed for the finite intersection of soft sets over P.

Then the collection τ is called a soft general topology over the general rough soft formal context P(simply called soft general rough topology). The triplet (G, S, τ) is called a soft general topological space over the general rough soft formal context P (simply called soft general rough topological space), and the members of τ are soft open sets in P, the relative complement (F, B) c = (F c, B) is said to be a soft closed set in P if (F, B) c∈ τ .

Based on P ,∀B, B ⊆ M, ∀X, X ⊆ U, the lower and upper rough approximations are defined as apr P X = {x ∈ U | ∃y ∈ B s.t. x ∈ F(y) ⊆ X}; aprPX = {x ∈ U | ∃y ∈ B s.t. F(y) ∩ X □= ∅}.

The operators X, aprPX are called the lower and upper general rough approximation operators on soft set (F, B), respectively.

Moreover, If apr P X = aprPX, we say that X is definable, otherwise, X is rough. posPX = apr P X is soft positive region of X over P; negPX = U − aprPX is soft negative region of X over P; bnPX = aprPX − aprP X is boundary region of X over P.

In the following, we simply say ”soft general rough set” over P as ”soft set” over P.

Obviously, ∀x ∈ B ⊆ M, F(x) ⊆ U is a parameterised family of subsets of U, and F(x) is the set of x- approximate elements which is a subset of U in P = (G, S).

Example 1: Let G = (U, M, R) be a general rough formal context, S = (F, B), B ⊆ M be a soft set over U, and

P = (G, S) be a soft general rough formal context, the triplet(G, S, τ) is a soft general rough topological space over the general rough soft formal context P, and τ = {∅,Ue}, then τ is a trivial soft general rough topology over G; if we denote τ as the collection of all soft sets which determined by the power set of U, that is, τ = {(Fi, M) | Fi : M → ℘(U)}, then τ is a discrete soft general rough topology over G.

Example 2: Let G = (U, M, R) be a general rough formal context, S = (F, B) be a soft set over U, and P = (G, S) be a soft general rough formal context, the triplet (G, S, τ) is a soft general rough topological space over the general rough soft formal context P, in which U = {h1, h2, h3, h4} is an initial universe set, M = {e1, e2} is a set of attributes, We can represent using table as: e1 e2 h1 1 1 h2 1 1 h3 0 1 h4 1 0 Tab1.

Let X = {h1, h3} ⊆ U, B = {e1}, because F(e1) = {h1} ⊆ X, we can get aprP X = {h1} = aprPX =, and posP (X) = {h1}, negP (X) = {h3}, X is definable in the soft general rough formal context P = (G, S).

And the collection τ =

{∅, X, e (F1, M1),(F2, M2),(F3, M),(F4, M),(F5, M),(F6, M)}

is a soft general rough topology on P, in which the subsets

M1 = {e1}, M2 = {e2}, and soft sets:

(F1, M1) = {({h2}, e1)}, (F2, M2) = {({h1}, e2)},

(F3, M) = {({h1, h2}, e1),(G, e2)},

(F4, M) = {({h1, h2}, e1),({h1, h3}, e2)},

(F5, M) = {({h2}, e1),({h1, h2}, e2)},

(F6, M) = {({h2}, e1),({h1}, e2)} .

Note: We can understand (F1, M1) = {({h2}, e1)} as

(F1, M1) = {({h2}, e1),(∅, e2)}, so, for simply, in the

following, we do not take the subset of the attributes set M.

Proposition 3.1: Let G = (U, M, R) be a general rough formal

context, S = (F, B) be a soft set over U, and P = (G, S) be

a soft general rough formal context, the triplet (G, S, τ) is a

soft general rough topological space over the general rough

soft formal context P, then

(1) ∅, Ge are closed soft sets over P.

(2) The union of any two soft closed sets in τ belongs to τ ,

that is, τ is closed for the finite union of soft closed sets over

P.

(3) The intersection of any number of soft closed sets in τ

belongs to τ , that is, τ is closed for the any intersection of

soft closed sets over P.

Proof: (i) Clearly, ∅

c = G, e Gec = ∅, so, ∅, Ge are closed soft

sets over P;

(ii) Let (F1, B1) and (F2, B2) be closed soft sets

over P, then (F1, B1)

c

,(F2, B2)

c∈ τ , by the

Definition 3.1, (F1, B1)

c⊓ (F2, B2)

c∈ τ , and

((F1, B1)

c⊓ (F2, B2)

c

)

c = (F1, B1) ⊔ (F2, B2) ∈τ ;

(iii) Suppose that {(Fi

, Bi) | i∈ I} be a family of

closed soft set in ∈ τ , then (Fi

, Bi)

c

is open soft

set in τ , for all i∈ I, so (

⊔

i∈I

(Fi

, Bi)

c

)

c∈ τ , and

(

⊔

$i \in I$

$(F_i$

$, B_i)$

c

$)$

$c = \sqcap_{i \in I} (F_i$

$, B_i) \in \tau$.

Example 3: Let G = (U, M, R) be a general rough

formal context, S = (F, B) be a soft set over U, and

P = (G, S) be a soft general rough formal context, the triplet

(G, S, τ) is a soft general rough topological space over the

general rough soft formal context P, then the collection

$\tau\alpha = \{F(\alpha) \mid (F, B) \in \tau, \alpha \in B \subseteq M\}$ for each $\alpha \in B$ is a

soft rough topology on P.

Obviously, because τ is a soft general rough topology over

P,

(i) by$\emptyset$,$Ue \in \tau$, we know $\emptyset\alpha$,$Ue\alpha \in \tau\alpha$;

(ii) By $(F_1, B_1),(F_2, B_2) \in \tau$ can infer $F_1(\alpha), F_2(\alpha) \in \tau\alpha$,

and $(F_1, B_1) \sqcap (F_2, B_2) \in \tau$ can infer $F_1(\alpha) \cap F_2(\alpha) \in \tau\alpha$;

(iii) By $(F_i$

$, B_i) \in \tau$ can infer $F_i \in \tau\alpha$, and $(F_i$

$, B_i) \in \tau$ can

infer$\cup$

$_i F_i(\alpha) \in \tau\alpha$.

Consider example 2, $\tau e1 = \{\emptyset, M, f \{h2\}, \{h1, h2\}\}$, $\tau e2$ =

$\{\emptyset, M, f \{h1\}, \{h1, h2\}, \{h1, h3\}\}$.

Proposition 3.2: Let G = (U, M, R) be a general rough

formal context, S = (F, B) be a soft set over U, and

P = (G, S) be a soft general rough formal context, the triplet

$(G, S, \tau_1),(G, S, \tau_2)$ be two soft general rough topological

spaces over the general rough soft formal context P, then

$(G, S, \tau_1 \cap \tau_2)$ is a soft topological space over P.

Proof: (i) Clearly, $\emptyset$, $Ge \in \tau_1 \cap \tau_2$;

(ii) Let $(F_1, B_1),(F_2, B_2) \in \tau_1 \cap \tau_2$, then

$(F_1, B_1),(F_2, B_2) \in \tau_1$, and $(F_1, B_1),(F_2, B_2) \in \tau_2$

since $(F_1, B_1) \sqcap (F_2, B_2) \in \tau_1$ and $(F_1, B_1) \sqcap (F_2, B_2) \in \tau_2$,

$(F_1, B_1) \sqcap (F_2, B_2) \in \tau_1 \cap \tau_2$;

(iii) Suppose that $\{(F_i$

$, B_i) \mid i \in I\}$ be a family of soft set

in$\in \tau_1 \cap \tau_2$, then $(F_i$

$, B_i) \in \tau_1$, and $(F_i$

$, B_i) \in \tau_2$, for all

$i \in I$, so $\sqcup$

$i \in I$

$(F_i$

$, B_i) \in \tau_1$, and $\sqcup$

$i \in I$

$(F_i$

$, B_i) \in \tau_2$, Hence

$\sqcup$

$i \in I$

$(F_i$

$, B_i) \in \tau_1 \cap \tau_2$.

Remark 1: Let $(G, S, \tau_1),(G, S, \tau_2)$ be two soft general

rough topological spaces over the rough soft formal context

P, however, $(G, S, \tau_1 \cup \tau_2)$ can be not a soft general rough topological space over P.

In fact, if (F1, B1),(F2, B2) ∈ τ1 ∪ τ2, then there

are four cases, case 1: (F1, B1),(F2, B2) ∈ τ1 − τ2;

case 2: (F1, B1),(F2, B2) ∈ τ2 − τ1; case 3:

(F1, B1),(F2, B2) ∈ τ1 ∩ τ2 ⊆ τ1 ∪ τ2, under these

three cases, (G, S, τ1 ∪ τ2) must be a soft rough topology

over P; case 4: (F1, B1) ∈ τ1, and (F2, B2) ∈ τ2, in this

case, (G, S, τ1 ∪ τ2) can be not a soft general rough topology

over P. For example:

In example 3, let τ1 = {∅, X, e (F

′

1

, M1),(F

′

2

, M2),(F

′

3

, M)}

is a soft general rough topology on T , in which (F

′

1

, M1) =

{({h1}, e1)},(F

′

2

, M2) = {({h2, h3}, e2)},(F

′

3

, M) =

{({h1}, e1),({h2, h3}, e2)}. Then τ ∪ τ1 = {∅, X, e (F1, M1),

(F2, M2),(F3, M),(F4, M),(F5, M),(F6, M),(F

′

1

, M1),

(F

′

2

, M2),(F

′

3

, M)}, consider (F4, M) ⊓ (F

′

3

, M) =

{({h1}, e1),({h3}, e2)} □∈τ∪ τ1, so τ ∪ τ1 is not a

soft general rough topology on P.

Proposition 3.3: Let G = (U, M, R) be a general rough

formal context, S = (F, B) be a soft set over U, andP = (G, S) be a soft general rough formal context, the triplet

(G, S, τ) is a soft general rough topological space over the

general rough soft formal context P, then ∀X ⊆ U,

aprP

(X) = ∪

a∈B{F(a) ⊆ U | F(a) ⊆ X};

aprP (X) = ∪

a∈B{F(a) ⊆ U | F(a) ∩ X □= ∅}.

Proof: This is easily obtained from the definition.

Let G = (U, M, R) be a general rough formal context,

S = (F, B) be a soft set over U, and P = (G, S) be a soft

general rough formal context, the triplet (G, S, τ) is a soft

general rough topological space over the general rough soft formal context P, then ∀X ⊆ U, using the idea of Pawlak's

rough set, we can define four basic classes of soft general

rough formal sets:

(1) X is roughly soft definable, if aprP

(X) □= ∅ and

aprP (X) □= U;

This means we can decide for some elements of U which

belong to X, and we are able to some elements which belong

to U − X.

(2) X is internally soft indefinable, if aprP

(X) = ∅ and

aprP (X) □= U;

This means we can decide for some elements of U which

belong to U − X, and we can't decide some elements which

belong to X.

(3) X is externally soft indefinable, if aprP

(X) □= ∅ and

aprP (X) = U;

This means we can decide for some elements of U which

belong to X, and we can't decide some elements which belong

to U − X.

(4) X is totally soft indefinable, if aprP

(X) = ∅ and

aprP (X) = U.

This means we can't decide for some elements of U which

belong to X, and we can't decide some elements which belong

to U − X.

Definition 3.2: Let G = (U, M, R) be a general rough formal

context, S = (F, B) be a soft set over U, and P = (G, S)

be a soft general rough formal context, the triplet (G, S, τ) is

a soft general rough topological space over the general rough

soft formal context P, X, Y ⊆ U, we define the lower soft

general rough relation, the upper soft general rough relation,

and the soft general rough relation, respectively, as following:

(1) X∼˙ P Y ⇔aprP

(X) = aprP

(Y);

(2) Xv˙ P Y ⇔aprP (X) = aprP (Y);

(3) X ≈P Y ⇔ Xv˙ P Y and X∼. P Y .

Clearly, all this relations are equivalent relations, and it is

easy to verify that all those relations satisfy the following

properties:

Proposition 3.4: Let G = (U, M, R) be a general rough formal

context, S = (F, B) be a soft set over U, and P = (G, S) be

a soft general rough formal context, the triplet (G, S, τ) is a

soft general rough topological space over the general rough

soft formal context P, X1, Y1, X, Y ⊆ U, then

(1) X∼˙ P Y ⇔ X∼˙ P (X ∪Y) ˙∼P Y ;

(2) X~˙ PX1, Y ~˙ P Y1, ⇔ (X ∪Y) ˙~P (X1 ∪ Y1);

(3) X~˙ P Y ⇒ (X ∪ U − Y) ˙~P U;

(4) X ⊆ Y, Y ~˙ P ∅⇒ X~˙ P ∅;

(5) X ⊆ Y,X ~˙ P U ⇒ X~˙ P U.

Correspondingly, relation ~. P has the similar properties.

Proposition 3.5 Let G = (U, M, R) be a general rough formal

context, S = (F, B) be a soft set over U, and P = (G, S) be

a soft general rough formal context, the triplet (G, S, τ) is a

soft general rough topological space over the general rough

soft formal context P, X, Y ⊆ U, we have the following

properties:

(1) aprP

(∅) = aprP (∅) = ∅;

(2) aprP

(U) = aprP (U) = U;

(3) X ⊆ Y ⇒aprP

(X) ⊆aprP

(Y);

(4) X ⊆ Y ⇒aprP (X) ⊆aprP (Y);

(5) aprP

(X ∩ Y)⊆aprP

(X) ∩ aprP

(Y);

(6) aprP

(X ∪Y)⊇aprP

(X) ∪aprP

(Y);

(7) aprP (X ∪ Y) = aprP (X) ∪aprP (Y);

(8) aprP (X ∩ Y) ⊆aprP (X) ∩ aprP (Y).

Proof: The proof is directed from the definition and the

proposition.

Remark 2: The including relation of (5) and (6) change ⊆ into

=, if and only if ∃a ∈ B, such that F(a) ⊆ X or, F(a) ⊆ Y ,

and F(a) * X ∩ Y .

Remark 3: In proposition 3.5, if we take R as a similar relation

(that is, R is reflexive and transitivity), and P = (G, S) is

a soft general rough formal context, then ∀X ⊆ U which

satisfies the properties as:

aprP

X ⊆ X, X ⊆aprPX, aprP

X ⊆aprP

aprP

X,

aprPaprPX⊆aprPX.

We can define the soft operations such as [12,24,30],:

Definition 3.3: Let G = (U, M, R) be a general rough formal

context, S = (F, B) be a soft set over U, and P = (G, S)

be a soft general rough formal context, the triplet (G, S, τ) is

a soft general rough topological space over the general rough

soft formal context P.

(1) The restricted intersection of (F1, B1) and (F2, B2) is

the soft general rough formal set (H, C) which is denoted as

(F1, B1) e (F2, B2) and is defined as (F1, B1) e (F2, B2) =

(H, C), where C = B1 ∩ B2, and ∀e ∈ C, H(e) = F1(e) ∩

F2(e).

(2) The extended intersection of (F1, B1) and (F2, B2) is the soft general rough formal set (H, C), where C = B1 ∪ B2, ∀e ∈ C, and denoted as (F1, B1) ⊓ε(F2, B2) = (H, C) = (H, B1 ∪ B2), where

H(e) =

F1(e), if e ∈ B1 − B2

F2(e), if e ∈ B2 − B1

F1(e) ∩ F2(e), if e ∈ B1 ∩ B2

(3) The restricted union of (F1, B1) and and (F2, B2) is defined as (F1, B1) ∪R (F2, B2) = (H, C) and, where C = B1 ∩ B2, and ∀e ∈ C, H(e) = F1(e) ∪ F2(e).

Theorem 1: Let G = (U, M, R) be a general rough formal context, S = (F, B) be a soft set over U, and P = (G, S) be a soft general rough formal context, the triplet (G, S, τ) is a

soft general rough topological space over the general rough

soft formal context P. Then we have

(1) aprP

((F1, B1) e (F2, B2)) ⊆aprP

(F1, B1) e

aprP

(F2, B2);(2) aprP

((F1, B1) ⊓ε (F2, B2)) ⊆aprP

(F1, B1) ⊓ε

aprP

(F2, B2);

(3) aprP ((F1, B1) e (F2, B2)) ⊆aprP (F1, B1) e

aprP (F2, B2);

(4) aprP ((F1, B1) ⊓ε (F2, B2)) ⊆aprP (F1, B1) ⊓ε

aprP (F2, B2);

(5) aprP

((F1, B1) ∪R (F2, B2)) ⊆aprP

(F1, B1) ∪R

aprP

(F2, B2);

(6) aprP

((F1, B1) ⊔ (F2, B2)) ⊇aprP

(F1, B1) ⊔

aprP

(F2, B2);

(7) aprP ((F1, B1) ∪R (F2, B2)) = aprP (F1, B1) ∪R

aprP (F2, B2);

(8) aprP ((F1, B1) ⊔ (F2, B2)) = aprP (F1, B1) ⊔

aprP (F2, B2);

(9) If (F1, B1) ⊆ (F2, B2), then aprP

(F1, B1) ⊆

aprP

(F2, B2), aprP (F1, B1) ⊆aprP (F2, B2).

Proof: Let P = (G, S) be a soft general rough formal

context as the theorem , Fi

: Bi → P(U), i = 1, 2 are two

set-value mappings over U . (F1, B1) and (F2, B2) are two

soft sets over P.

(1) Let (F1, B1) e (F2, B2) = (H, C), then C = B1 ∩ B2

and H(x) = F1(x) ∩ F2(x), ∀x ∈ C, using the defi-

nition, aprP

((F1, B1) e (F2, B2)) = aprP

(H, C), where

aprP

H(x) = aprP

(H(x)) = aprP

(F1(x) ∩ F2(x)), because

aprP

(F1(x) ∩ F2(x)) ⊆aprP

(F1(x)) ∩ aprP

(F2(x)) for all

x∈ C = B1 ∩ B2 and so we deduce that

aprP

((F1, B1) e (F2, B2)) ⊆aprP

(F1, B1) e aprP

(F2, B2).

(2) Let (F1, B1) ⊓ε (F2, B2) = (H, C), then C = B1 ∪

B2, ∀x ∈C

H(x) =

F1(x), if x ∈ B1 − B2

F2(x), if x ∈ B2 − B1

F1(x) ∩ F2(x), if x ∈ B1 ∩ B2

using the definition, aprP

((F1, B1) ⊓ε (F2, B2)) =

aprP

(H, C), where

aprP

(H(x)) =

aprP

(F1(x)), if x ∈ B1 − B2

aprP

(F2(x)), if x ∈ B2 − B1

aprP

(F1(x) ∩ F2(x)), if x ∈ B1 ∩ B2

By proposition, aprP

(F1(x) ∩ F2(x)) ⊆aprP

(F1(x)) ∩

aprP

(F2(x)), so, for all x ∈ C.

Hence, aprP

((F1, B1) ⊓ε (F2, B2)) ⊆aprP

(F1, B1) ⊓ε

aprP

(F2, B2).

(3) This is similar to the proof of (1).

(4) This is similar to the proof of (2).

(5) Let (F1, B1) ∪R (F2, B2) = (H, C), then

C = B1 ∩ B2, ∀x ∈ C and H(x) = F1(x) ∪ F2(x),

using the definition,

aprP

H(x) = aprP

(F1(x) ∪ F2(x)) = aprP

(F1(x)) ∪

aprP

(F2(x)).

By proposition 3.3, aprP

((F1, B1) ∪ (F2, B2)) ⊇

aprP

(F1, B1) ∪aprP

(F2, B2) Hence, aprP

((F1, B1) ∪R

(F2, B2)) ⊒aprP

(F1, B1) ∪R aprP

(F2, B2).

(6) Let (F1, B1) ⊔ (F2, B2) = (H, C), then

C = B1 ∪ B2, ∀x ∈ C and H(x) = F1(x) ∪ F2(x),

where

H(x) =

F1(x), if x ∈ B1 − B2

F2(x), if x ∈ B2 − B1

F1(x) ∪ F2(x), if x ∈ B1 ∩ B2

using the definition, aprP

((F1, B1) ⊔ (F2, B2)) =

aprP

(H, C), where

aprP

H(x) =

aprP

(F1(x)), if x ∈ B1 − B2

aprP

(F2(x)), if x ∈ B2 − B1

aprP

(F1(x) ∪ F2(x)), if x ∈ B1 ∩ B2

By proposition 3.3, aprP

(F1(x) ∪ F2(x)) ⊒aprP

(F1(x)) ∩

aprP

(F2(x)), so, for all x ∈ C,

aprP

H(x) =

aprP

(F1(x)), if x ∈ B1 − B2

aprP

(F2(x)), if x ∈ B2 − B1

aprP

(F1(x)) ∪aprP

(F2(x)), if x ∈ B1 ∩ B2

Hence, aprP

((F1, B1) ⊔ (F2, B2)) ⊒aprP

(F1, B1) ⊔

aprP

(F2, B2).

(7) This is similar to the proof of (5).

(8) This is similar to the proof of (6).

(9) Suppose (F1, B1) ⊆ (F2, B2), by definition 3.2, F1(x) ⊆

F2(x), ∀x ∈ B1, using proposition 3.2, aprP

(F1(x)) ⊆

aprP

(F2(x)), and aprP (F1(x)) ⊆aprP (F2(x)) for all

x∈ B1. Hence, we can conclude that aprP

(F1, B1) ⊆

aprP

(F2, B2), aprP (F1, B1)⊆aprP (F2, B2).

Remark 3 we know the properties (5) and (6) can't change

⊆into =, if ∃a3 ∈ B, such that F(a3) = F(a1) ∩ F(a2)

whenever F(a2) ∩ F(a3) □= ∅, in this case, the "inclusion"

will be "equal".

4. Conclusion

In this paper, we discussed soft topological space over the soft general rough formal context, defined the general rough approximation operations over the soft general topological space which associates with the rough set, formal context and soft sets together using the arbitrary relation between the objects set and attributes set.We also discussed the general rough properties of soft topological space based on general rough soft formal context which associates with the rough set, formal context and soft sets together using the arbitrary relation between the objects set and attributes set. That is, we investigated the general rough properties combining soft sets with rough sets, formal context and topology space, some different types of hybrid models are presented, which is general rough properties with soft topology based on the soft general rough formal context.

That offers a new method and tool in data analysis. Therefore, we can deal with lots of data more easily and do more efficient decision in data mining, information system, human reasoning and so on.

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