Impact of Inventory Policies on Supply Chain Dynamics

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Abstract

The fluctuations in the supply chains over time are of significance in research as well as industrial situations. The bullwhip effect which shows the dynamics in the supply chain is a well documented phenomenon. This article presents the importance of considering the inventory policies in supply chain planning to minimize the bullwhip effect. A similar work available in the literature by (Jakšic and Rusjan, 2008) has considered the effect of replenishment policies on the bullwhip effect, but has not considered the Work In Progress (WIP) inventory. We have done the performance analysis to show the role of work in progress inventory in supply chain policy planning. Our policy is named as "Viswanadh's policy", which will increase efficiency of supply chain. We have adopted the control engineering approach to find the system response of our policy. By comparing the system frequency response of both the policies, we found our policies to be better than their policies. Under the given circumstances the improvement in our policies was 15% and 5% respectively.

Keywords: Supply Chain Management, Bullwhip Effect, Replenishment Policies, Linear Control Theory.

1. INTRODUCTION

Supply Chain policies control inventory level within supply chain process. The type of inventory policy has a significant effect on the variability of order quantities and inventory levels at the various stages of a supply chain. In particular, high order variability causes companies to hold excess capacity, to use overtime production, and to use premium shipping, altogether, high variability. This variability of demand when moves from downstream to upstream in a supply chain is called bullwhip effect. The term bullwhip is not a new concept. Though it was first documented by Forrester, the name Bullwhip or whipsaw was coined by (Lee et al. 1997a). It shows a small variance in the demands of the downstream end-customer may cause dramatic variance in the upstream supplier's side. More fundamentally, bullwhip refers to the amplification of end-customer order signals. The diaper sales at Procter and Gamble and the printer sales by Hewlett-Packard which depict variance amplification are typical cases demonstrating the supply chain dynamics. The concept of bullwhip effect is taught to the students worldwide using a business simulation game called the 'Beer Game'' developed at MIT. Demand signal processing, order batching, shortage gaming, lead times and price fluctuations were recognized (Lee et al. 1997b) as the major causes of the bullwhip effect. (Jakšic and Rusjan, 2008). It is documented that supply chain dynamics is sure to occur when using order-up-to policy. We are focusing our attention on replenishment policies for which smoothing of demand pattern may accomplish the lessening or even elimination of the supply chain dynamics. This paper has demonstrated that the replenishment policies used in inventory management is better if they consider more information. In this paper we are showing the effect of consideration of Work In Progress (WIP) inventory on the system response.

The control engineering methodology is used in this paper to explain the occurrence of the bullwhip effect. It differs from the more common statistical inventory control approach as was used in (Lee et al. 1997a, b) and (Chen et al. 2000). Linear control theory has been applied widely in engineering for many years. For an introduction to linear control theory, we refer the reader to (Nagrath and Gopal, 2003) and (Nise, 2000) for a brief overview of its historic background. In an inventory management context, linear control theory was first applied by (Simon, 1952) and (Vassian, 1955). While Simon considered a continuoustime representation to study the steady-state behavior of an inventory system, Vassian applied a discrete-time representation to provide an order rule based on past customer orders and current inventories In recent years linear control theory has been widely used for developing inventory policies and for studying the bullwhip effect.

This paper is discussed in six sections. In section two, we have discussed the methodology. Replenishment policies and their transfer functions with their frequency response graphs are dealt with in section three. We have discussed about the results in section four and conclusions in section five.

1.1 Nomenclature

- α Exponential Smoothing constant. ($0<\alpha<1$)
- β Parameter of inventory position smoothing. ($0<\beta<1$)
- γ Parameter of order quantity smoothing. ($0<\gamma<1$)
- D_t Observed Customer Demand
- $\hat{D}_t$ Demand forecast for the next period
- $\hat{D}_{t-1}$ Demand forecast made in the previous period t-1
- FR Frequency Response
- O_t The order quantity at t
- O_{t-1} Last placed order quantity.
- IP_t Current inventory position
- IP_t^T Target inventory position.
- WIP Work in Progress Inventory
- w Frequency.
- k Defines a desired service level times the ratio of the standard deviation over the forecast demand
- T_L Replenishment lead time ($0 < T_L < 1$).
- NS_t Net Inventory position at time t.
- z z-transform operator.

2. METHODOLOGY

The linear control theory is the basis used in our study. Transfer functions were derived and corresponding frequency response graphs were plotted. We have used MATLAB7.0 software for plotting the frequency response. Reader can refer to (Rudrapratap, 2006) for more details on MATLAB7.0.

2.1 Transfer Function

We have used the term "control system" to refer to an amalgamation of elements. The function of control system can be mathematically described using its transfer function G(t), that is equated as the ratio of the control system's output e2(t) and input e1(t):

$$G(t)=e2(t)/e1(2) \qquad (1)$$

(D'Azzo and Houpis, 1966) have reported that if the control system is linear, the analysis is made easy with the use of the Laplace transformation and a move from the time space 't' into the space 's', the space of the Laplace operator. The transfer function is defined as a ratio of the Z-transform of the output signal to the Z-transform of the input signal and is written as a ratio of two polynomials with z being a variable:

$$G(z)=\frac{b_0+b_1z+b_2z^2+......+b_qz^q}{a_0+a_1z+a_2z^2+......+a_qz^q} \qquad (2)$$

2.2 The Graph of Frequency Response

Sinusoidal inputs of different frequencies are considered for drawing the Frequency Response graph (FR) of a replenishment rule. It implies that our aim is to know the output (orders) when the input (demand) is sinusoidal. As we are considering a linear system, though the amplitude and the phase angle may have changed, the output will be a sine wave with same frequency. It is well known from Shannon's Sampling Theorem, that sampled data systems can only detect inputs of frequencies up to π radians per sampling interval unit, hence the graph is only required for the frequencies 0 to π. For all these frequencies, technically, the FR graph is made by letting z =exp (iw) in the transfer function and calculating the modulus of the vector in the complex plane. Because of the fact that any real life demand data can be seen as composed of different sinusoids, it makes sense to analyze responses to different sine waves. The FR graph immediately yields insight into the dynamic behavior of the replenishment rule, without making any assumptions on the distribution of the demand pattern. It will be used to make predictions on whether or not the replenishment rule will lead to variance amplification. The variance amplification of orders over demand constitutes the bullwhip effect. So the amplitude frequency response plot gives us the magnitude of supply chain dynamics for a sinusoidal demand patterns of frequencies [0, π/T]. A sinusoidal input with frequency w = 0 corresponds to constant demand and is amplified by neither policy. A sinusoidal input with frequency $w = \pi$ radians corresponds to alternating demand between periods and is amplified.

3. REPLENISHMENT POLICIES

A supply chain having a single manufacturer and a single retailer are considered. Even before the customer demand D_t is noted and filled, the retailer receives the order from the manufacturer, during each period t. Backlogging takes place for unsatisfied demands. The latest inventory level is observed by the retailer and the demand for the next stage is forecasted. Then retailer places the replenishment order O_t with the manufacturer. The replenishment time is fixed and is denoted by L which represents the time between order placement by retailer and the replenishment at the retailer echelon. Therefore the receiving of an order happens at the end of period t + L, where t is the time period at which order is placed.

3.1 Policy of Demand Forecasting and Rules of Replenishment

In this study we have considered two policies considered by (Jakšic and Rusjan, 2008) but we have added work in progress (WIP) inventory to those two policies. All through the paper it is assumed that the retailer is using a well-known method of simple exponential smoothing to estimate a demand forecast for the next period $\hat{D}_t$, that is:

$$\hat{D}_t = \hat{D}_{t-1} + \alpha(\hat{D}_t - \hat{D}_{t-1}) \tag{3}$$

Observe that, with the notation used, D_t represents the observed customer demand from the previous period, which we tried to predict by the demand forecast made in the previous period t-1, $\hat{D}_{t-1}$.

3.2 Replenishment Policies

The policies by (Jakšic and Rusjan, 2008) are shown below in *Policy I* and *Policy II*. As the present paper is an extension of the aforementioned paper, the assumptions and terminology used are the same, but the policies were redesigned by adding work in progress (WIP) inventory in the proposed policies which were discussed below as *Proposed Policy I* and *Proposed Policy II* and its detailed block diagram is shown in Figure 1. For more explanation of blocks in Figure 1 the reader can refer to (Jakšic and Rusjan, 2008).

Jakšic and Rusjan's Policy Set

Policy I:

$$O_t = \hat{D}_t + (1-\gamma)(O_{t-1} - \hat{D}_t) + \beta(IP_t^T - IP_t) \tag{4}$$

this policy is named as (R, γO, βIP) policy.

Where β is parameter of inventory position smoothing and γ is parameter of order quantity smoothing.

Policy II:

For the second policy β is 0< β<1 and γ =1 and is named as for (R, βIP) policy.

The policy is

$$O_t = \hat{D}_t + \beta(IP_t^T - IP_t) \tag{5}$$

Viswanadh's Policy Set

The proposed new policy set was named as Viswanadh's policies. In the proposed policy the assumptions were taken as same as taken by (Jakšic and Rusjan, 2008) , but the inventory position (IP_t) is replaced by Net Inventory (NS_t)

Proposed Policy I

Here the Net inventory Position (NS_t) is defined as Current Inventory position (IP_t) plus Work in progress (WIP) inventory i.e.

$NS_t = IP_t +$ work in progress (WIP) inventory

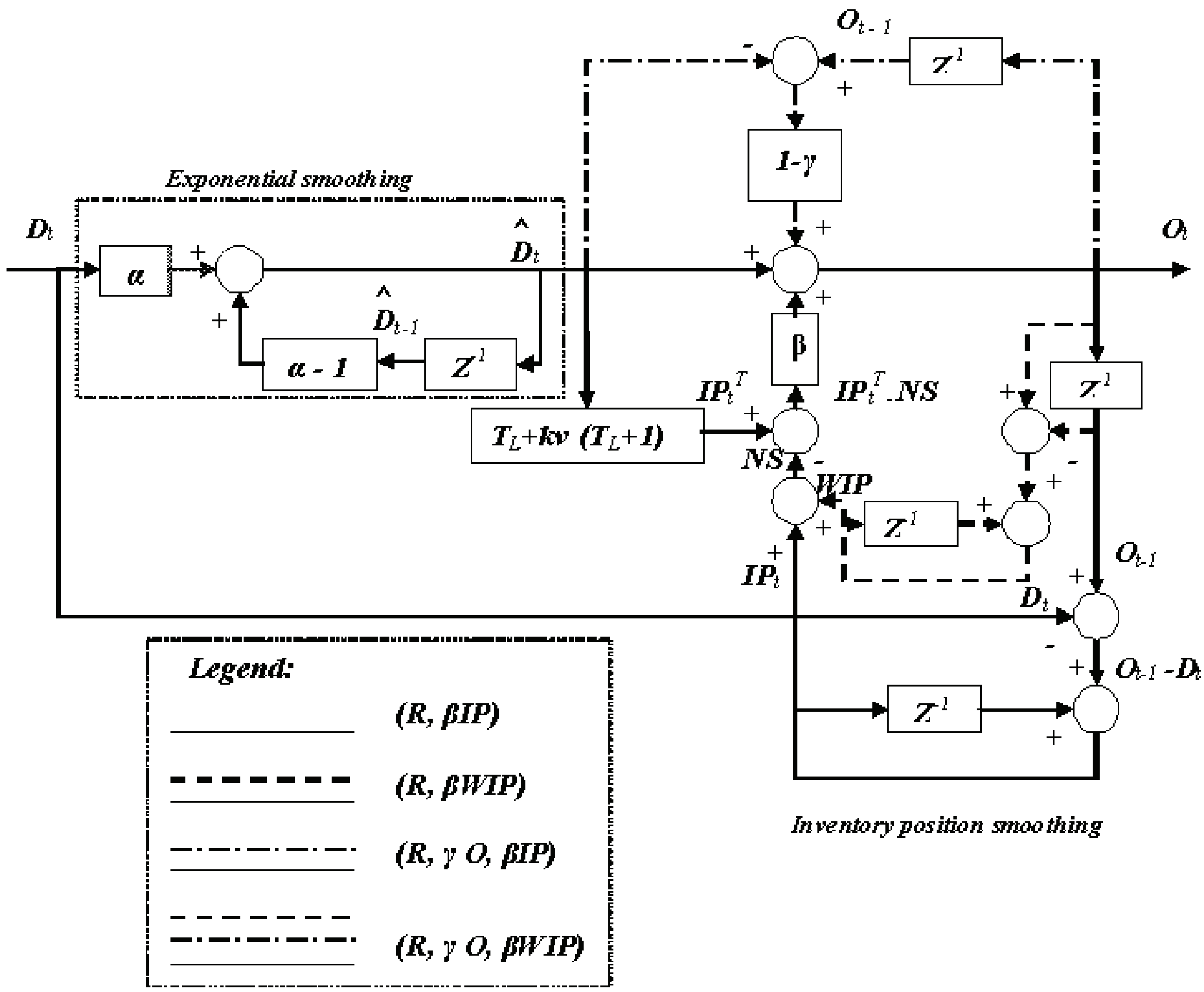
Then the policy is

$$O_t = \hat{D}_t + (1-\gamma)(O_{t-1} - \hat{D}_t) + \beta(IP_t^T - NS_t) \tag{6}$$

This policy is named as (R, γO, βWIP) policy.

Where β is parameter of inventory position smoothing and γ is parameter of order quantity smoothing and NS is net stock.

Figure 1: Block diagram of replenishment policies.



Proposed Policy II:

For the second policy β is 0< β<1 and γ =1 and is named as for (R, βWIP) policy.

The policy is

$$O_t = \hat{D}_t + \beta(IP_t^T - NS_t) \quad (7)$$

3.3 Transfer Function Calculation

We use the z-transform to represent the inventory system in the frequency domain. The z-transform is defined as

$$Z\{x(t)\} = X(z) = \sum_{k=0}^{\infty} x(k).z^{-k} \quad (8)$$

Where z is a complex variable and x(k) is the value of a time series x(.) in period k. More information on discrete-time systems and the z-transform can be found in (Franklin et al., 2002) and (Oppenheim et al., 2005). The frequency domain is an alternative representation to the time domain. It is often analyzed because some important systems properties can be more easily identified in the frequency domain rather than in the time domain. Figure 1 shows the block diagram of the replenishment policies. The representation shows various building blocks:

$Z\{\int_{\tau=0}^{t} x(\tau)d\tau\} = \frac{z}{z-1}X(z)$ is the z-transform of the time-discrete summation, $Z\{x(t-T)\} = z^{-T}X(z)$ is the z-transform of a time delay of T periods, and $\frac{\alpha.z}{z-1+\alpha}X(z)$ is the z-transform of a single-parameter, exponentially smoothed forecast with smoothing factor α applied to the time series x(t).

Jakšic and Rusjan’s policy Set:

Policy I:

Transfer function of the (R, γ O, βIP) policy is:

$$\frac{O(z)}{D(z)}=\frac{((T_L+k\sqrt{T_L+1})\beta+\gamma)(z-1)(\alpha z^2)+\beta z^2(z-(1-\alpha))}{[\beta z+(z-(1-\gamma))(z-1)][z-(1-\alpha)]} \quad (9)$$

Policy II:

Transfer function of the (R, βIP) policy is:

$$\frac{O(z)}{D(z)}=\frac{((T_L+k\sqrt{T_L+1})\beta+1)(z-1)(\alpha z^2)+\beta z^2(z-(1-\alpha))}{[\beta z+(z(z-1)][z-(1-\alpha)]} \quad (10)$$

Viswanadh’s Policy Set

Proposed policy I:

Transfer function of the (R, γ O, βWIP) policy is:

$$\frac{O(z)}{D(z)}=\frac{((T_L+k\sqrt{T_L+1})\beta+\gamma)(z-1)(\alpha z^2)+\beta z^2(z-(1-\alpha))}{[\beta z^2+(z-(1-\gamma))(z-1)][z-(1-\alpha)]} \quad (11)$$

Proposed Policy II:

Transfer function of the (R, γ βWIP) policy is:

$$\frac{O(z)}{D(z)}=\frac{((T_L+k\sqrt{T_L+1})\beta+1)(z-1)(\alpha z^2)+\beta z^2(z-(1-\alpha))}{[\beta z^2+(z(z-1)][z-(1-\alpha)]} \quad (12)$$

Frequency Response Graphs

Frequency response graphs are drawn for α=0.4, γ =0.5, β=0.5, T_L=2 and k=0.5. The figure 2 shows the Frequency Response graph of (R, γ O, βIP) and (R, γ O, βWIP)

Figure 3 shows the Frequency Response graph of (R, βIP) policies and (R, βWIP)

4. DISCUSSION

The frequency response plots for all the policies are drawn on same scale and it is very clear from the plots that new policy set amplification are less than the existing policy set as both new proposed policy’s FR plots are covered under the existing policy set response plots. So, We considered Base as 1 for both policy sets as all the policies amplificion was same at *w*=0 and is 1.

Figure 2: Frequency Response Graph of (R, γ O, βIP) and (R, γ O, βWIP)

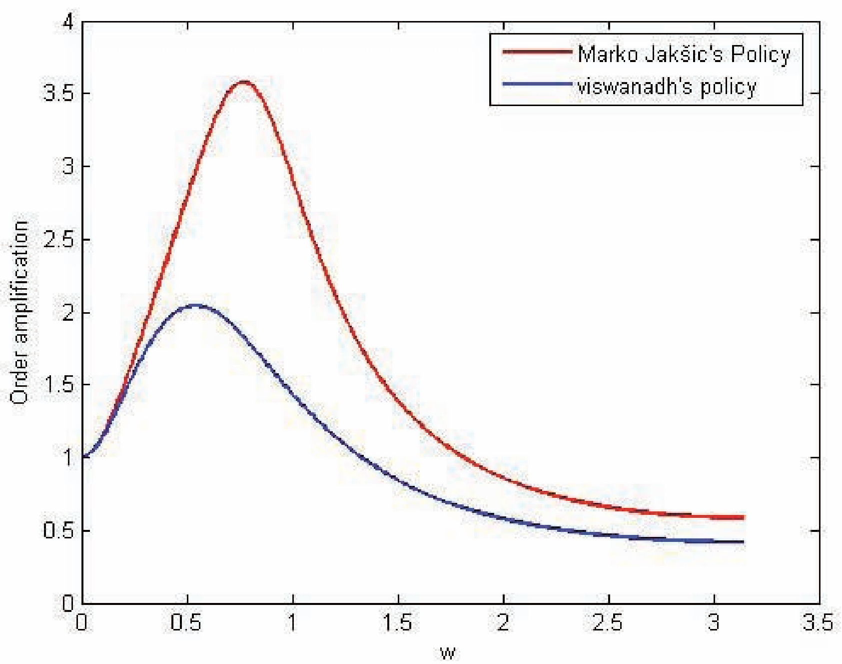


Figure 3: Frequency Response graph of (R, βIP) policies and (R, βWIP)

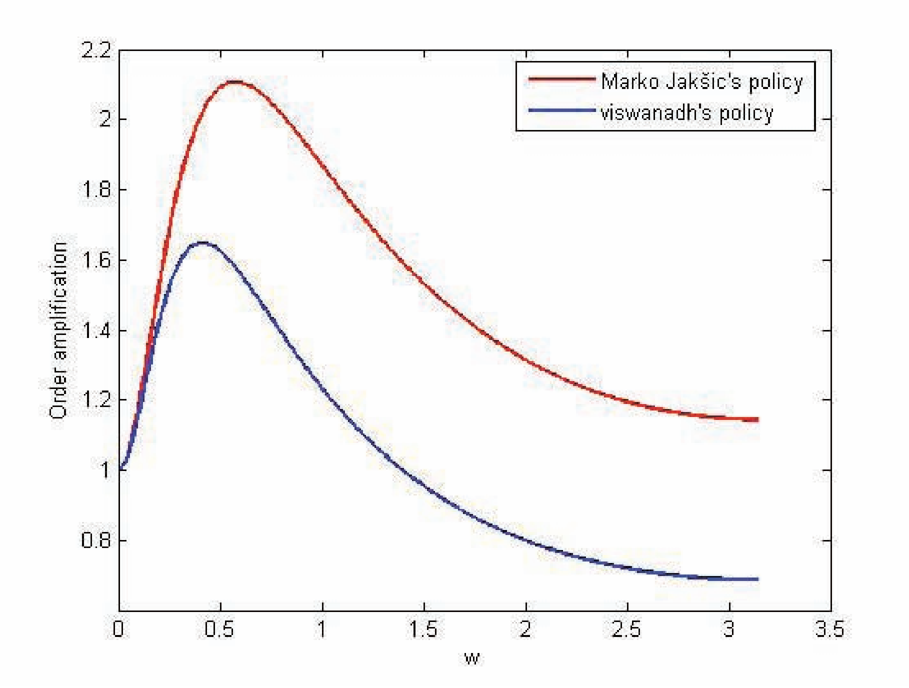


Table 1: Comparison of policies

Policies Comparing	*Policy name*	*Marko Jakšic's policies Amplification*	*Viswanadh's policies Amplification*	*% Improvement#*
Policy I Vs Proposed policy I	(R, γ O, βIP) policy Vs (R, γ O, βWIP) policy	3.6	2.1	15
Policy II Vs Proposed policy II	(R, βIP) policy Vs (R, γ O, βWIP) policy	2.15	1.65	5

Above, table 1 shows the comparison of order amplification of the policies discussed

% improvement is calculated by (Viswanadh's policy amplification – Jakšic and Rusjan's amplification / Base)*100 (here Base = 1 as for all policies the amplification was same at w=0 and is 1.)

From the above table it can be seen that *Policy set I* and *Policy set II* have the order amplifications more than those in *proposed policy set I* and *Proposed Policy set II*. Even in our *(R, γO, βWIP)* policy amplification is less than the (Jakšic and Rusjan, 2008) *(R, βIP)* policy.

5. CONCLUSION

We have considered the impact of work-in-progress inventory in the. supply chain policy planning. The policies, (R, γO, βIP), (R, βIP) , (R, γO, βWIP) and (R, βWIP) are stable. The policies proposed by (Jakšic and Rusjan, 2008) have order amplification of 3.6 and 2.15 respectively. whereas, proposed policies have order amplification of 2.1 and 1.65 respectively. The percentage improvement is 15% and 5% respectively. So we can conclude that Work In Progress (WIP) inventory is effective in increasing the supply chain performance (i.e. reducing the bullwhip effect from the above results). That is, if we consider more information in designing of a policy, it is more effective. So supply chain managers can benefit by keeping an eye on information available such that they can reduce cost incurred in inventory management.

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