The Study of Arc Routing Problem Arising in Small Package Delivery Industry

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Abstract

In the small package shipping industry, companies try to differentiate themselves by providing high levels of customer service. In practice, each service provider is encouraged to follow a master route—a predesigned sequence of street addresses—over an extended planning horizon (more than one day). The objective here is to construct efficient master routes. Currently, a Deterministic Arc-Routing Problem (DARP) model is used to solve the problem. However, this approach ignores the uncertainty in the street segment presence probability—the probability that a street segment requires (i.e., there is a demand for) a visit on a particular day. We have considered a new model, namely, the Probabilistic Arc-Routing Problem (PARP) model which deals with the street segment presence probabilities. PARP attempts to minimize the expected length of the master route. It assumes that the street segment presence probabilities are independent. The limitation of this model is it requires excessive amounts of computation time. Our computational results show that PARP may produce more efficient master routes than DARP by taking demand uncertainty into account.

Keywords: Arc-routing problem, Deterministic arc-routing model, Probabilistic arc-routing problem, Vehicle routing problem

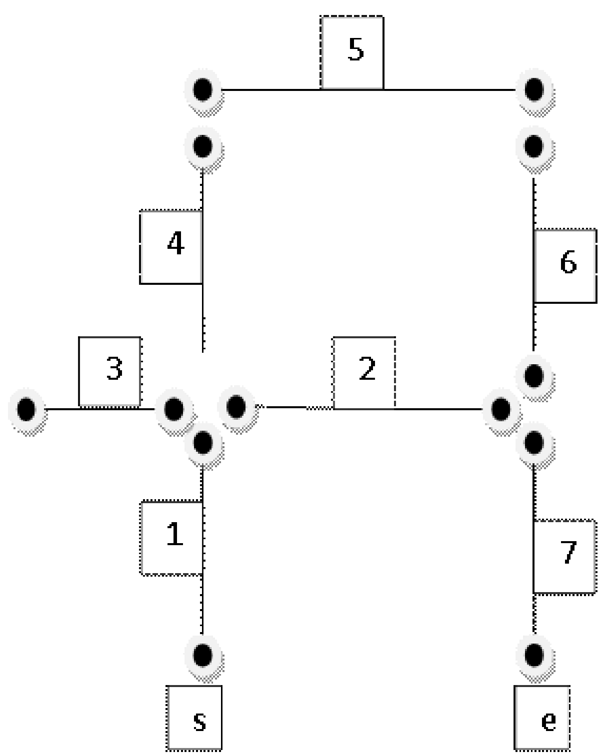
1. Introduction

Small-package shipping firms rely on daily local delivery and pick-up routes to service their customer base. At the operational level, each Service Provider (SP) is responsible for a specific delivery area (e.g., a service provider's delivery area may contain street segments from a single zip code). In practice, an SP is encouraged to follow a master route, which defines a sequence of street segments and the direction in which each street segment is to be traversed for his/her delivery area. Street segments are defined by address ranges. For instance, a street segment may contain building numbers 1 to 100 on Pithampur Street. On the street network, a street segment can be either one way or two ways. On any given day, the exact set of customers to be served along a given street segment may vary. Servicing the customers in the same order each day (according to a master route of the delivery area) has various advantages for the SPs, including gaining familiarity with their service routes and arriving at regular customers at about the same time each day. In addition, this practice improves the efficiency of delivery because packages are loaded into the vehicles in accordance with the master routes. For instance, packages with destinations located on the street segments that appear early in the master route are placed in the front portion of the cargo area where the SP can easily reach them. Our overall objective is to construct efficient master routes for the service areas. The yearly revenues of the major small-package shipping firms in the Pithampur area are quite high, thus underscoring the economic importance of efficient local service routes. The issue of planning daily service where the set of customers may vary each day was first recognized by Jaillet (1985), who proposed the probabilistic travelling salesman problem (PTSP) where each potential customer has a given presence probability on any given day. The problem is to find a master route through all of the nodes that will minimize the total expected (daily) cost of servicing all of the customers. In the context of small-package local operations, the number of possible different street addresses for customer delivery may be really quite large, so the PTSP may not be a practical model. It may be more useful to aggregate the set of possible customers into clusters (Campbell 2006). We propose to partition them into a set of street segments, where each segment has a presence probability (probability of requiring service) on a given day. Also, the available historical service data may only be available in terms of street segments, not individual delivery points whose individual presence probabilities may be quite small and difficult to estimate.

Given an extended planning horizon (more than one day), if the set of street segments requiring service every day remains unchanged, we only need to solve an un--capacitated arc-routing problem once during the entire time horizon. However, in reality, the street segments that need to be visited can vary on a daily basis. Currently, the problem is often approached in a deterministic manner over a single day. More specifically, an arc-routing problem is solved. The resulting master route is used over the entire planning horizon. On a particular day, the route is realized following the predesigned sequence while skipping the street segments that do not require service. We will refer to this approach as the deterministic arc-routing problem (DARP). Essentially, DARP belongs to a family of problems known as the mixed rural postman problem (Laporte 1997). One major problem with this approach, as pointed out by (Jaillet 1985, 1988), stems from the fact that a good solution when all required street segments are present may not remain a good solution when some street segments are skipped.

We use an example in Figure 1 to illustrate this point. In this example, we use separate starting and ending locations. This assumption makes the problem more general. For instance, if a problem requires the vehicle to start and end at the depot, we only need to make the two locations identical. This assumption is also consistent with real-world practice because the service providers are often not required to return to the depot immediately after they finish serving their delivery area. Instead, they may be directed to pick up packages from a neighbouring delivery area before returning to the depot. Given two master routes t1 = (s, 1, 3, 4, 5, 6, 2, 7, e,) and t_2 = (s, 1, 2, 6, 5, 4, 3, 7, e,), where s is the starting location and e the ending location, observe that t1 and t2 have the same deterministic length. Now consider that on a particular day, street segment 2 does not require a service, the delivery route following the sequence specified in t_1 is r_1 = (s, 1, 3, 4, 5, 6, 7, e,) whereas that following t_2 is r_2 = (s, 1, 6, 5, 4, 3, 7, e,). Clearly, r_1 is shorter than r_2.

Figure 1 A Small Example with Seven Street Segments



(Street segment 2 has a small presence probability; s is the starting location, and e is the ending location.)

This uncertainty as to whether a street segment requires service on a particular day suggests that it may be beneficial to study the problem in a probabilistic context. One approach models the problem of finding a suitable master route for an extended planning horizon as a probabilistic arc-routing problem (PARP) where each street segment has a corresponding presence probability on any given day just as in the PTSP. The next section elaborates upon this problem description. In the context of the deterministic, and probabilistic arc-routing models, the remainder of this paper discusses the advantages and disadvantages of using these two different models for finding master routes in small-package delivery service systems where the customer sets vary from day to day.

Section 2, gives precise descriptions of the two arc-routing problems: the DARP, the PARP, and some local search-based solution approach for the PARP. Section 3, addresses suitability of the arc-routing models in small-package delivery service operations. Section 4, describes the implementation and testing of these solution approaches on test problems, some of which are drawn from actual industrial data. Section 5, summarizes our results and analyzes the use of deterministic versus probabilistic arc-routing models in small-package delivery routing practice. Because these master routes are utilized on a daily basis in the small-package shipping industry, any minor improvement in the master route length/cost can translate into significant cost savings over a significant time horizon (e.g., one year).

2. Description Of Arc-Routing Problems

In this section, we describe two arc-routing problems: DARP, PARP; and some local search-based solution approaches for the PARP. All of these arc-routing models have the following common inputs: the starting and ending locations (in case the two locations coincide, it becomes the depot), a set of arcs (street segments), the length of each street segment, the length of the shortest path between an endpoint of any segment to an endpoint of any other segment, and the length of the shortest path to/from the ending/starting location from/to the endpoint of any street segment.

Deterministic Arc-Routing Problem

The DARP has the following description. Given the common inputs, and a set of street segments (arcs) that must be serviced (traversed), find the master route of minimum length, which starts at the starting location, traverses all the arcs, and returns to the ending location. DARP belongs to a well-known class of arc-routing problems known as the mixed rural postman problem (MRPP). Many solution approaches for this class of problems rely on transforming it into traveling salesman problems (Laporte 1997). Comprehensive surveys on MRPP, as well as the arc-routing problem in general, can be found in Eiselt, Gendreau, and Laporte (1995) and Assad and Golden (1995). In addition, Corberan, Mart, and Romero (2000) present an approximate algorithm based on the resolution of some flow and matching problems as well as a tabu search heuristic to solve the MRPP. Note that once we have specified the sequence order for visiting the arcs that must be traversed and the direction in which these arcs are traversed, then it is straightforward to calculate the total route length. After traversing one arc, we always use the shortest path from the end point of the just-traversed arc to the start of the next arc to be traversed. Because these shortest path lengths and the street segment lengths are part of the common input for the arc-routing models, the total route length can be easily computed. For our computational tests, the major small package shipper that we worked with provided a sophisticated state-of-the-art procedure for solving the DARP.

Probabilistic Arc-Routing Problem:

The PARP has the following description. Given common inputs, a set of street segments (arcs) that must be serviced (traversed), and the presence probabilities (probabilities of each segment requiring a visit on a particular day), find the master route of minimum expected length, which starts from the starting location, traverses all the arcs, and returns to the ending location. We now discuss the calculation of the expected length. Our expression is derived from Bertsimas, Jaillet, and Odoni (1990). Without loss of generality, we consider the master route t = (s, 1, 2... n, e), where s is the starting location and e the ending location. Given the presence probability pi (probability that street segment (arc) i requires a visit on a particular day), we define qi = (1 – pi) as the probability that arc i does not require a visit. We use i0 and i1 to represent the entry point and the exit point of i, whose length is represented by l (i0, i1). Also, let d (i_1, j_0) be the shortest path from street segment i to street segment j on the street network. We assume that the street segment presence probabilities are independent. This assumption is based on our analysis of real world industrial data where it is often the case that no prevalent correlations among deliveries are found. The expected length of t can be computed with the following expression:

$$E[L(\tau)] = \sum_{i=1}^{n} d(s, i_0) p_i \prod_{k=1}^{i-1} q_k \quad (1)$$

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} d(i_1, j_0) p_i p_j \prod_{k=i+1}^{j-1} q_k \quad (2)$$

$$+ \sum_{i=1}^{n} [d(i_1, e]) p_i \prod_{k=i+1}^{n} q_k \quad (3)$$

$$+ \sum_{i=1}^{n} l(i_0, i_1) p_i \quad (4)$$

$$d(s, e) \prod_{k=1}^{n} q_k \quad (5)$$

The first component of the equation is the expected cost of using the path from the starting location to a street segment i, whereas the third component is that of using the path from a street segment i back to the ending location. The second component is the expected cost associated with using the path between street segments i and j. The fourth component is the expected cost of using street segment i. The last component is the cost of traveling from s to e if no arcs between them are realized. The expected cost of the path is based on the probability that the street segments at both ends of the path are realized, the probability that none of the street segments in between are realized, and the length of the path (i.e., the distance from the exit point of the starting segment to the entry point of the ending segment). The expected cost of using a street segment depends on the probability of it being realized (presence probability) and its distance (length from the entry point to the exit point of the street segment. Although the uncertainty in the street segment presence probability is an important issue, it seems to have been largely ignored by the academic literature on arc routing problems. In our literature search, we have come across only two papers, by Mohan, Gendreau, and Rousseau (2007, 2008), that discuss the issue of uncertainty in presence probability in the context of an Eulerian tour problem.

Example of PARP Characteristics

The description of the PARP is straightforward, but understanding the difference between a deterministic arc-covering problem and the PARP is more subtle. It is sometimes difficult to see how the probability that an arc requires services on a particular day can affect the configuration of the optimal route. In order to illustrate some of these concepts, consider the following example. Let $i_1, i_2, i_3, \cdots, i_n$ and $j_1, j_2, j_3, \cdots, j_n$ be the 2n nodes in the problem. The sets of arcs are:

1. $(i_\downarrow k, i_\downarrow(k+1))$ k = 1. (*n* -1)
2. (i_n, i_1)
3. $(j_\downarrow k, j_\downarrow(k+1))$ k = 1... (n-1)
4. (j_n, j_1)
5. (i_k, j_k) k = 1... (n-1)
6. $(j_\downarrow k, i_\downarrow(k-1))$ k = 1... (n-1)
7. (j_1, i_n)

All arcs lengths are one. In addition,

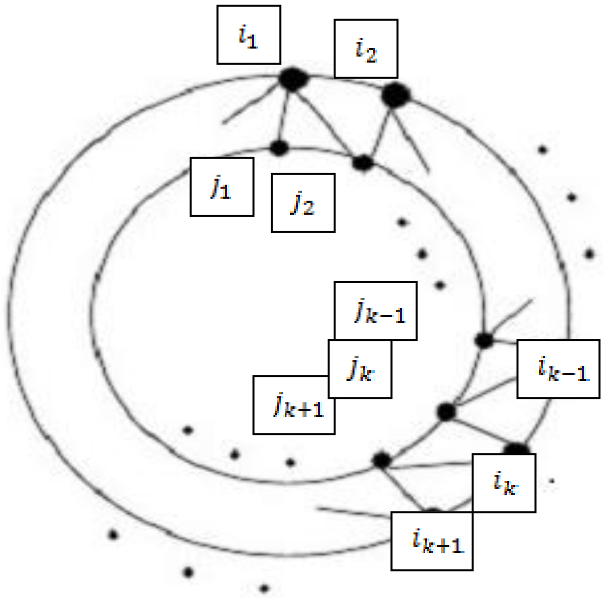
1. Arc sets 1 and 2: $(i_k, i_{k+1})\ k = 1, \dots, (n-1)$ and (i_n, i_1); all arcs in the outer ring must be traversed with presence probability one;
2. Arc sets 3 and 4: $(j_k, j_{k+1})\ k = 1, \dots, (n-1)$ and (j_n, j_1) ; all arcs in the inner ring must be traversed with presence probability p;
3. All other arcs do not need to be traversed. See Figure 2 for a depiction of the example.

Probabilistic Local Search Procedure

For the PARP, we used a solution procedure that adapted local search approaches to our probabilistic context. Our solution heuristic incorporates the presence probabilities using two local search procedures, namely, 1-p-Shift and 2-p-Opt. They act as local improvement techniques for the current solution method, which is essentially an efficient TSP heuristic based on the Lin-Kernighan (Helsgaun 2000) algorithm. We now describe 1-p-Shift and 2-p-Opt in the context of arc routing. Be rtsimas and Howell

(1993) provide a clear description of 1-p-Shift and 2-p-Opt, which are designed primarily for the PTSP problem. In the PARP, we consider street segments (arcs) instead of nodes.

Figure: 2



Accordingly, the local search needs to be adjusted. In particular, we need to decide on the directions (i.e., from which end point to enter) of the street segments. Consider the starting route given in Figure 3, where the dashed arcs represent the arcs that must be covered and arc h has nodes h0 and h1 as its end points. The solid directed arcs between a pair of dashed arcs represent the path taken between the two dashed arcs. For the sake of simplicity, assume the route starts and ends at the same location: the depot (node 0). The two dotted arcs represent the path from the depot to an arc that must be traversed and the path from an arc that must be traversed to the depot. Note that once an ordering of all the arcs that must be traversed and the direction in which each such (dashed) arc is traversed is obtained, the route is completely specified. That is, the remaining arcs (the solid and dotted arcs) are completely specified. Figures 4 and 5 illustrate two possible options to implement the 1-p-Shift and involve moving arc i from its position on the starting route into the position between arc j and arc (j +1). Figure 5 represents the situation when arc i is traversed in the same direction as it was in the starting route of Figure 3. Figure 5 represents the situation when arc i is traversed in the opposite direction as it was in the starting route. The 2-p-Opt for PARP is more straightforward and similar to the classical 2-Opt procedure where two arcs are removed and two new arcs are inserted in order to form a new route. Consider its implementation on arcs $(i – 1)$, i, j, and $(j + 1)$. We simply remove the two arcs connecting arcs $(i −1)$ and i and arcs j and $(j + 1)$, reverse the directions of all the arcs in between $(i – 1)$ and $(j + 1)$, and insert two new arcs as illustrated in Figure 6.

Note that for the special case of $i = j$, the 2-*p*-Opt is equivalent to reversing the direction in which arc i traversed. Each time a 1-*p*-Shift or 2-*p*-Opt movement is proposed, the algorithm calculates the change in the expected length. If there is an improvement in the expected length of the solution, then the movement is accepted; otherwise, the movement is rejected.

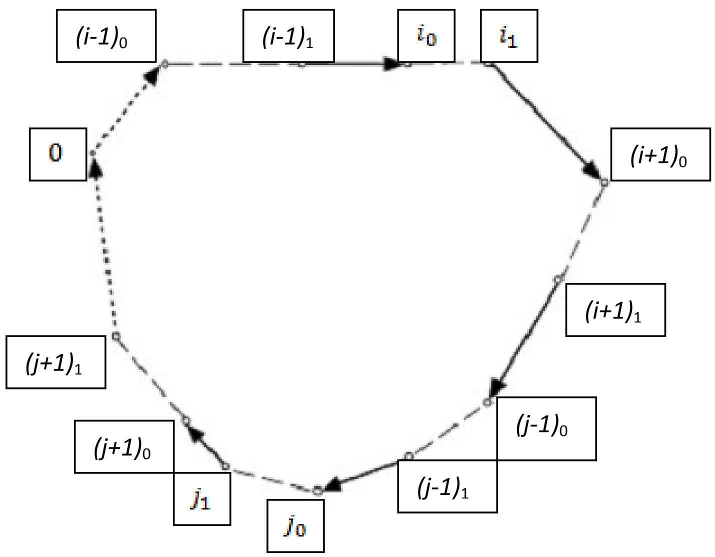
Solution Procedure.

- Input: a list of street segments (arcs) that requires service, presence probabilities for each street segment on the list, an initial master route stating how to traverse the list of street segments, and an iteration limit. Evaluate the expected length of the initial master route. Initialize the total number of iterations to zero.
- Apply a probabilistic 1-Opt or 2-Opt improvement technique to the current master route. Evaluate the expected length of the new master route. If the expected length of the new master route is less than

that of the current master route, the new master route becomes the current master route;

- Increase the total number of iterations by one. If the total number of iterations is less than the iteration limit, go back to Step 1. Otherwise, Stop. Each proposed solution change of Step 1 requires that the expected cost of the new route must be calculated. We identified various techniques to reuse the computation of the expected value for the current solution in order to reduce the computation time. However, in our implementation of the probabilistic local search, we did not utilize such computational reduction techniques. Even with these techniques, the computational burden of these expected value computations is quite substantial. For the PTSP, various researchers (for example, see the survey by Campbell and Thomas 2007) have noted the substantial burden in computing expected value computations for large-scale problems greater than about 100 nodes even with some proposed possible remedies.

Figure 3 The original Tour



3. Suitability Of The Arc-Routing Models In Small-Package Delivery Industries:

As mentioned earlier, arc-routing models are well suited to model the large number of customer locations (about 1,000 on average for the five service routes used in our computational experiments) that can require service over a significant planning horizon (e.g., for a small-package local routing system over a 30-day planning horizon). Instead of having a very large number of customer nodes in the PTSP, we represent the customer locations as a moderate number of street segments to be covered. Finding the optimal solution to a PTSP with hundreds or thousands of nodes is rather challenging with currently available solution techniques (see Campbell and Thomas 2007). In our computational results for the arc-routing models, we will see that one of our suggested solution procedures can effectively handle the arc-routing model derived from a 30-day model. In addition, it may be much easier to estimate presence probabilities for street segments instead of individual addresses whose individual service frequencies may be quite small, even when the corresponding street segment service frequency is relatively large. Another issue is the suitability of using arc- routing models for small-package local service operations. Suppose that an arc representing 1 to 100 Maple Street must be traversed. Assume that there are actually three customer locations, at 1, 38, and 70 Maple Street that must be serviced. The description of all the arc-routing models states that the entire arc or street segment must be traversed. In reality, the three customer locations on Maple Street will be covered if the sub-segment from 1–70 Maple Street is traversed. Thus, the arc-routing model may overestimate the mileage and route that must be used to cover the customer locations. However, this overestimation should not affect our analysis in determining the relative suitability of the arc-routing models in evaluating and identifying the preferred master route. If we wish to use an arc-routing model in an operational context for a small-package shipping firm, it is probably necessary to relax the constraint that the entire street segment must always be covered in the arc-routing model. Another issue is that some customer locations or their corresponding street segments may have implicit time windows associated with them.

Figure 4 Arc Shift i position to j position: Direction of i remains unchanged

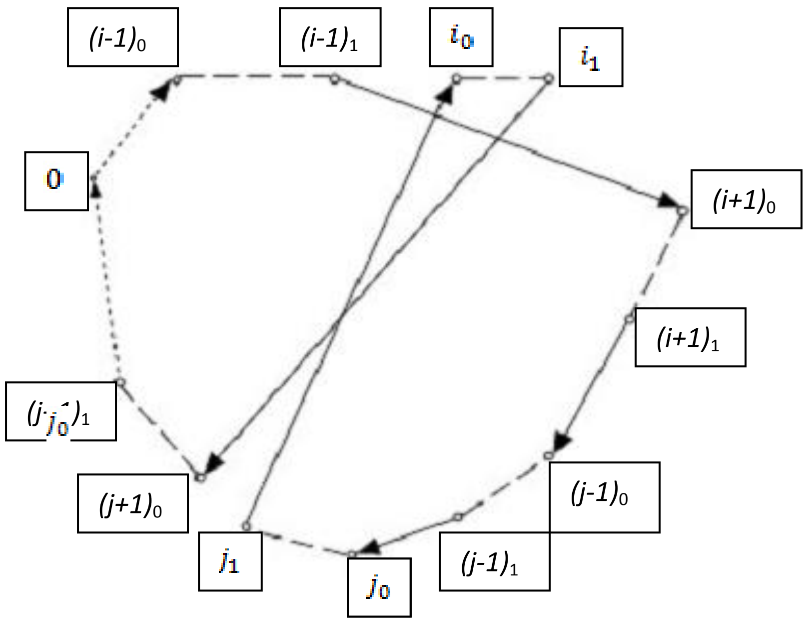
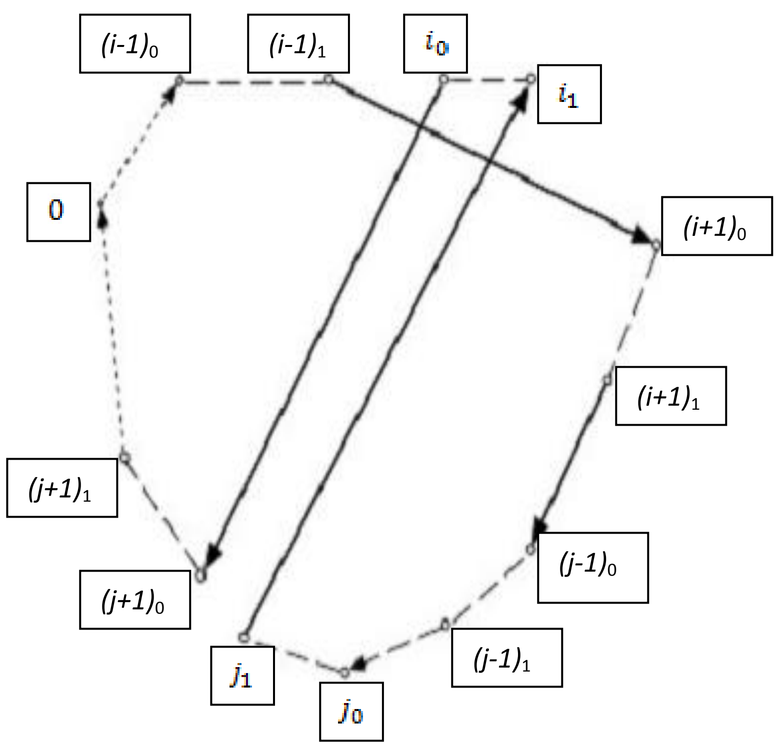
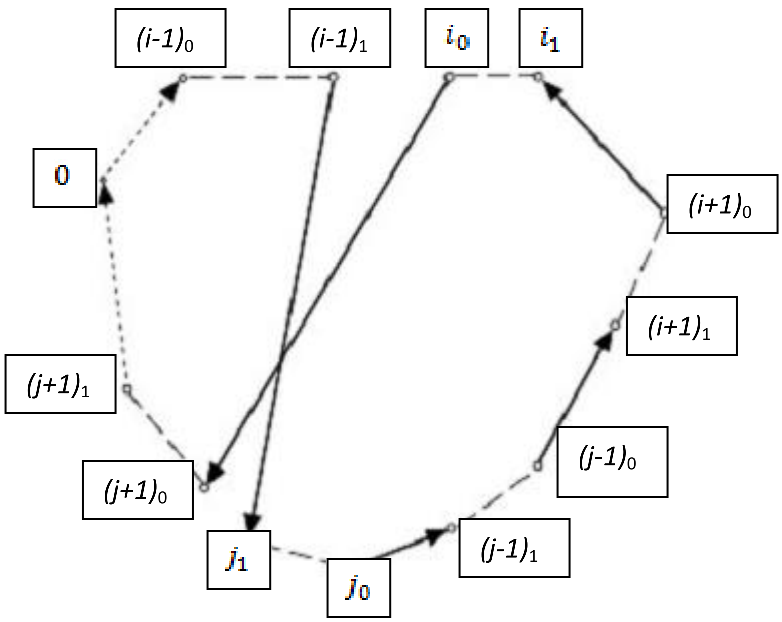


Figure 5 Arc Shift i position to j position: Direction of i is Switched



Customer locations may correspond to commercial stores whose operations have evolved to expect their small-package deliveries at around the same time each day. The most common type of time window is when the customer deliveries are expected to be before lunchtime (say 1:00 p.m.). We point out that our approach can be easily generalized to address this issue by partitioning the set of arcs to be covered into two subsets, the arcs corresponding to customer locations that normally expect to be serviced before lunch or after lunch. Then we can solve two separate routing problems (corresponding to the two subsets) and then combine the two routes into a solution for the entire arc-routing problem (we assume that after lunch the vehicle picks up from where it was before lunch). Note that in order to utilize such a partitioning strategy, a great deal of information is required about the customer service history, or expected delivery time.

Figure 6 An Example of 2-p-opt: Reverse the direction of Arcs between i-1 to j+1



4. Computational Results

We implemented the solution procedures for the PARP described in the previous section in VC++ 6.0 and tested them on a computer with Pentium IV 2 GHz and 1.24 GB RAM. As discussed previously, the major small-package shipping firm that we worked with supplied a sophisticated state-of- the-art procedure for obtaining solutions to the DARP. These solutions are also used as the initial solutions for PARP. This section discusses the results of the computational tests with these solution procedures and their implications concerning the utility of these two arc-routing models for small-package shipping firm local operations. The next subsection describes two sets of test problems— one is drawn from actual industrial data provided by the major small-package shipping firm, whereas the other is computer generated—used for evaluating the performance of these two solution techniques. The second subsection describes the results of the computational tests and some implications of these results.

Generating Test Problems:

We first describe the set of test problems drawn from actual industrial data. We collected data on five local service routes used by a major small-package shipping firm. Each local service route encompasses both commercial and residential areas and required a single service provider to handle the daily work. The service routes were located in three different states. The major small-package shipping firm provided the street segments and their lengths, as well as the shortest path length between any two end points of any two street segments, and between the starting/ ending location and the end point of any street segment. All of these lengths are derived from the underlying street network of the service territory associated with each route. For each local service route, we also collected daily customer demand data over a historical study interval consisting of 20–30 days. In other words, we solve for a daily master route based on 20–30 observed (historical) realizations. Table 1 gives some summary statistics for these service routes.

Table 1 (Service Routes)

Routes	Number of days in historical study interval	Number of unique street segments served during historical study interval
Service Route 1	30	230
Service Route 2	30	228
Service Route 3	20	226
Service Route 4	30	179
Service Route 5	30	160

For each local service route, we derived a corresponding test problem, referred to as Service Routes 1 to 5. The list of street segments that must be traversed corresponded to the list of unique street segments serviced during the historical study interval. The presence probability for a street segment is the ratio of the number of days during the historical study interval when the street segment had at least one customer demand to the number of days in the historical study interval. For the number of days and the series of street segment sets corresponding to the set of street segments that must be traversed during each day, we used the number of days in the historical study interval and the daily set of streets segments that must be traversed. Next, we create the set of computer-generated test problems, referred to as Problem Sets 1 to 5. Each problem has 200 street segments (including the starting and ending locations) and a 30-day study interval. The street segments are randomly placed on a 50 × 50 square grid. The coordinates for the starting and ending locations are (24.5, 0) and (25.5, 0), respectively. Euclidean distances are used as the lengths of the street segments as well as the shortest path length between any two end points of any two street segments and between the starting/ending location and the end point of any street segment. Each street segment presence probability is randomly selected from a uniform distribution on the interval (0, 1). The daily realized street segment data is generated according to the presence probabilities. For example, if the presence probability for a street segment is 0.5, then we randomly select 15 (=30×0.5) days and create a service request for this segment on each of these 15 days.

Empirical Evaluation of Master Routes:

We used the 5 test problems described in the previous subsection to perform various types of evaluations and comparisons. For each test problem, we obtained two master routes (one master route solution obtained by solving each of the two arc-routing models). We obtained the solutions by using the solution procedures described in the previous section. The computation times for the probabilistic local search (PARP) solution procedure are given in Tables 2. Then we performed two types of evaluations using these two arc-routing model solutions. First, we evaluated the two solutions using the total route-length criteria of the DARP. Note that, due to proprietary considerations, we use a normalized cost instead of real mileage. As expected, the DARP master route was the best in terms of the total route length criteria. See Table 3 for this evaluation. Next, we evaluated the two solutions using the expected length criteria of the PARP. Table 4 shows that the PARP solutions are better than the DARP solution using expected length criteria. These two evaluations show that using the standard total route length (DARP) criteria can be misleading in terms of evaluating master routes.

Table 2 (Computing time for PARP solution procedure)

Routes	Problem Sets	
	Actual Industrial Data(CPU time in Secs.)	Computer- Generated Data (CPU time in Secs.)
Service Route 1	12,553	5,978
Service Route 2	11,918	5,699
Service Route 3	11,825	6,057
Service Route 4	7,852	5,325
Service Route 5	4,761	6,010

The DARP solution is about 10% better in terms of the standard total length criteria than the other solution. However, in terms of the expected route-length (PARP) criteria, the DARP solution is generally about 2% to 5% worse than the PARP solutions. These results confirm that using the standard deterministic single-period criteria of total route length is not a good way to evaluate the master routes because it does not take into account the daily variation in customer demands.

Table 3 (Master Route Quality Using DARP objective Function)

Routes	Actual Industrial Data		Computer – Generated Data	
	DARP Solution (Normalized Cost)	PARP Solution (Normalized Cost)	DARP Solution (Normalized Cost)	PARP Solution (Normalized Cost)
Service Route 1	1.000	1.1305	15.8673	17.0117
Service Route 2	1.000	1.1873	16.9527	20.0605
Service Route 3	1.000	1.1651	17.3456	19.0176
Service Route 4	1.000	1.0956	16.2723	19.0053
Service Route 5	1.000	1.1335	18.0013	20.1252

The probabilistic models expected route-length criteria are much better at accounting for the daily variation in customer demands. The results of Tables 2 to 4 indicate that both criteria are comparable in evaluating the quality of master routes. For realistically sized problems, the PARP and its solution procedure may require excessive amounts of computation time.

Table 4 (Master Route Quality Using PARP objective Function)

Routes	Actual Industrial Data		Computer – Generated Data	
	DARP Solution (Normalized Length)	PARP Solution (Normalized Length)	DARP Solution (Normalized Length)	PARP Solution (Normalized Length)
Service Route 1	1.000	0.9753	11.4362	10.9843
Service Route 2	1.000	0.9621	11.7853	11.1463
Service Route 3	1.000	0.9883	11.5546	10.8752
Service Route 4	1.000	0.9785	11.3894	10.6723
Service Route 5	1.000	0.9842	11.9364	11.4627

5. Conclusion

We have considered the local routing problem for small-package shipping where customer demands vary daily. In this context, node-routing problems such as the PTSP may not be appropriate set of customers served over a multiday time horizon may be quite large. Instead, arc-routing problems where a set of arcs instead of nodes must be traversed offer more tractable decision models. We introduced three types of arc-routing problems and described heuristic solution approaches for two of them. Our computational results with test problems based on actual small-package shipping firm data as well as on computer-generated data confirm that the standard deterministic single-period arc-covering model does not produce the most desirable types of master routes. The multiday and the probabilistic arc-routing problems produce better master routes by taking into account daily customer demand variation via multi-time-period or probabilistic models. Our computational results also show that the multiday model (with a moderate number of days) is much simpler to solve than the probabilistic model because it avoids the burden of expected length calculations required by the probabilistic model. because the In the context of small-package shipping firm planning operations, the deterministic arc-routing problem (DARP) is convenient to use in obtaining a master route because it requires only a limited set of input parameters: the starting and ending locations, a set of arcs (street segments), the length of each street segment, the length of the shortest path between an end point of any segment to an end point of any other segment, and the length of the shortest path from/to the starting/ending location to/from the end point of any street segment, and is a relatively simple model. However, our results indicate that noticeable (ranges from 2% to 5%) improvements can be obtained in master route quality by using a somewhat more complex model such as a multiday or probabilistic arc-routing problem that takes into account the daily variation in customer demand instead of the simpler deterministic and single-period arc-routing problem. As we have discussed previously, because small-package local service operations occur on all weekdays throughout the entire state, such improvements could represent substantial economic benefits in the cost of small package local service operations. This Monte Carlo sampling derived sample average approximation problem approach to solving the PARP can also be utilized for other type of problems with an expected value objective function, such as the probabilistic traveling salesman problem. We intend to pursue this possible new approach to solving the PTSP and related models as an area of future research. We intend to analyze this new approach and determine the number of days required to be a reasonable approximation to the probabilistic model (i.e., PARP) in terms of the master route produced.

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