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Using Nonlinear Kalman Filter to Estimate the State of Nonlinear Semi-Active Suspension System

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Abstract

A nonlinear method is used to estimate the state of the nonlinear semi-active suspension system. To estimate the state of the nonlinear semi-active suspension system, a nonlinear method is required. In this study, two nonlinear estimators including the Extended Kalman Filter (EKF) and the Unscented Kalman Filter (UKF) are used. EKF uses first order Taylor expansion while the UKF performs stochastic linearization to approximate the nonlinear system. A comparison between true value and state estimation of nonlinear semi-active suspension system based on EKF and UKF have been done and by the aid of these estimations, Sky – Hook controller and output feedback PD controller are designed. Simulations show the effectiveness of using two nonlinear Kalman filters in estimating the state of a nonlinear suspension system.

Keywords: Sky-Hook, Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), State Estimation, Suspension Model, Output feedback PD Controller

1. Introduction

Keeping the tires on the road surface to ensure the stability of vehicle, vehicle handling and isolation the vehicle frame from road roughness, road isolation, are two main functions of vehicle suspension system. The main concerns in the field of suspension system are to achieve improvements in these two tasks. Computer-controlled suspensions try to reach, a trade-off between riding comfort and vehicle handling at a small cost penalty [1]. Three different vehicle suspension systems are defined: passive, semi-active and active. A spring and damper forms the passive suspension system. The indexes of these two elements are constant in the passive suspension system. Road disturbances are absorbed by these elements in order to minimize the driver position variation and hence increasing ride comfort [2]. Passive suspension have good performance only in a limited range of operating conditions and the primary purpose of adopting the active and semi – active suspension system is to broaden this optimal range. In the last two decades many different active and semi – active suspensions have been used. The Sky – Hook control strategy is widely accepted in order to improve the vehicle comfort [3], [4], [5].

The basic idea of this strategy is based on a fictitious damper which is connect the vehicle frame and a stationary sky and since this configuration is impossible to reach in reality, in action this strategy is typically approximated using a two-state control strategy [6], [7] or continuously variable shock absorbers [5], [8], [9], [10]. The state must be calculated exactly and in real time for efficient control.

The development of Kalman filter was in order to estimate the parameters or states in linear system. It has many different applications such as Global Positioning System (GPS), object tracking, etc. [11]. But almost all of the real systems are nonlinear, so the extended Kalman filter (EKF) was proposed to address the problem. EKF uses first order Taylor expansion. It computes the Jacobian matrix of the model and because of this, bad estimation would be gained In the case of severely nonlinear systems. In order to address this problem I the systems with severe nonlinearity the unscented transform had been applied by Julier and Uhlmann, which is stochastic linearization

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through the use of weighted statistical linear regression process. The unscented Kalman filter (UKF) principle generates a series of sigma points to approximate the nonlinear system using unscented transform [12]. Since UKF doesn't calculate the Jacobian matrix, it would perform better when the nonlinearity of the system increases. In this study the nonlinear suspension model is given in section 2. Extended Kalman Filter and the Unscented Kalman Filter are section 3, 4 respectively. Sky-Hook controller and Output feedback PD controller is treated in section 5. Results of simulation are shown in section 6. The conclusion will be added at the end.

2. Nonlinear Suspension Model

Analyzing the fundamental concepts of vehicle dynamics can be based on the simplified vehicle model, since in many analyses the vehicle acts as a unit and there is no need to consider the numerous components separately. Turning maneuver is an instance for such an analysis. The other instance is quarter car model as a simple nonlinear model of a suspension system. A quarter car semi-active suspension model is shown in figure1 and its dynamics can be represented by:

$$m_s Z_s = -f_k - f_b \qquad (1)$$

$$m_u Z_u = f_k - f_b - k_t\ Z_u - Z_r \qquad (2)$$

In which Zs stand for sprung mass absolute displacement, Zu, unsprung mass absolute displacement, Zr, indicate road disturbance, kt, tire stiffness, f_k , the spring force and f_b is the damper force.

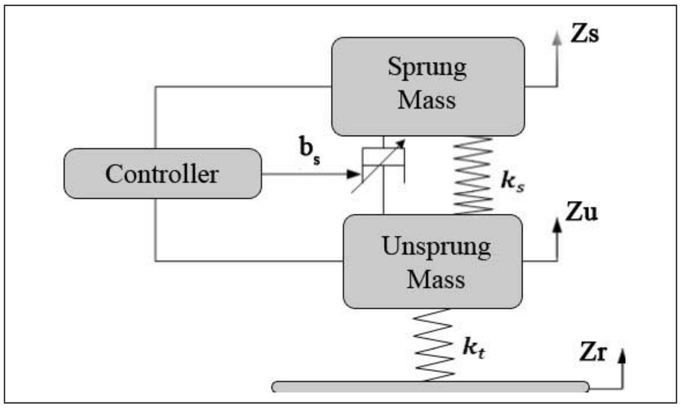
Ando Y [13] suggested a method to bring the nonlinearity to fk and f_b. And as a result to equations in the case of large deflection or some added noise, the nonlinearity should be considered in the equations since the damper and spring demonstrate nonlinear behavior. In the following study, the method which is suggested by Ando Y [13] is used to consider the nonlinear form of f_k and f_b.

$$f_k = k_s\ Z_s - Z_u\ + 0.3k_s(Z_s - Z_u)^3 \qquad (3)$$

$$f_b = b_s\ Z_s - Z_u\ + 0.2b_s\ Z_s - Z_u\ ^2 sgn\ Z_s - Z_u \qquad (4)$$

In which the suspension stiffness and damping coefficient are shown by k_s and b_s respectively. The damping coefficient as the manipulated variable of the control system is time varying but in semi-active suspension is also bounded by physical damper constraints,

Figure 1. Semi-Active Quarter Car Suspension Model



3. Extended Kalman Filter

Since the Kalman filter is useless for estimating a state or parameter in a nonlinear system, extended Kalman filter would approximate the nonlinear system by using the Taylor expansion. The simple nonlinear model system and measurement equations are:

$$x_k = f(x_k) + w_k \qquad (5)$$

$$z_k = h(x_k + 1) + v_k \qquad (6)$$

In which w_k and v_k stand for processing noise and measurement noise respectively. These noises are white and independent of each other. The function *f ()* would use the previous estimated value to predict the state and the other function *h ()*, predict measurement from predicted state in previous step. The nonlinearity of the system prevents from applying the f and h to covariance directly, so the first order of Taylor expansion is used

The algorithm of EKF is shown step by step.

A. Initialization, t = 0;

The filter is initialized by:

$$x_0^+ = E\ x_0$$

$$P_0^+ = E\ (x_0 - x_0)(x_0 - x_0)^T \qquad (7)$$

B. Linearization and Prediction

The time update of the state estimation and estimation error covariance performs as:

$$A_{k-1} = \frac{\partial f_{k-1}}{\partial x}|_{x_{k-1}^+} \qquad (8\text{-}1)$$

$$L_{k-1} = \frac{\partial f_{k-1}}{\partial w}|_{x_{k-1}^+} \qquad (8.2)$$

$$H_k = \frac{\partial h_k}{\partial x}|_{x_k^-} \quad (8.3)$$

$$M_{k-1} = \frac{\partial h_k}{\partial v}|_{x_k^-} \quad (8.4)$$

$$x_k^- = f_{k-1}(x_{k-1}^+) \quad (9)$$

$$P_k^- = A_{k-1}P^+{}_{k-1}A_{k-1}{}^T + L_{k-1}Q_{k-1}L_{k-1}{}^T \quad (10)$$

In which the process noise covariance matrix is shown by Q.

C. Calculate Kalman Gain

Kalman gain would be computed by calculating partial derivatives (8-3), (8-4) and (10) as:

$$K_k = P_k^- H_k{}^T (H_k P_k^- H_k{}^T + M_k R_k M_k{}^T)^{-1} \quad (11)$$

In which R is measurement noise covariance matrix.

D. Update

The measurement update of the state estimation and estimation error covariance perform as:

$$x^+{}_k = x_k^- + K_k(z_k - h(x_k^-)) \quad (12)$$

$$P^+{}_k = P_k^- - K_k H_k P_k^- \quad (13)$$

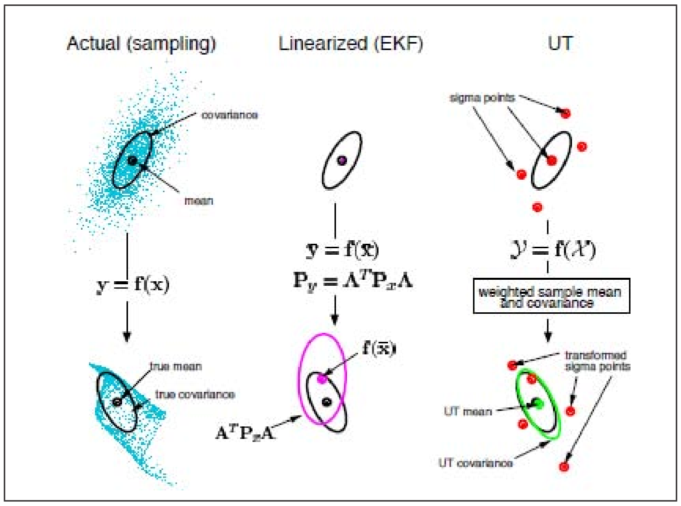
In which z_k and x^+_k are measurement input and output of the algorithm respectively. x^-_k and P_k would update recursively[14].

Taylor expansion does not match well with nonlinear system in case of severely nonlinear or complex system. In such cases Unscented Kalman Filter (UKF) which is based on a stochastic linearization by using a weighted linear regression process.

4. Unscented Kalman Filter

As it mentioned previously the UKF would be applied when the system exhibit sever nonlinear behavior. UKF doesn't process the nonlinearity with linearization; it means that it doesn't calculate the transition matrix A or H, using linearization, which is the advantage of this filter. UKF uses 2n+1 points which are called sigma points and depend on user parameter k, to calculate mean and covariance in a system of n dimension. The simple example for a 2-dimensional system is shown in Figure 2 in which the true mean and true covariance propagation using Monte-Carlo sampling is shown in the left plots, the results using Unscented Transform is the center one. EKF uses Jacobian to get the results while UT get a close result to true function by using only 5 sigma points [14].

Figure 2. A Simple Example of 2-Dimensional System. Actual, EKF and UT [14]



Assume the simple nonlinear model in (5), (6). The UKF algorithm would be applied as follows [15].

E. Initialization, t = 0;

The UKF initialize as follows:

$$x^+ 0 = E\, x_0 \quad (14)$$

$$P^+ 0 = E\,(x_0 - x_0)\,(x_0 - x_0)^T$$

F. Calculate the Sigma Point and Weights

$$(x_i, w_i)\ \ (x_{k-1}, P_{k-1,k}) \quad (15)$$

Assume , then the sigma points and weights would be obtained in following steps:

$$\mathcal{X}_1 = x \quad (16)$$

$$\mathcal{X}_{i+1} = x + u_i \qquad i = 1,2,3,\dots,n$$

$$\mathcal{X}_{i+n+1} = x - u_i \qquad i = 1,2,3,\dots,n$$

$$\mathcal{W}_0 = \frac{n}{n+k} \quad (17)$$

$$\mathcal{W}_i = \frac{k}{2(n+k)} \qquad i = 1,2,3,\dots,n$$

$$\mathcal{W}_{i+n} = \frac{k}{2(n+k)} \qquad i = 1,2,3,\dots,n$$

The is the row vector of U which satisfy. Matrix $U^T U = n + k\, P_x$ could be obtained by using Cholesky decomposition. k is a constant value that usually satisfies. $n + k = 3.2n$

+ 1 sigma points and its weights are generated using the unscented transformation.

G. Prediction

$$x_k^-, P_k^- = UT(f(\mathcal{X}_i), \mathcal{W}_i, Q) \quad (18)$$

$$z_k^-, P_k^- = UT(h(\mathcal{X}_i), \mathcal{W}_i, Q) \quad (19)$$

In which UT is a function of unscented transformation. The process of this transformation is shown below:

$$x_k^- = \sum_{i=1}^{2n+1} \mathcal{W}_i f(\mathcal{X}_i) \quad (20)$$

The predicted measurement at time *k:*

$$z_k = \sum_{i=1}^{2n+1} \mathcal{W}_i h(\mathcal{X}_i) \quad (21)$$

In order to take the measurement noise in the estimation of the covariance of the predicted measurement, R should be added to the equation:

$$P_k = \sum_{i=1}^{2n+1} \mathcal{W}_i \left(f(\mathcal{X}_i) - z_k^-\right)\left(f(\mathcal{X}_i) - z_k^-\right)^T + R \quad (22)$$

H. Update

The cross covariance between state, and measurement, is estimated by:

$$P_{xz} = \sum_{i=1}^{2n+1} \mathcal{W}_i \left(f(\mathcal{X}_i) - x_k^-\right)\left(h(\mathcal{X}_i) - z_k\right)^T \quad (23)$$

Following normal Kalman filter equations are used to perform the measurement update of the state estimation:

$$K_k = P_{xz} P_z^- \quad (24)$$

$$x^+_k = x_k^- + K_k (z_k - z_k) \quad (25)$$

$$P^+_k = P_k^- - K_k P_z K_k^T \quad (26)$$

K_k is Kalman gain, z_k is the measurement value that is input to the algorithm and is the output value. To make algorithm recursively P_k and will be used in next generation input.

5. Controller Design

Sky – hook policy and output feedback PD controller are used as follows:

I. Ideal Sky-Hook Model

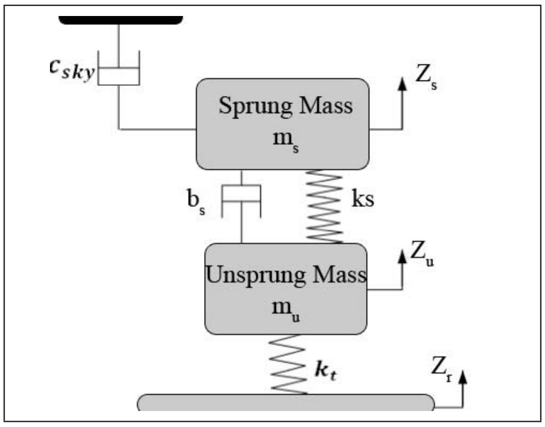
Sky-hook policy was first introduced by Karnopp [3] in which a virtual damper is placed between the frame of the vehicle and a virtual inertial reference frame. This reference frame is attached to the sprung mass as a way to hold up the vibratory motion of the vehicle frame. This attachment shows schematically in Figure. 3.

The fourth–order Sky-hook for the nonlinear state equations are:

$$m_s Z_s = -f_k - f_b - c_{sky} Z_s \quad (27)$$

$$m_u Z_u = f_k - f_b - k_t (Z_u - Z_r) \quad (28)$$

Figure 3. Ideal Sky-hook Model



Since the mentioned configuration is far from reality, the following control policy is used so that the performances of the ideal sky – hook model address the real problem.

$$b_s = c_{max} \quad if \ Z_s (Z_s - Z_u) \geq 0 \quad (29)$$

$$b_s = c_{min} \quad if \ Z_s (Z_s - Z_u) \leq 0 \quad (30)$$

However, as it stated in Yokoyama [16], this strategy is not always useful since lack of feedback to compensate for the error between the desired damping coefficient and the actual one and also sometimes the damper is not able to produce the desired amount of the damping force.

Inserting (3), (4) into (5), (6) and change the state variable $Z_s, Z_s, Z_u, Z_u, Z_r, c_{sky}, Z_s$, to $x1, x2, x3, x4, d, u_a$ simple form of the nonlinear state space model is achieved.

$$x_1 = x_2 \quad 31$$

$$x_2 = -\frac{f_k}{m_s} - \frac{b_s}{m_s} - \frac{u_a}{m_s} \quad 32$$

$$x_3 = x_3 \quad 33$$

$$x_4 = \frac{f_k}{m_u} + \frac{b_s}{m_u} - \frac{k_r}{m_s}\ x_3 - d \quad 34$$

$$Z = 1 \quad 0 \quad 0 \quad 0 \ X \quad 35$$

Nonlinear suspension parameters of Ford Fiesta MK2 have been used through this paper [17]. All parameter values are arranged and shown in table 1.

Table 1. Model Parameters

m_s (kg)	m_u (kg)	k_s (N/m)	k_t (N/m)	b_s (N.sec/m)
216.75	28.85	21700	184000	1200
Sky-hook damping		Max damping		Min damping
1000N.sec/m		1900N.sec/m		500N.sec/m

J. Output Feedback PD Controller

The output feedback controller would be designed to stabilize the system. Using PD controller and considering the only one output (x1), the controller will be:

$$U = -k_1 x_1 - k_2 x_2 \quad (36)$$

And the state space equation is as follow:

$$x = A + B \ -k_1 \ -k_2 \quad 0 \quad 0 \quad x \quad (37)$$

The state feedback controller should be designed so that the inequality satisfies and hence the Re[λ *A* + *BK*] < 0 system is asymptotically stable. At last, poles are selected by trial and error to perform good results [2].

6. Simulation and Results

Assume that only one sensor which is the vehicle height sensor exist in simulation to measure vehicle position (x1), All states would be estimated and finally compare with true values using this measurement. The road profile is shown in Figure. 4.

Process and measurement noise must be considered when EKF or UKF is used to estimate state or parameter. Process noises are due to causes like wind, friction, temperature and etc. While the accuracy of sensor causes measurement noise. Process noises lead system to operate more accurate and make its behavior more real while measurement noise influence the exact feedback signal value and enter some error into calculations.

Covariance matrix *Q* and *R* which has been used in this study are as follows:

Q = *diag* (1,10000,1,10)

R = 0.0.1

State x1 value of passive suspension, semi – active suspension system and semi – active suspension system with output feedback PD controller are shown in Figure. 5.

Despite using one sensor to measure the car position, both nonlinear Kalman filters can estimate the other state well.

The estimation state x1 under road profile disturbance in semi-active suspension is shown in Figure. 6. In this figure EKF and UKF and PD controller without Kalman filter estimation have been shown to estimate the state x1 value.

Figure. 6 represents the estimation state x1 under. In this figure EKF and UKF estimation has been used to estimating the state x1 value.

7. Conclusion

In order to estimate the states of semi-active nonlinear suspension system, using the Kalman filters results in high accuracy under disturbance condition. It has been observed that by using the only one sensor to measure the position of vehicle, height sensor, all other states also estimated well. Simulation results show the performance of EKF and UKF are well demonstrated and these values are used in PD controller design. However, in the case of more complex systems, the UKF would be a better choice.

Figure 4. The Road Profile

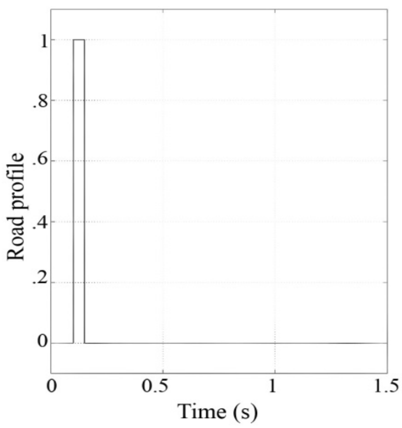


Figure 5. The State x1 Value of Passive, Semi-Active and Semi-Active with PD Controller.

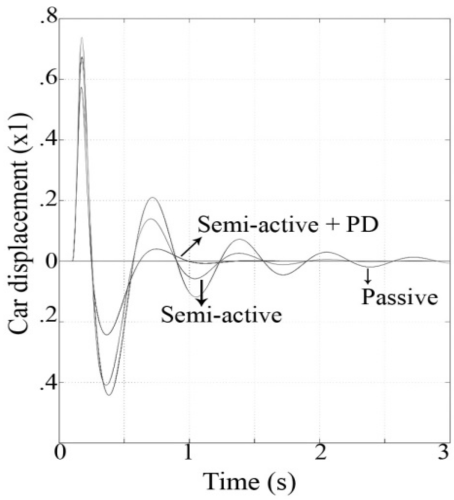
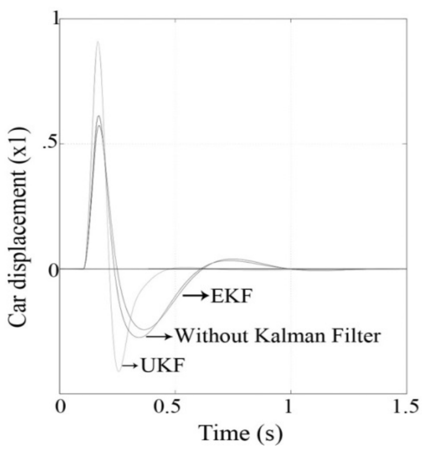


Figure 6. The State x1 Value of Semi-Active with PD Controller and EKF and UKF Estimation and Without Estimator.



References

[1] Akutain, X. C., Vinolas, J., Savall, J. & Castro, M. (2007). Comparing the performance and limitations of semi-active suspensions. *International Journal of Vehicle Systems Modelling and Testing*, 2(3), 296-314.

[2] Yim, S., Seok, J. & Lee, J. (2012). *State Estimation of the Nonlinear Suspension System Based on Nonlinear Kalman Filter*. 12th International Conference on Control, Automation and Systems. Korea.

[3] Karnopp, D. C., Crosby, M. J. & Harwood, R. A. (1974). Vibration control using semi-active force generators. *Transactions on ASME, Journal of Engineering for Industry*, 96(2), 619-626.

[4] Ahmadian, M. & Pare, C. A. (2000). A quarter-car experimental analysis of alternative semi-active control methods. *Journal of Intelligent Material Systems and Structures*, 11(8), 604–612.

[5] Hong, K. S., Sohn, H. C. & Hedrick, J. K. (2002). Modified skyhook control of semi-active suspension: A new model, gain scheduling and hardware-in-the loop tuning. *Transactions on ASME, Journal of Dynamic Systems, Measurement and Control*, 124(1), 158-167.

[6] Ahmadian, M., Song, X. & Southward, S. (2004). No-jerk skyhook control methods for semi-active suspensions. *Journal of Vibration and Acoustics*, 126(4), 580-584.

[7] Emura, J., Kakizaki, S., Yamaoka, F. & Nakamura, M. (1994). Development of the semi-active suspension system based on the Sky-hook damper theory. *Society* of *Automotive* Engineers, (pp. 17-26). Warrendaie, U.S.A.

[8] Savaresi, S. M., Silani, E. & Bittanti, S. (2005). Acceleration-driven-damper (add): an optimal control algorithm for comfort-oriented semi-active suspensions. Transactions on ASME, *Journal of Dynamic Systems Measurements and Control*, 127(2), 218-229.

[9] Alleyne, A. & Hedrick, J. K. (1993). Adaptive control for active suspension. *Advanced Automotive Technology*, 52, 7-13.

[10] Williams, R. A. (1993). *Automotive Active Suspensions Part 1: Basic Principles*. Proceedings of the *Institution* of *Mechanical* Engineers, 211(6), 415-426.

[11] Baker, R. C. & Charlie, B. (2011). *Improved Unscented Kalman Filter for Bounded State Estimation*. Electronics, Communications and Control (ICECC) (pp. 2101-2104).

[12] Thrun, S., Burgard, W. & Fox, D. (2005). *Probabilistic Robotics*. The MIT Press.

[13] Ando, Y. & Suzuki, M. (1996). Control of active suspension systems using the singular perturbation method. *Control Engineering Practice*, 4(3), 287-293.

[14] Jamei, M., Mahfouf, M. & Linkens, D. (2000). *A GA Tuned Fuzzy Controller for a Nonlinear Active Suspension System*. Proceedings of the 7th UK Workshop on Fuzzy Systems (2, 143-146).

[15] Wan, E. A. & Merwe, R. V. D. (2010). *The Unscented Kalman Filter for Nonlinear Estimation.* Adaptive Systems for Signal Processing, Communications, and Control Symposium, AS-SPCC (pp. 153-158).

[16] Yokoyama, M., Hedrick, J. K. & Toyama, S. (2001). *A Model following Sliding Mode Controller for Semi-Active Suspension Systems with MR Dampers*. Proceedings of American Control Conference (pp. 2652-2657).

[17] Asadsangabi, B., Eghtesad, M., Daneshmand, F. & Vahdati, N. (2006). Hybrid sliding mode control of semi-active suspension system. *Smart Materials and Structures Journal*, 18(12), 125027. DOI:10.1088/0964-1726/18/12/125027