Stock Index Return Forecasting and Trading Strategy Using Hybrid ARIMA-Neural Network Model

Manish Kumar* and **M. Thenmozhi****

Abstract

This study presents the application and development of hybrid methodology that combines both ARIMA and Artificial Neural Network model to take advantage of the unique strengths of both linear and non-linear modeling to model and predict the stock market index returns. The performance of the hybrid ARIMA Neural Network model is compared with the performance of ARIMA and Neural Network model. The performance of the models are evaluated in terms of widely used statistical metrics, correctness of sign and direction change and various trading performance measures like annualized return, Sharpe ratio, maximum drawdown, annualized volatility, average gain/loss ratio, etc. via a trading strategy. The findings of the study reveal that the hybrid ARIMA Neural Network model developed is the best Forecasting model to achieve greater accuracy and yields better trading results.

Keywords: ARIMA, Artificial Neural Network, Forecasting, Stock market trading

JEL Codes: C22, C45, C52, E17, G15

1. Introduction

In the last two decades, forecasting financial time series have been attempted using different linear and non-linear methods. The most popular and traditional time series model is Box-Jenkins or ARIMA model. The ARIMA approach is both simple and yields accurate results which explains its wide use. Many authors, e.g. Virtanen and Paavo (1987), Pagan and Schwert (1990), Leseps and Morell (1997), Crawford and Fratantoni (2003), etc. have used ARIMA model as proposed by Box-Jenkins to forecast different time series such as stock index returns, exchange rates, etc. and compared it with different models like Markov Switching, Regime Switching GARCH, etc. The results show that ARIMA model performed well compared to other models.

However, the major limitation of the ARIMA model is the pre-assumed linear form of the model. The approximation of linear models to complex real-world financial time series problem is not always satisfactory. Financial time series are considered as highly non-linear where the mean and variance of the series changes overtime. Grudnitski and Osburn (1993) in their study stated that there is noisy non-linear process present in the prices. Moreover, Refenes *et al*. (1994) in their study also indicated that traditional statistical techniques for forecasting have serious limitations with respect to applications with non-linearities in the data set such as stock indices. Hence, detecting this hidden non-linear relationship and the application of non-linear methods may help in improving the forecasting

*__Manish Kumar,__ Research Scholar, Department of Management Studies, Indian Institute of Technology, Madras, Chennai, India
**__M. Thenmozhi__ of Management Studies, Indian Institute of Technology, Madras, Chennai, India

accuracy. Recent developments in the theory of Neural Computation provide interesting mathematical tools for such a new kind of financial analysis. One of the popular and powerful tools in this area is Artificial Neural Network. The major advantage of neural networks is their flexible non-linear modeling capability (Donaldson and Kamstra, 1996). Neural networks have flexible non-linear function mapping capability, which can approximate any continuous function with arbitrarily desired accuracy. Due to their success in financial forecasting, neural networks have been adopted as an alternative method in the prediction of stock prices, exchange rates, etc.

A number of studies (Refenes *et al.* (1987), Kimoto *et al.* (1990), Takashi *et al*. (1990), Kryzanowski *et al.* (1993), McCluskey (1993)**,** Bansal and Vishwanatahn (1993), Refenes (1994), Donaldson and Kamstra (1996), Zirilli (1997)) have investigated Neural Network model for predicting the stock market and the results support the importance of the model. Castiglianc (2001) and Phua *et al.* (2003) have used Neural Network to forecast stock index increment. Yao *et al*. (2002) used Neural Network for forecasting option price; Jasic and Wood (2004) examined the daily stock market indices of S&P 500, DAX, TOPIX and FTSE for profitability of trades based on Neural Network prediction. Thus, many studies have shown that neural networks are better and can serve as a better prediction model that can overcome many of the drawbacks associated with the traditional techniques.

In the Indian context, Thenmozhi (2001) examined the feasibility of Neural Network in predicting the movement of the daily and weekly returns of BSE Index. The architecture used four inputs, which are the four consecutive daily returns and one output being the prediction of return on the fifth day. The study uses multiplayer perceptron with backpropagation algorithm. The results show that predictive powers of both the models (daily and weekly return) were low. Pant and Rao (2003) in their work used ANN for estimating the daily return of the BSE Sensex using randomized backpropagation. The study was based on the daily price time series of BSE Sensex. It used four different architectures of three-layer Neural Network that consist of three-input parameters and one output parameter. Results indicate that ANN based forecasting method is superior to the naïve strategy of holding the stocks. Manish and Thenmozhi (2004) used backpropagation neural networks and compared it with a linear ARIMA model for forecasting exchange rate like INR/USD and the Stock index return. Results indicate that ANN based forecasting method is superior to the linear ARIMA model.

The recent researches have focused on using hybrid model or combining various models of forecasting to improve the forecasting accuracy. The idea behind the model combination is to use the unique advantageous features of each model to accurately analyse different patterns in the data (Reid (1968) and Bates and Granger (1969)). The study of Newbold and Granger (1974), Makridakis *et al.* (1982), Makridakis (1989), Clemen, (1989), Palm and Zellner (1992) and Makridakis *et al.* (1993) suggests that by combining several different models, forecasting accuracy can often be improved. In addition, the combined model is more robust and flexible with regard to the possible structure change in the data.

There have been some studies suggesting hybrid models, combining the ARIMA model and neural networks. An important motivation to combine different forecasting is that one cannot identify the true process of the time series, i.e. the time series under examination is generated from a linear or non-linear underlying process. Moreover, the time series data often contain both linear and non-linear patterns. Hence, different models may be tried in approximating the underlying process. However, the final selected model is not necessarily the best for future uses due to many potential influencing factors such as sampling variation, model uncertainty, and structure change. Therefore, combining different models can increase the chance to capture different patterns in the data with increased accuracy and improve forecasting performance drastically. By combining different methods, the problem of model selection can be eased with little extra effort and this can serve as a universal model thus saving time and effort (Zhang, 2003).

Voort *et al.* (1996) used this combination to forecast short-term traffic flow. Their technique used a Kohonen self-organizing map as an initial classifier; with each class having an individually tuned ARIMA model associated with it. Su *et al.* (1997) used the hybrid model to forecast a time series of reliability data with growth trend. Their results showed that the hybrid model produced better forecasts than either the ARIMA model or the Neural Network by itself. Wedding and Cios (1996) described a combining methodology using radial basis function networks and the Box–Jenkins models. Luxhoj *et al.* (1996) presented a hybrid econometric and ANN approach for sales forecasting. Pelikan *et al.* (1992) and

Ginzburg and Horn (1994) proposed to combine several feed-forward neural networks to improve time series forecasting accuracy. Zhang (2003) used the hybrid methodology to forecast the three well-known data sets—the Wolf's sunspot data, the Canadian lynx data, and the British pound/US dollar exchange rate data. Experimental results with real data sets indicate that the combined model can be an effective way to improve forecasting accuracy achieved by either of the models used separately. Hence, there is strong evidence in the literature that hybrid models are more robust and are more accurate over the individual models.

Recently, Tugba and Casey (2005) using Zhang (2003) approach showed that the combined forecast can underperforms significantly compared to its constituents' performances. They demonstrated these using nine monthly time series data sets, Auto Regressive (AR) linear and Time Delay Neural Network models (TDNN). The last 12 values were reserved for testing, the preceding 12 values for validation, whilst the rest were used for training. For five of the nine data sets, the linear AR and TDNN models outperform the ARIMA Neural Network hybrids, albeit with similar levels of performance for two of these data sets. They concluded that despite the popularity of hybrid models, which rely upon the success of their components, single models themselves can be sufficient.

Although, different hybrid ARIMA-ANN model has been developed, in earlier studies related to hybrid models, auto, regressive terms have been used as input to the Neural Network. The residuals of ARIMA model has been modeled using Neural Network. Zhang (2003) in his study assumed that the non-linear patterns will always be present in the residuals of the linear ARIMA model, which can be modeled using artificial neural networks. Moreover, there is an assumption that the relationship between the linear and non-linear components is additive and this may degrade performance if the relationship is different, e.g. multiplicative. Such assumptions are likely to lead to unwanted degeneration of performance if the opposite situation occurs Tugba and Casey (2005). Clemen (1989) and Granger and Ramanathan (1984) in their study states that the lack of success using the combination models may be attributed to the performance of benchmark models. The performance of the benchmark models was so much weaker than that of the neural network models that it is unlikely that combining relatively 'poor' models with an otherwise 'good' one will outperform the 'good' model alone. Hence, the result of the recent study on the hybrid ARIMA-ANN model is mixed.

Moreover, the other key problems associated with these studies are as follows. These studies use simulated or artificial data set for the analysis and the number of observation for training and the test data were very low (Zhang (2003). The degree of accuracy and the acceptability of certain forecasting models are measured by the estimates' deviations from the observed values, i.e. MAE, RMSE, etc. but turning point forecast capability using sign and direction test has not been considered ((Zhang (2003), and Tugba and Casey (2005)). Leung *et al*. (2000) in their study suggested that depending on the trading strategies adopted by investors, forecasting methods based on minimizing forecast error may not be adequate to meet their objectives. In other words, trading driven by a certain forecast with a small forecast error may not be as profitable as trading guided by an accurate prediction of the direction or sign of return. Hence, the competing models must be evaluated not only in terms of MAE, RMSE etc., but also in terms of sign and direction test. The other drawback of the previous studies is that, none of the studies evaluated their models based on the trading performance. Statistical measures of performance are often inappropriate for financial applications. The forecast error may have been minimized during model estimation, but model with a small forecast error may not be as profitable as a model selected using financial criteria such as risk adjusted measure of return Leung *et al.* (2000) Evaluations of models using financial criteria through a trading experiment may be more appropriate.

Although, there are studies addressing the issue of forecasting financial time series such as stock market index most of the empirical findings are associated with the developed financial markets (UK, USA, and Japan). However, few studies exist in the literature which predicts the financial time series of emerging markets. Nowadays, many international investment bankers and brokerage firms have major stakes in overseas markets. Harvey (1995) found emerging market returns are more likely to be influenced by local information than developed markets; in fact, emerging market returns are generally more predictable than developed market returns. Indian stock markets have received relatively little attention until recently. Now there is more interest and research on Indian market data due to the country's rapid growth and potential opportunities for investors. Since the establishment of National Stock Exchange (NSE), the financial markets in this Asian country have attracted considerable global investments.

Given this notion, this study examines the applicability of hybrid ARIMA-neural network models for predicting the daily return of the S&P CNX NIFTY Index and compares it with isolated ARIMA and neural network model. The study differs from earlier studies in several ways. Firstly, the study develops the hybrid ARIMA-ANN models. In the first stage of this study, the ARIMA and an artificial neural network model is used to forecast the variable of interest. In second stage hybrid ARIMA-ANN models are developed. The hybrid ARIMA-ANN model is similar to the Zhang (2003). Secondly, the different competing models are rigorously compared using two approaches. Firstly, the study examines the out-of-sample forecasts generated by different competing models employing non penalty-based performance criteria such as Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE) and Mean Absolute Error (MAE) and performance criteria based on direction and sign change such as Directional Symmetry (DS), Correct Up trend (CU) and Correct Down trend (CD) goodness of Forecast Measures Thirdly, the different competing models are also examined in terms of trading performance and economic criteria *via* a trading experiment. For example, the study uses the return forecasts from the different models in a simple trading strategy (buy when the forecast is positive and sell when forecast is negative) and compare pay-offs to determine which model can serve as a useful forecasting tool.

Thus, the major contribution of this study will be (1) to find out the appropriate neural network and ARIMA model for NIFTY return series; (2) to find out the appropriate hybrid ARIMA-ANN for NIFTY return series; (3) to demonstrate and verify the predictability of S&P CNX NIFTY Index return by applying the hybrid ARIMA-neural network models; (4) to compare the performance of the hybrid model with that of individual ARIMA and neural network model in terms of forecasting accuracy using non penalty-based performance criteria such as Root Mean Square Error (RMSE), Normalized Mean Square Error (NMSE) and Mean Absolute Error (MAE) and performance criteria based on direction and sign change such as Directional Symmetry (DS), Correct Up Trend (CU) and Correct Down trend (CD); (5) to evaluate the three models in terms of trading performance via a trading experiment.

The remaining portion of this paper is organized as follows. The data used in the study, the details of hybrid approach and the benchmark models are introduced in Section 2. The empirical results from the real data sets are discussed in Section 3. Finally, Section 4 contains the concluding remarks.

2. Data and Methodology

The study is based on the daily closing prices for the S&P CNX NIFTY Index. The series span the period from 1st January 2000 to 31st March 2005 totaling a 1,319 trading days. The data is divided into two periods- the first period runs from 1st January, 2000 to 26th December, 2003 (1,000 observations) used for model estimation and is classified as in-sample, while the second period runs from 27th December, 2003 to 31st March, 2005 (319 observations) is reserved for out-of-sample forecasting and evaluation. The division amounts to approximately 25 per cent being retained for out-of-sample purposes.

The use of data in levels in the stock market has many problems: stock market price movements are generally non-stationary and quite random in nature, and therefore not very suitable for learning purposes. To overcome these problems, the NIFTY series is transformed into rates of return. Given the price level $P_1, P_2, \ldots, P_t$, the rate of return at time t is formed by: $R_t = (P_t/P_{t-1}) - 1$. An advantage of using a returns series is that it helps in making the time series stationary, a useful statistical property.

2.1 Forecasting Methodology

The premise of this research is to examine the use of hybrid models in NIFTY returns forecasting and trading models. Their performance is compared with univariate linear ARIMA model and a non-linear backpropagation neural network. As all of these methods are well-documented in the literature, an outline of the methods is given below.

2.1.1 ARIMA Methodology

Popularly known as Box-Jenkins (BJ) methodology, but technically known as ARIMA methodology, assumes that the future values of a time series have a clear and definite functional relationship with current, past values and white noise. The mixed auto regressive model of order (p,q) denoted as ARMA (p,q) is defined as

$$Z_t = \theta + \Phi_1 Z_{t-1} + \Phi_2 Z_{t-2} + \Phi_p Z_{t-p} + \Phi_0 a_t + \Phi_1 a_{t-1} + \Phi_2 a_{t-2} + \Phi_q a_{t-q}$$

Where Z_t is the time series and a_t is an uncorrelated random error term with zero mean and constant variance and θ represents a constant term.

The time series models are based on the assumption that the time series involved are stationary. But many a time series are not stationary, that is they are integrated. If a time series are integrated of order 1 (i.e., it is I (1)), and its first difference is I (0), it is said to be stationary. Similarly, if a time series is I (2), its second difference is I (0). In general, if a time series is I (d) after differencing it d times. Then I (0) series will be obtained.

If a time series is differenced d times to make it stationary and then ARMA (p, q) model is applied to it, then the original time series is ARIMA (p, d, q), that is an autoregressive integrated moving average time series.

The Box-Jenkins models are implemented using E-Views 4. The correlogram, which are simply the plots of Autocorrelation Functions (ACFs) and Partial Autocorrelation Functions (PACFs) against the lag length, is used in identifying the significant ACFs and PACFs. The lags of ACF and PACF whose probability value is less than 5% are significant and are identified. The possible models are developed from these plots for the NIFTY Index returns series. The best model for forecasting is identified by considering the information criteria, i.e. Akaike Information Criteria (AIC) and Schwarz Bayesian Information Criteria (SBIC). It is also an accepted statistical paradigm that the correctly specified model for the historical data will also be the optimal model for forecasting. Hence, it is reasonable to compare the hybrid model and the best neural network results with those of ARIMA models.

2.1.2 Neural Network Methodology

In this study, one of the widely used ANN models, the feed forward neural network is used for financial time series forecasting. Usually, the NN model consists of an input layer, an output layer and one or more hidden layers. The hidden layers can capture the non-linear relationship between variables. Each layer consists of multiple neurons that are connected to neurons in adjacent layers.

A neural network can be trained by the historical data of a time series in order to capture the non-linear characteristics of the specific time series. The model parameters (connection weights and node biases) will be adjusted iteratively by a process of minimizing the forecasting errors. For time series forecasting, the final computational form of the ANN model is as

$$Y_t = ao + \ Y_t = ao + \sum_{j=1}^{q} w_j f(a_j + \sum_{i=1}^{p} w_{ij} Y_{t-i}) + \varepsilon_t$$

where $a_j (j = 0,1,2,\dots q)$ is a bias on the j^{th} unit, and $w_{ij} (i = 1,2,\dots, p; j = 1,2,\dots q)$ is the connection weight between layers of the model, $f(.)$ is the transfer function of the hidden layer, p is the number of input nodes and q is the number of hidden nodes. Actually, the ANN model in (2) performs a non-linear functional mapping from the past observation (Y_{t-1}, Y_{t-2},…, Y_{t-p}) to the future value Y_t, i.e.,

$$Y_t = \varphi(Y_{t-1}, Y_{t-2}, Y_{t-3}, \dots, Y_{t-p}, v) + \xi_t$$

where v is a vector of all parameters and φ is a function determined by the network structure and connection weights. Thus, in some senses, the ANN model is equivalent to a Nonlinear Auto Regressive (NAR) model.

2.1.3 Model Formulation

This study employs a three-layer backpropagated neural network to forecast NIFTY Index returns. The return series of NIFTY Index are fed to the neural network model to forecast the next period return in this model. For example, the inputs to a 5–x–1 neural network are NX_{i-4}, NX_{i-3}, NX_{i-2}, NX_{i-1} and NX_i while the output of the neural network is NX_{i+1}, the next day's NIFTY return, where NX_i stands for the current day's NIFTY return. The architecture of the neural network is denoted by *X-Y-Z*. The *X-Y-Z* stands for a neural network with *X* neurons in input layer, *Y* neurons in hidden layer, and *Z* neurons in output layer. Only one output node is deployed in the output layer since one-step-ahead forecast is made in this study. The number of input nodes and hidden nodes are not specified *a priori*. This will be selected through experiment. This study uses tansigmoid function for the nodes in the input layer for backpropagated neural network, while tansigmoid function and pure linear function are used at hidden layers and output layers.

The number of input nodes is probably the most critical decision variable for a time series-forecasting problem since it contains important information about the data. In this study, the number of input nodes corresponds to the number of lagged returns observations used to discover the underlying pattern in a time series and to make forecasts for future values. Currently, there is no theory suggesting the appropriate number of input nodes. But ideally it would be better to have a small number of essential nodes, since this can unveil the unique features embedded in the data. Too few or too many input nodes can affect either the learning or prediction capability of the network. This study resorts to experimentation in the network construction process. The network construction process has been evaluated with six levels of the number of input nodes ranging from 1 to 6.

The number of hidden nodes plays a very important role too. These hidden neurons enable the network to detect the feature, to capture the pattern in the data, and to perform complicated non-linear mapping between input and output variables. Hornik *et al.* (1989) in their theoretical work found that single hidden layer is sufficient for the network to approximate any complex non-linear function with any desired accuracy. Most authors use only one hidden layer for forecasting purposes. This study employs three-layer BPN to forecast the daily returns of NIFTY returns. Five levels of hidden nodes, 1, 2, 3, 4 and 5 have been experimented. The combination of six input nodes and five hidden nodes yields a total of 30 different neural network architectures. These in turn are being considered for each in-sample training set for the NIFTY returns the backpropagation neural network models.

This study uses backpropagation algorithm to train the BPN. Backpropagation is the most widely used algorithm for supervised learning with neural networks. The study uses MATLAB 6.5 to build and train neural network. The MATLAB program works with default parameter values of weight, assigned by the MATLAB.

2.1.4 The Hybrid Methodology

This study develops the hybrid models to forecast the S&P CNX NIFTY Index return. The forecasting method using hybrid models initiates with the basic time series data on NIFTY Index return. It may be reasonable to consider a time series to be composed of a linear autocorrelation structure and a non-linear component. A hybrid model comprising a linear and a non-linear component has been employed in the experiments (Zhang, 2003): It is represented as

$$Y_t = L_t + N_t$$

where L_t denotes the linear component and N_t denotes the non-linear component. These two components have to be estimated from the data. These data then enter the first stage of the ARIMA to account for a linear component; hence the residuals from the linear model will contain only the non-linear relationship. Let e_t denote the residual components at time t from the linear model, then

$$e_t = Y_t - \hat{L}_t$$

where $\hat{L}_t$ is the forecast value for time *t*. Any significant non-linear pattern in the residuals will indicate the limitation of the ARIMA. By modeling residuals using ANNs, non-linear relationships can be discovered. With n input nodes, the ANN model for the residuals will be

$$e_t = f(e_{t-1}, e_{t-2,}\dots, e_{t-n}) + \varepsilon_t$$

where f is a non-linear function determined by the neural network and ε_t is the random error. Denote the forecast from ANN as $\widehat{N}_t$, the combined forecast will be

$$\hat{Y}_t = \hat{L}_t + \widehat{N}_t$$

The proposed methodology of the hybrid system by Zhang (2003) consists of two stages. In the first stage, an ARIMA model is fitted to the time series data to capture the linear part of the problem. In the second stage, an appropriate neural network model is developed to forecast the residuals from the ARIMA model. The hybrid model exploits the unique feature and strength of ARIMA model as well as ANN model in determining different patterns. So, the above hybrid ARIMA neural network model uses the following: (a) forecast residuals $\widehat{N}_t$ (results of ARIMA model) of neural network and (b) the forecast $\hat{L}_t$ (results of ARIMA model).

The optimal architecture of hybrid model that captures the non-linear patterns of residuals of ARIMA model is formed in the same way as discussed in the model formulation of neural network methodology.

2.2 Forecasting Accuracy and Trading Simulation

To compare the performance of the models, it is necessary to evaluate them on previously unseen data. This situation is likely to be the closest to a true forecasting or trading situation. To achieve this, all models were maintained with an identical out-of-sample period allowing a direct comparison of their forecasting accuracy and trading performance.

2.2.1 Out-of-Sample Forecasting Accuracy Measures

This study uses six widely used statistical metrics such as Mean Absolute Percentage Error (MAPE), Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Directional Symmetry (DS), Correct Up trend (CU) and Correct Down trend (CD) to evaluate the forecasting capabilities between the three models. RMSE, MAPE and MAE measure the deviation between actual and forecast value. The smaller the values of MAE, MAPE and RMSE, the closer are the predicted time series values to that of the actual value. It is observed that RMSE, MAE or MAPE functions that are used for financial forecasting models may not make sense in the financial context. Caldwell (1992) gives a general review for the performance metrics. Yao *et al.* (1996) use the correctness of the trend to judge the performance of neural network forecasting model. So this study uses additional evaluation measures, which includes the calculation of correct matching number of the actual and predicted values with respect to sign and directional change. DS measures correctness in predicted directions while CU and CD measure the correctness of predicted up and down trend, respectively, in terms of percentage. Higher value of these metrics indicates better direction and time information. The statistical performance measures used to analyze the forecasting techniques are presented in Appendix 1.

This study also uses other measures to test the model's ability to predict turning points. A correct turning point forecast requires that:

$$\text{Sign}\,(\hat{Y}_t - \hat{Y}_{t-t}) = \text{Sign}\,(Y_t - Y_{t-1})$$

Where Y_t and $\hat{Y}_t$ represents the actual and predicted value at time t.

The ability of a model to forecast turning points can be measured by a fourth evaluation method developed by Cumby and Modest (1987). This model defines a forecast direction variable F_t and an actual direction variable A_t such that

$$A_t = 1 \text{ if } \Delta Y_t > 0 \text{ and } A_t = 0 \text{ if } \Delta Y_t \le 0$$

$$F_t = 1 \text{ if } \Delta\hat{Y}_t > 0 \text{ and } F_t = 0 \text{ if } \Delta\hat{Y}_t \le 0$$

Where ΔY_t is the amount of change in actual variables between time t–1 and t; and $\Delta\hat{Y}_t$ is the amount of change in forecasting variables between time t–1 and t.

Cumby and Modest (1987) suggest the following regression equation:

$$F_t = \alpha_0 + \alpha_1 A_1 + \varepsilon_t$$

where ε_t is error term; and α_1 is the slope of this linear equation. Here, α_1 should be positive and significantly different from 0 in order to demonstrate those F_t and A_t have a linear relationship. This reflects the ability of a forecasting model to capture the turning points of a time series.

2.2.2 Out-of-Sample Trading Performance Measures

Statistical performance measures are often inappropriate for financial applications. Typically, modeling techniques are optimized using a mathematical criterion, but ultimately the results are analyzed on a financial criterion upon which it is not optimized. In other words, the forecast error may have been minimized during model estimation, but the evaluation of the true merit should be based on the performance of a trading strategy.

Hence, this study uses a simple trading strategy to evaluate the performance of different models. The operational detail of the trading is as follows. This study considered an index in place of a single stock to avoid (or average out) the impact of company-specific news on the prediction of only one stock, given that the prediction is performed by taking into account past prices only. In the simulated market set up for experimenting the proposed methodology, a virtual trader can buy or sell stock index fund on the stock index concerned, and both short and long positions can be taken over the index.

Assume that a certain amount of seed money is used in this trading experiment. The seed money is used to buy stock index funds when the prediction shows a rise in the stock index price. To calculate the profit, the stock index funds are bought or sold at the same time. It should be noted that the price of the stock index fund is directly proportional to the index level so that the virtual investor can gain from both a fall and rise of the stock index price. The trading strategy is to go long when the model predicts that the stock index price will rise, i.e. the forecast is positive and a sell otherwise. Then the stock index funds will be held at hand until the next turning point that the model predicts.

For many traders and analysts, market direction is more important than the value of the forecast itself, as in financial markets money can be made simply by knowing the direction the series will move. The trading performance measures used to analyze the forecasting techniques are presented in Appendix 2. Some of the more important measures include the Annualized return, Annualized volatility, Sharpe ratio, maximum drawdown and average gain/loss ratio. The Sharpe ratio is a risk-adjusted measure of return, with higher ratios preferred to those that are lower; the maximum drawdown is a measure of downside risk and the average gain/loss ratio is a measure of overall gain, for which a value above one is preferred.

The application of these measures may be a better standard for determining the quality of the forecasts. After all, the financial gain from a given strategy depends on trading performance, not on forecast accuracy.

3. Results

3.1 Summary Statistics

The mean, median, standard deviation, skewness, and kurtosis for the NIFTY Index return are given in Table 1. The analysis shows that the sample mean of daily returns of NIFTY returns is not statistically different from zero. The measure of skewness and kurtosis indicates that the distributions of the return series are different from the standard normal distributions. They reveal a slight skewness and high kurtosis, which 'is common in financial time series data'

Table 1 **Descriptive Statistics of NIFTY Returns**

Mean	*Median*	*Std. Dev*	*Skewness*	*Kurtosis*	*Observations*
.000306	.001202	.015392	–.621756	8.409610	1319

3.2 Stationarity Test

The Augmented Dickey Fuller test and Philip Perrons test statistics as given in Table 2 indicate that the rate of return of the NIFTY Index is stationary as the absolute value of statistics is greater than the critical value and thus, the time series is suitable for modeling.

Table 2 **Unit Root Test for the NIFTY Return Series**

Augmented Dickey Fuller Test		*Phillip Perron Test*	
Statistic	*Critical Value*	*Statistic*	*Critical Value*
–15.64312	–3.4382	–32.62406	–3.4382

3.3 ARIMA Model

The correlogram is used to identify the number of significant spikes of ACF or PACF of the NIFTY Index return series. The lags of ACF and PACF whose probability value is less than 5% are significant and are identified. Several ARMA specifications were tried out. After considering all possible models and looking at AIC and SBIC as given in Appendix 3**,** the ARIMA (1 1 2) model are identified for NIFTY return.

In order to verify the adequacy of ARIMA model, the study uses one of the popular diagnostics test known as Breusch-Godfrey LM Test. Here the test is used to check the presence of serial correlation in the residuals. It allows us to examine the relationship between residuals and several of its lagged values at the same time. The Null Hypotheses to be tested is "there is no serial correlation". If the predictability value is greater than 5% then we can accept the Null Hypotheses (at 95% confidence levels) which means there is no serial correlation in the series. The Breusch-Godfrey LM Test for serial correlation of residuals as shown in Table 3 suggests that, in case of NIFTY return the ARIMA model captures the entire serial correlation and the residual do not exhibit any serial correlation.

Table 3 **Breusch-Godfrey Serial Correlation LM Test for the NIFTY return**

F-Statistics	*Probability*	*Obs*R Square*	*Probability*
1.004433	0.366624	2.016921	0.364780

3.4 Neural Network Model

The combination of six input nodes and five hidden nodes yields a total of 30 different neural network architecture which are being considered for each in-sample training set for NIFTY return and the residuals of ARIMA model. The best network architecture thus obtained from this experiment for NIFTY return and residuals of ARIMA model on the basis of least error (MSE) associated with the model is 3-2-1, i.e. three input nodes in input layer, two nodes in hidden layer and one node in output layer. The Neural network model 3 – 2 – 1 provides better fit to the NIFTY returns and ARIMA residuals series.

3.5 Hybrid Model

One hidden layer is used to develop the hybrid models. The study experimented with different nodes or neurons in hidden layer, which varies from one to five for the different hybrid models. The output layer has one neuron or node, which is the forecast value. The study uses MSE to select the architecture. The hybrid model which uses forecast results of ARIMA and the forecast residual (results of ARIMA) of neural network has three hidden neurons in hidden layer and three input neurons in input layer.

3.6 Forecast Evaluation

Out-of-Sample Forecasting Accuracy Results

For the NIFTY return series one-period-ahead forecast were produced by the three models namely hybrid modes, ARIMA and neural networks. The predictive performance of the three models is summarized in Table 4.

it is observed that hybrid model outperforms the other two models. MAE and RMSE achieved by the hybrid model is quite low indicating that there is a smaller deviation between the actual and predicted values in hybrid model.

Between neural network and ARIMA models, the former performs better in terms of the three most commonly used criteria i.e. MAE, RMSE and NMSE. The results of the hybrid model show that by combining two models together, the overall forecasting errors can be reduced considerably.

In terms of other performance metrics like correct up (CU) and correct down (CD), hybrid models yields better performance than the other models. It is really the directional symmetry (DS) measure that singles out the neural network model as the 'best' performer, predicting most accurately 52.38 per cent of the time. These three criteria provide a good measure of the consistency in prediction of the time series direction.

Between hybrid model, neural network and ARIMA models, the latter performs worst almost all of the times in terms of performance metrics like direction sign and change and non penalty based measure like MAE, NMSE and RMSE. A majority decision rule would therefore select the hybrid model as the overall 'best' model.

Turning Point Evaluation

Table 4 **Out-of-Sample Prediction Accuracy**

Model	*Performance Metrics*					
	MAE	*RMSE*	*NMSE*	*DS*	*CU*	*CD*
Hybrid Model	0.011063	0.015916	0.937088	0.514285	0.540541	0.481928
Neural Network	0.011107	0.016153	0.965219	0.523809	0.527027	0.481928
ARIMA	0.011125	0.016275	0.979853	0.425396	0.405405	0.439759

The main purpose of any financial time series modeling is to determine how well forecasts from estimated models perform based on the non penalty based measure of performance such as MAE, RMSE and NMSE. The forecasting accuracy statistics provide very conclusive results. A glance at these values shows the superiority of hybrid model over the two other models. Comparing the forecasting performance of the three models in terms of MAE, NMSE and RMSE for the NIFTY return time series;

The turning point evaluation method using Cumby and Modest (1987) regression equation is shown in Table 5 for all the models.

The *t* ratio of the slope coefficient α_1 of all the models shows that it is statistically different from zero for the NIFTY Index return time series. This implies that all models have good turning point forecasting ability.

Table 5 **Results of turning point forecasting capability of the five models**

Method	ARIMABP	NN	ARIMA
α_0 (t-ratio)	.242 (9.862)	.224 (9.071)	.342 (15.49)
α_1 (t-ratio)	.474 (13.85)	.513 (14.95)	.291 (9.34)

Null hypothesis of the α_0 or α_1 existence of the is equal to zero. The *t* ratio of the slope coefficient α_1 is statistically different from zero. This implies that for the out-of-sample period, the model had turning point forecasting power.

Out-of-Sample Trading Performance Results

The performance of the hybrid model is encouraging. However, predictability does not necessarily imply profitability. Hence, the model that can assure profitability on using particular type of strategy needs to be identified. It has to be known if the investor can make profit, if so how much and on what risk. In order to evaluate the performance of the three models 'trend following' strategies was used via a trading simulation. The trading simulation allows virtual investors to trade on the Nifty Index using Index funds. The investors choose to buy or sell each day based on the next day's return of the three models. The procedure to create the buy and sell signals is quite simple: a buy signal is produced if the forecast return is positive and a sell otherwise. If a buy signal is created, then the investor will invest the amount proportional to the index level. A comparison of the trading performance results of the three models is presented in Table 6.

Table 6 **Trading Performance Results**

Trading Measure of Performance	Hybrid Model	Neural Network	ARIMA
Annualized Return	81.40%	55.65%	33.00%
Cumulative Profit	100.56%	68.75%	40.77%
Annualized Volatility	25.80%	26.07%	26.22%
Sharpe Ratio	3.15	2.13	1.25
Maximum daily profit	12.24%	12.23%	12.23%
Maximum daily loss	–03.94%	–04.89%	–08.29%
Maximum drawdown	–14.96%	–17.13%	–20.53%
% winning trades	50.24%	55.44%	52.45%
% losing trades	49.77%	44.55%	47.54%
Number of up periods	175	163	167
Number of down periods	140	152	148
Number of transactions	213	193	183
Total trading days	315	315	315
Avg gain in up periods	1.29%	1.29%	1.18%
Avg loss in down periods	–.90%	–.94%	–1.06%
Avg gain/loss ratio	1.43	1.38	1.11
Profits T-statistics	55.99	37.88	22.33
Number of periods daily returns rise	169	169	169
Number of periods daily returns fall	146	146	146
Number of winning up periods	95	90	100
Number of winning down periods	79	72	66
% winning up periods	59.00%	53.25%	59.17%
% winning down periods	51.30%	49.31%	45.20%

The results of the hybrid model are quite impressive. It generally outperforms the benchmark models, i.e. neural network and ARIMA models in terms of overall profitability with annualized return of 81.40 per cent, cumulative return of 100.56 per cent, annualized volatility of 25.80 per cent and in terms of risk-adjusted performance with a Sharpe ratio of 3.15. ARIMA model perform worst both in terms of overall profitability with annualized return of 33.00 per cent, and in terms of risk-adjusted performance with a Sharpe ratio of 1.25. The hybrid model has the lowest downside risk as measured by maximum drawdown at –14.96 per cent, while ARIMA model has the highest downside risk at –20.53 per cent.

The hybrid model predicted the highest number of winning down periods at 79. The ARIMA model forecast the highest number of winning up periods at 100; however the hybrid model was 'second best' for this measure. Interestingly, all models were more successful at forecasting a rise in the NIFTY returns series, as indicated by a greater percentage of winning up periods to winning down periods. The ARIMA model predicted correctly a rise in the NIFTY returns series with 59.17 per cent, the hybrid model was the second best for this measure. The hybrid model successfully forecasted a fall in the NIFTY return series with 51.30 per cent, the neural network model was the second best for this measure.

The hybrid model has the highest number of transactions at 213, while the ARIMA model has the lowest at 183. In addition, the hybrid model has the highest average gain/loss ratio at 1.43, highest maximum daily profit at 12.24 per cent and lowest maximum daily loss at 3.94 per cent, while the ARIMA model has the lowest average gain/loss at 1.11 and highest maximum daily loss at 8.29 per cent. A simple neural network model outperforms the hybrid model and ARIMA models in terms of percentage of winning and per cent of losing trades with a value of 55.44 per cent and 44.55 per cent respectively. As with statistical performance measures, financial criteria clearly single out the hybrid model as the one with the most consistent performance: it is therefore considered the 'best' model for this particular application.

Zhang (2003) in their study found that hybrid ARIMA-NN model outperform the individual neural network and ARIMA model. However, the studies uses only non penalty based criteria (MAE, RMSE etc) to evaluate the forecast model. The turning point forecast capability test has not been considered. Moreover, these studies does not evaluated their models based on the performance of trading. The present study generally supports the finding of the Zhang (2003) and contradicts the findings of Tugba and Casey (2005). The results validate the findings with real financial time series data and also using by evaluating the performance of models using a trading strategy.

4. Conclusion

This study reports an empirical work which investigates the usefulness of hybrid (ARIMA and neural network) model in forecasting and trading the S&P CNX NIFTY Index return. The linear ARIMA model and the non-linear ANN model are used in combination, aiming to capture different forms of relationship in the time series data. The performance of the hybrid model was measured statistically and financially via a trading simulation. The logic behind the trading simulation is that, if profit from a trading simulation is compared solely on the basis of statistical measures, the optimum model from a financial perspective would rarely be chosen. The hybrid model was benchmarked against traditional forecasting techniques such as ARIMA and non-linear technique like neural network to determine any added value to the forecasting process.

The empirical results with the NIFTY returns clearly suggest that the hybrid model is able to outperform each component model used in isolation. A neural network architecture of 3-2-1 and ARIMA (1 1 2) is the best identified model for forecasting the returns of NIFTY Index. With the prediction, significant profits were obtained for a chosen testing period.

The results show that useful prediction could be made for NIFTY without the use of extensive market data or knowledge. It also shows how an 81.40 per cent annual return and a Sharpe ratio of 3.15 could be achieved by using the hybrid model. The present results indicate that the hybrid ARIMA neural network models is important in out-of-sample forecasting and trading performance, and are in line with Wedding and Cios (1996) and Zhang (2003) who found that hybrid model works well and found to outperform the isolated models. The results are in contrary with the results of Tugba and Casey (2005).

Thus, the study shows that hybrid ARIMA-neural network model outperforms in forecasting stock index returns both in terms of forecasting accuracy and in generating trading returns. Probably this type of hybrid model could be used by policy makers in forecasting financial and economic data, apart from traders, borrowers, FIIs and arbitrageurs developing trading models that leads to better investment decision and returns.

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Appendix 1

1. NMSE $= \dfrac{\sum (Y_t - \hat{Y}_t)^2}{\sum (Y_t - \bar{Y}_t)^2}$

2. MAE $= \dfrac{\sum |Y_t - \hat{Y}_t|}{N}$

3. RMSE $= \sqrt{\dfrac{\Sigma (Y_t - \hat{Y}_t)^2}{N}}$

4. DS $= \dfrac{100}{N} \sum_t d_t$, $d_t = \begin{cases} 1 \text{ if } (Y_t - Y_{t-1})(\hat{Y}_t - \hat{Y}_{t-1}) \\ 0 \text{ Otherwise} \end{cases}$

5. CU $= \dfrac{\sum_t d_t}{\sum_t k_t}$

$$d_t = \begin{cases} 1 \text{ If } (\hat{Y}_t - \hat{Y}_{t-1}) > 0; (Y_t - Y_{t-1})(\hat{Y}_t - \hat{Y}_{t-1}) \geq 0 \\ 0 \text{ Otherwise} \end{cases},$$

$$k_t = \begin{cases} 1 \text{ If } (Y_t - Y_{t-1}) > 0 \\ 0 \text{ Otherwise} \end{cases}$$

6. CD $= \dfrac{\sum_t d_t}{\sum_t k_t}$

$$d_t = \begin{cases} 1 \text{ If } (\hat{Y}_t - \hat{Y}_{t-1}) < 0; (Y_t - Y_{t-1})(\hat{Y}_t - \hat{Y}_{t-1}) \geq 0 \\ 0 \text{ Otherwise} \end{cases},$$

$$k_t = \begin{cases} 1 \text{ If } (Y_t - Y_{t-1}) < 0 \\ 0 \text{ Otherwise} \end{cases}$$

Where Y_t and $\hat{Y}_t$ represents the actual and predicted value at time t and N is the number of predictions.

Appendix 2

1. Annualized return : $R^A = 255 \times \dfrac{1}{N} \sum_{t=1}^{N} R_t$

2. Cumulative return : $R^c = \dfrac{1}{N} \sum_{t=1}^{N} R_t$

3. Annualised volatility : $\sigma^A = \sqrt{255} \times \sqrt{\dfrac{1}{N-1} \sum_{t=1}^{N} (R_t - \bar{R})^2}$

4. Sharpe ratio : R^A/σ^A

5. Maximum daily profit : Maximum value of R_t over the period

6. Maximum daily loss : Minimum value of R_t over the period

7. Maximum drawdown : Maximum negative value of $\Sigma(R_t)$ over the period

$$MD = \min_{t=1\ldots N} (R_t^c - \max_{i=1\ldots t}(R_t^c))$$

8. % Winning trades : $WT = 100 \times \frac{\sum_{t=1}^{N} F_t}{NT}$

   where $F_t = 1$ transaction profit$_t > 0$

9. % Losing trades : $LT = 100 \times \frac{\sum_{t=1}^{N} G_t}{NT}$

   where $G_t = 1$ transaction profit$_t < 0$

10. Number of up periods : Nup = number of $R_t > 0$
11. Number of down periods : Ndown = number of $R_t < 0$
12. Number of transaction : $NT = \sum_{t=1}^{N} L_t$

    where $L_t = 1$ if trading signal$_t$ = trading signal$_{t-1}$
13. Total trading days : Number of R_t's
14. Avg. gain in up periods : AG = (sum of all $R_t > 0$)/Nup
15. Avg. loss in down periods : AL = (sum of all $R_t < 0$)/Ndown
16. Profit T-statistics : T-statistics = $\sqrt{N} \times \frac{R^A}{\sigma^A}$
17. Number of periods daily returns rise : $NPR = \sum_{t=1}^{N} Q_t$

    where $Q_t = 1$ if $Y_t > 0$ else $Q_t = 0$
18. Number of periods daily returns falls : $NPF = \sum_{t=1}^{N} S_t$

    where $S_t = 1$ if $Y_t < 0$ else $S_t = 0$
19. Number of winning up periods : $NWU = \sum_{t=1}^{N} B_t$

    where $B_t = 1$ if $R_t > 0$ and $Y_t > 0$ else $B_t = 0$
20. Number of winning down periods: $NWD = \sum_{t=1}^{N} E_t$

    where $E_t = 1$ if $R_t > 0$ and $Y_t < 0$ else $E_t = 0$
21. Winning up periods (%) : WUP = 100*(NWU/NPR)
22. Winning down periods (%) : WDP = 100 * (NWD/NPF)

Appendix 3

Model	*AIC*	*SBIC*
ARIMA (1 1 1)	–5.567444	–5.552709
ARIMA (2 1 1)	–5.571542	–5.551880
ARIMA (1 1 2)	–5.572746	–5.553099
ARIMA (2 1 2)	–5.571903	–5.547325
ARIMA (1 1 3)	–5.570930	–5.546371
ARIMA (2 1 3)	–5.571832	–5.552339

** ARIMA (1 1 2) has the lowest AIC and SBIC value